Scalable Rational Secret Sharing
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Starting Example
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Setting

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- The shares are distributed among the players.
- Each share on its own should not reveal the secret.
- Combining all the shares reconstructs the secret.
- The players are selfish and rational.
- Each player prefers to
  1. learn the secret by him self
  2. learn the secret together with other
  3. not learn the secret at all.
Starting Example
Secure Secret Sharing

A secure secret sharing scheme distributes shares so that anyone with fewer than \( n \) shares has no additional information about the secret than someone with no shares at all.
Classical secure secret sharing Scheme

**Shamir’s Scheme**: A polynomial of degree $n - 1$ can be reconstructed using $n$ points.
Scalable Rational Secret Sharing

| Introduction |
| Setting |

Classical secure secret sharing Scheme

Shamir’s Scheme: A polynomial of degree $n - 1$ can be reconstructed using $n$ points.

- Encode the secret as the first coefficient of a random polynomial $f$ of degree $n - 1$
- The shares are points $(i, f(i))$ on the polynomial
- Any $n$ players can reconstruct the polynomial using interpolation
The Problem with Selfish Players

A selfish player will never send his share to the other players, as he has to fear, that he will not get the other players shares back.
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⇒ We need to adapt the protocol.
Solution
Our Goals

- If our group of $n$ agents follow the protocol they will all learn the secret.
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▶ If our group of \( n \) agents follow the protocol they will all learn the secret.
▶ No player can improve substantially by deviating from the protocol.
▶ The protocol is scalable:
  ▶ message complexity per agent: \( O(1) \)
  ▶ time complexity: \( O(\log |agents|) \)
Outline

- **Dealer’s Protocol**
  - is only active at the beginning
  - prepares the input for the player’s protocol

- **Player’s Protocol**
  - is played in rounds
  - in round $X$ the secret is revealed
Dealer’s Protocols
Dealer’s Protocol

- assign all players to leaves
Dealer’s Protocol

- assign all except for one player to remaining nodes
- give each player his share list $S_i$ and a list of potential secrets $L$
Player’s Protocols
Player’s Protocol – Up-Stage

- children send their shares to parent
Player’s Protocol – Up-Stage

- verify $S_1[t]$
- verify $S_2[t]$
- construct new share $\nu$
Player’s Protocol – Up-Stage

- children send their shares to parent
Player’s Protocol – Up-Stage

- verify \( v \)
- verify \( S_3[t] \)
- construct \( h_t \)
Player’s Protocol – Down-Stage

- send $h_t$ to children
Player’s Protocol – Down-Stage

- verify $h_t$
- send $h_t$ to children
Player’s Protocol

- verify $h_t$
- all players know $h_t$
- if $h_t = 0$ then $t = X$
Analysis
There are only two ways in which a player can deviate from the protocol:

1. Send fake messages
2. Leave the protocol before the secret was revealed
Verification by Tag Values

- Dealer prepares
  - $T_i^w$: a tag list for the sending node $w$
  - $H_j^{w',w}$: a list of verification tokens for the receiving node $w'$
- Node $w$ sends a tag $\bar{g}$ from $T_i^w$ along with its share $v$.
- Node $w'$ asserts

$$c = a \cdot v + b \cdot \bar{g}$$

where $(a, b, c) \in H_j^{w',w}$.

- All entries in the lists $S$, $T$ and $H$ are elements taken from a finite field $F_q$. 
Verification by Tag Values

- children send their shares and tags to parent

$S_1[t], g_t^{leaf_1}$

$S_2[t], g_t^{leaf_2}$

$S_1, L, T_1^{leaf_1}$

$S_2, L, T_2^{leaf_2}$
Verification by Tag Values

- verify that $c_{\text{leaf}_i}^t = a_{\text{leaf}_i}^t \cdot S_i[t] + b_{\text{leaf}_i}^t \cdot \overline{g}_t$
- construct new share $\nu$
Send a Fake Message

Proposition (3.1)

The probability that a faked message will satisfy the verification function is \( \frac{1}{q-1} \).
Send a Fake Message

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*The probability that a faked message will satisfy the verification function is* $\frac{1}{q-1}$.

- Send a fake message $v'$ with the corresponding $g'$ (not known by the sender).
Exit Protocol

If at any time during a round $t$ of the player’s protocol some player $i$ catches some other player cheating, $i$ outputs the current secret $L[t]$ and leaves the protocol.
Leave the Protocol

- Don’t transmit the fact $h_t = 0$.
- Guess the real secret with a sufficiently high probability.
Leave the Protocol

Lemma (3.2)

A player deviating from the protocol cannot increase his expected payoff by more than \( \epsilon \) unless his probability of successfully learning the secret by deviating is at least
\[
p = \frac{(U_- - U_- + \epsilon)}{(U_+ - U_-)}.
\]

Lemma (3.3)

A player who initially received a list of length \( \alpha \) has at most \( \frac{1}{\alpha - t} \) chance of (correctly) guessing the position of the secret on round \( t \) if it has not already been revealed.
Ensuring Trustfulness

We need $p \geq \frac{(U-U_-+\epsilon)}{(U_+-U_-)}$ for cheating to be profitable and we know that $p = \frac{1}{\alpha-t}$.
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\[
\alpha - t \geq \frac{(U_+ - U_-)}{(U - U_- + \epsilon)} > Y
\]

\( \Rightarrow \) Choose padding \( Y \geq \frac{(U_+-U_-)}{(U-U_-+\epsilon)} \) to ensure truthfulness.
Proof of Efficiency

- The expected number of messages sent by each player is $O(1)$.
- The expected number of bits sent is $O(\log(q))$.
- The expected overall latency is $O(\log(n))$. 
Conclusions
Summary

- An algorithm for rational secret sharing, where no player can improve substantially by deviating from the protocol.
- The mechanism is scalable in terms of latency and number of messages sent by each player.
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<thead>
<tr>
<th></th>
<th>Scalable RSS</th>
<th>Previous Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>messages per player per round</td>
<td>$O(1)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>latency per round</td>
<td>$O(\log n)$</td>
<td>$O(n)$</td>
</tr>
<tr>
<td>$E[# \text{ rounds}]$</td>
<td>$O(1)$</td>
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Questions?