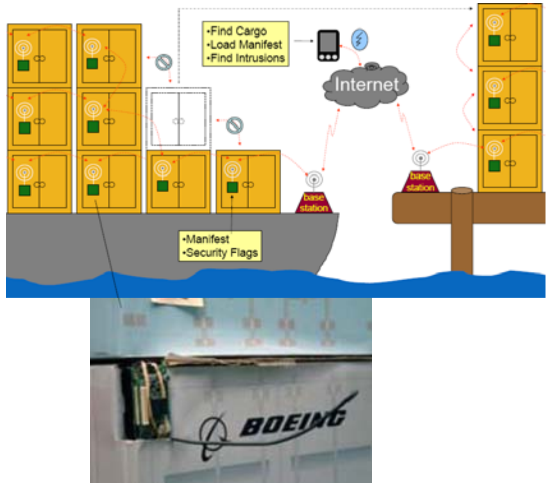
Topology Control

Chapter 3

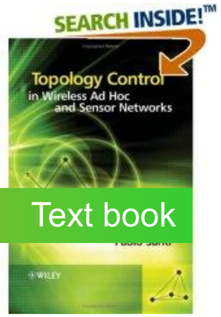
ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Inventory Tracking (Cargo Tracking)

- Current tracking systems require line-of-sight to satellite.
- Count and locate containers
- Search containers for specific item
- Monitor accelerometer for sudden motion
- Monitor light sensor for unauthorized entry into container



Rating



- Area maturity

  First steps — Text book

- Practical importance

  No apps — Mission critical

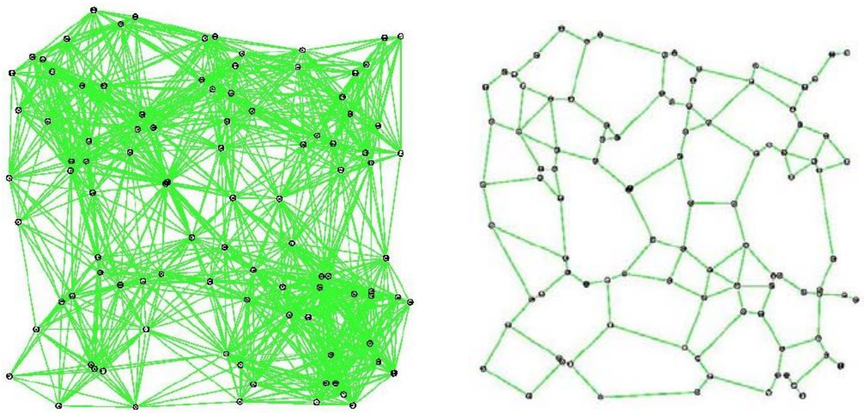
- Theoretical importance

  Not really — Must have

Overview – Topology Control

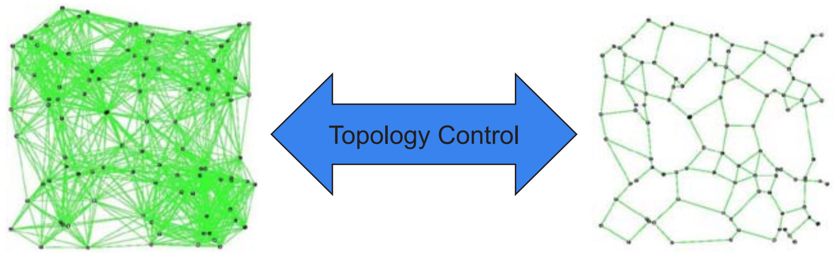
- Gabriel Graph et al.
- Practical Topology Control: XTC
- Interference

Topology Control



- Drop long-range neighbors: Reduces interference and energy!
- But still stay connected (or even spanner)

Topology Control as a Trade-Off



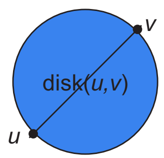
Network Connectivity
Spanner Property

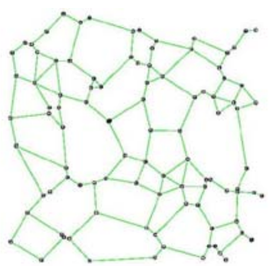
$d(u,v) \cdot t \geq d_{TC}(u,v)$

Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
Planarity
Symmetric Links
Less Dynamics

Gabriel Graph

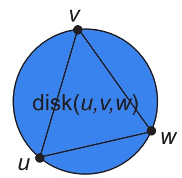
- Let disk(*u*,*v*) be a disk with diameter (*u*,*v*) that is determined by the two points *u*,*v*.
- The Gabriel Graph GG(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is an edge between two nodes *u*,*v* iff the disk(*u*,*v*) including boundary contains no other points.
- As we will see the Gabriel Graph has interesting properties.

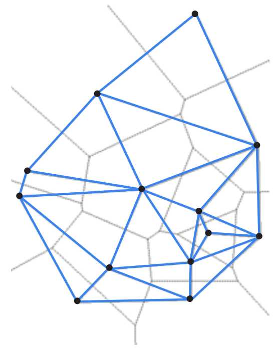




Delaunay Triangulation

- Let disk(*u*,*v*,*w*) be a disk defined by the three points *u*,*v*,*w*.
- The Delaunay Triangulation (Graph) DT(*V*) is defined as an undirected graph (with *E* being a set of undirected edges). There is a triangle of edges between three nodes *u*,*v*,*w* iff the disk(*u*,*v*,*w*) contains no other points.
- The Delaunay Triangulation is the dual of the Voronoi diagram, and widely used in various CS areas; the DT is planar; the distance of a path (s,…,t) on the DT is within a constant factor of the s-t distance.



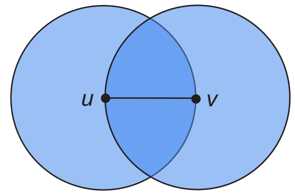


Other planar graphs

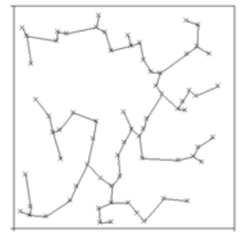
Do you know other Topologies? 

Other planar graphs

- Relative Neighborhood Graph RNG(V)
  - An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



- Minimum Spanning Tree MST(V)
  - A subset of *E* of *G* of minimum weight which forms a tree on *V*.

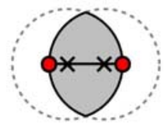


Properties of planar graphs

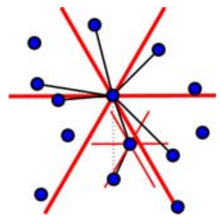
- Theorem 1:
  MST(V) $\subseteq$ RNG(V) $\subseteq$ GG(V) $\subseteq$ DT(V)
- Corollary:
  Since the MST(V) is connected and the DT(V) is planar, all the graphs in Theorem 1 are connected and planar.
- Theorem 2:
  The Gabriel Graph contains the so called "minimum energy path" (for any path loss exponent $\alpha \geq 2$)
- Corollary:
  GG(V) $\cap$ UDG(V) contains the Minimum Energy Path in UDG(V)

More examples

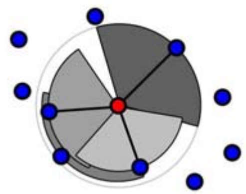
- β-Skeleton
  - Generalizing Gabriel (β = 1) and Relative Neighborhood (β = 2) Graph



- Yao-Graph
  - Each node partitions directions in k cones and then connects to the closest node in each cone



- Cone-Based Graph
  - Dynamic version of the Yao Graph. Neighbors are visited in order of their distance, and used only if they cover not yet covered angle

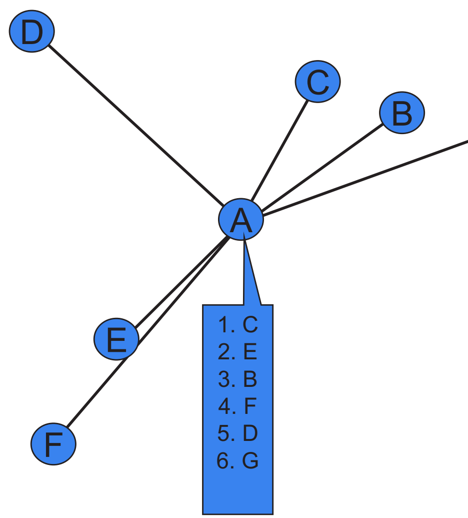


Lightweight Topology Control

- Topology Control commonly assumes that the node positions are known.

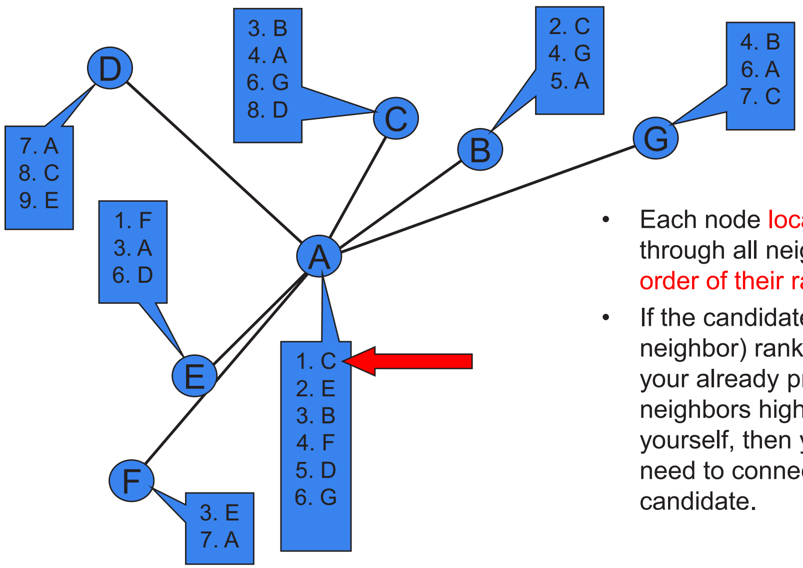
What if we do not have access to position information?

XTC: Lightweight Topology Control without Geometry



- Each node produces "ranking" of neighbors.
- Examples
  - Distance (closest)
  - Energy (lowest)
  - Link quality (best)
  - Must be symmetric!
- Not necessarily depending on explicit positions
- Nodes exchange rankings with neighbors

XTC Algorithm (Part 2)



- Each node locally goes through all neighbors in order of their ranking
- If the candidate (current neighbor) ranks any of your already processed neighbors higher than yourself, then you do not need to connect to the candidate.

XTC Analysis (Part 1)

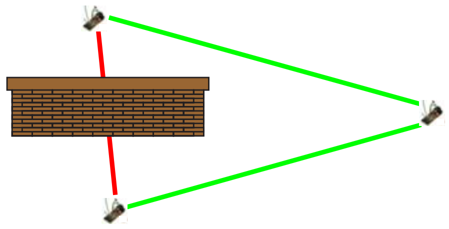
- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.

- Proof:
  - Assumption 1) $u \rightarrow v$ and 2) $u \not\leftarrow v$
  - Assumption 2) $\Rightarrow \exists w$: $\underbrace{\text{(i) } w \prec_v u \text{ and (ii) } w \prec_u v}$

Contradicts Assumption 1)

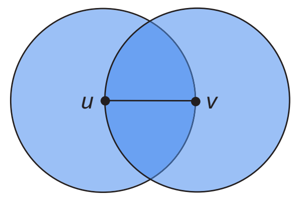
XTC Analysis (Part 1)

- Symmetry: A node u wants a node v as a neighbor if and only if v wants u.
- Connectivity: If two nodes are connected originally, they will stay so (provided that rankings are based on symmetric link-weights).
- If the ranking is energy or link quality based, then XTC will choose a topology that routes around walls and obstacles.

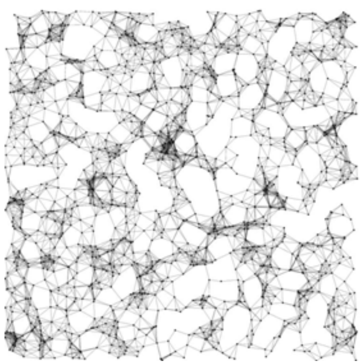


XTC Analysis (Part 2)

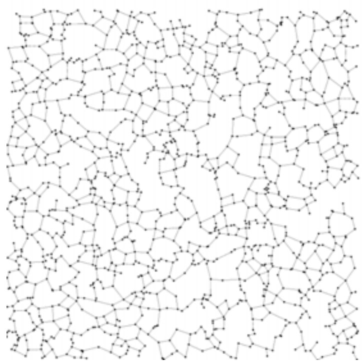
- If the given graph is a Unit Disk Graph (no obstacles, nodes homogeneous, but not necessarily uniformly distributed), then …
- The degree of each node is at most 6.
- The topology is planar.
- The graph is a subgraph of the RNG.
- Relative Neighborhood Graph RNG(V):
  - An edge e = (u,v) is in the RNG(V) iff there is no node w with (u,w) < (u,v) and (v,w) < (u,v).



XTC Average-Case

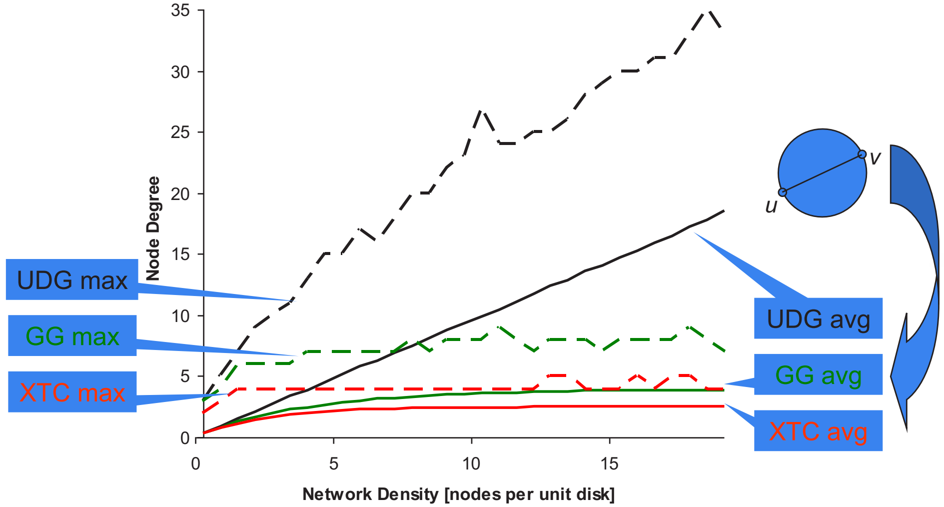


Unit Disk Graph

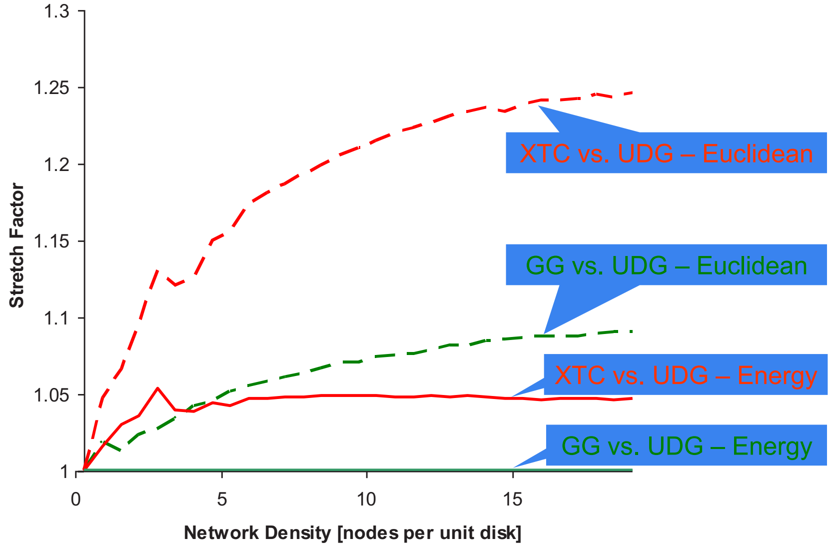


XTC

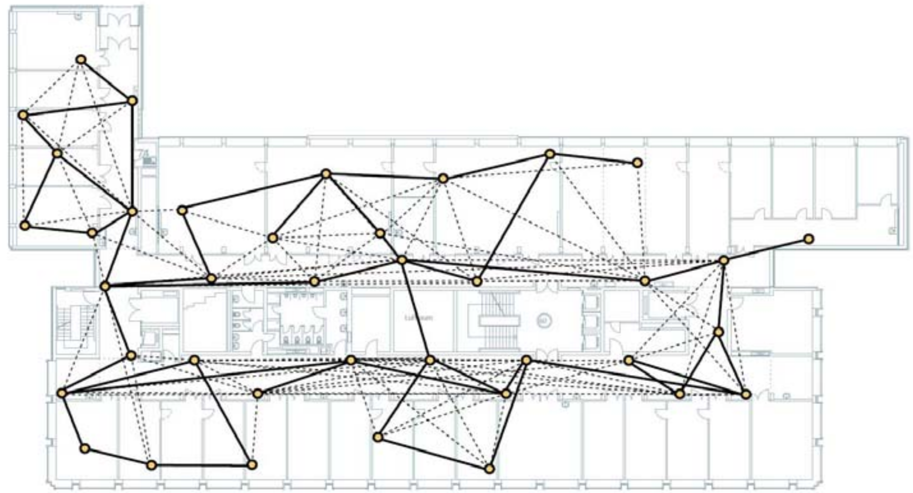
XTC Average-Case (Degrees)



XTC Average-Case (Stretch Factor)

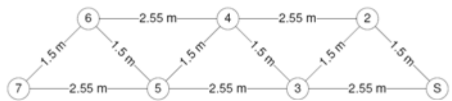


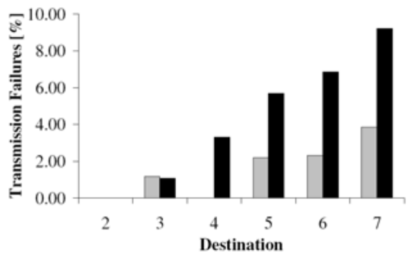
Implementing XTC, e.g. BTnodes v3

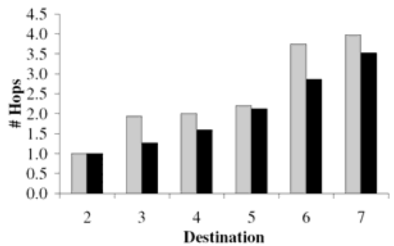


Implementing XTC, e.g. on mica2 motes

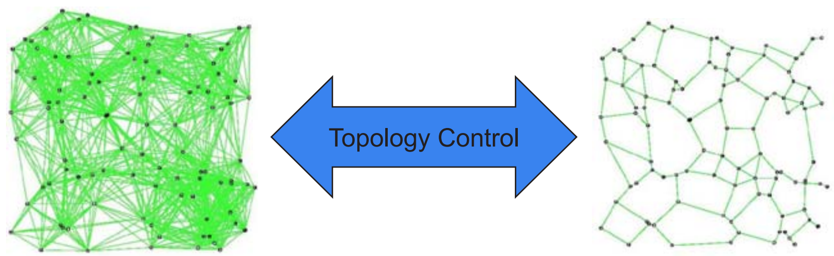
- Idea:
  - XTC chooses the reliable links
  - The quality measure is a moving average of the received packet ratio
  - Source routing: route discovery (flooding) over these reliable links only
  - (black: using all links, grey: with XTC)







Topology Control as a Trade-Off



Network Connectivity
Spanner Property

Conserve Energy
Reduce Interference
Sparse Graph, Low Degree
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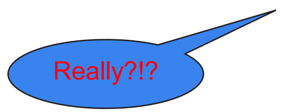
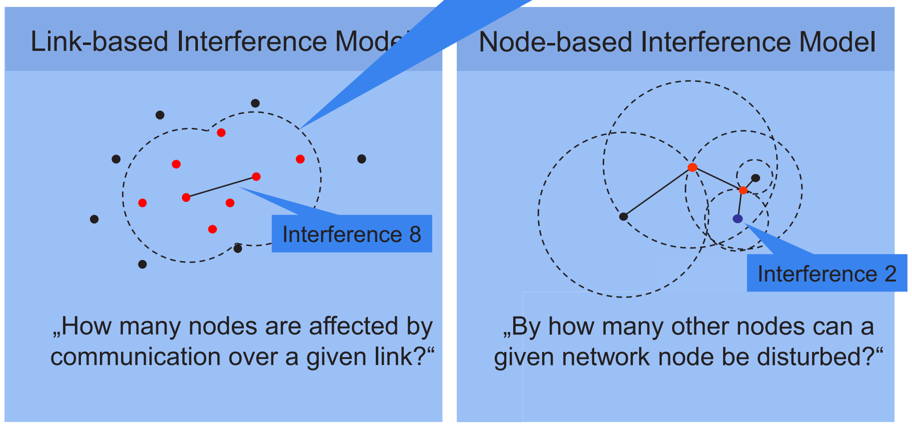
Less Dynamics



What is Interference?

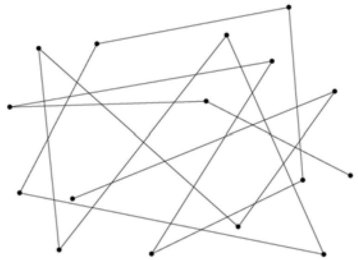
Exact size of interference range does not change the results



- Problem statement
  - We want to minimize maximum interference
  - At the same time topology must be connected or spanner

Low Node Degree Topology Control?

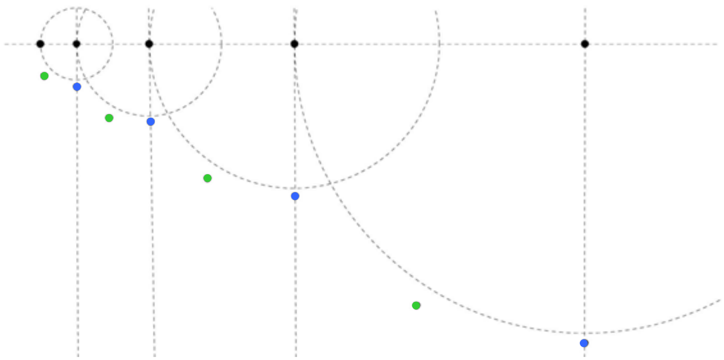
Low node degree does not necessarily imply low interference:



Very low node degree but huge interference

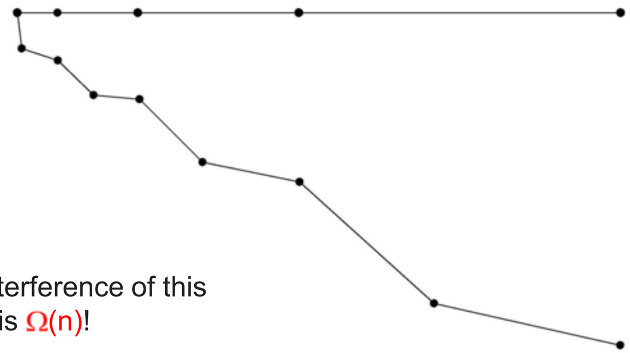
Let's Study the Following Topology!

…from a worst-case perspective



Topology Control Algorithms Produce…

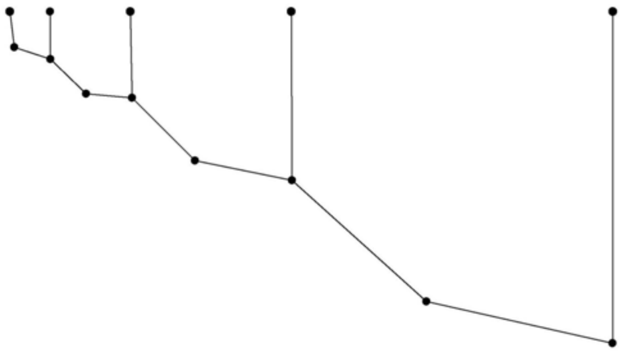
- All known topology control algorithms (with symmetric edges) include the nearest neighbor forest as a subgraph and produce something like this:



- The interference of this graph is $\Omega(n)$!

But Interference…

- Interference does not need to be high…

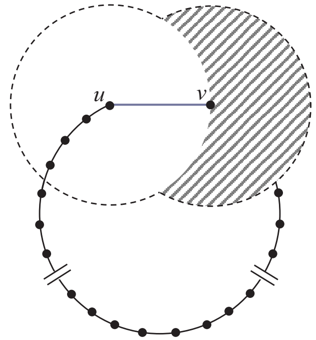


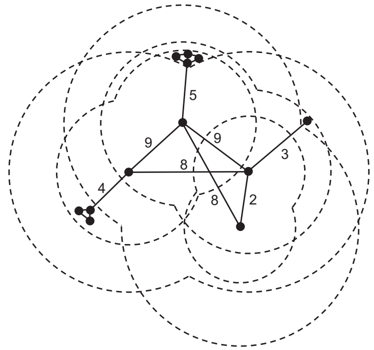
- This topology has interference O(1)!!

Link-based Interference Model

There is no local algorithm that can find a good interference topology

The optimal topology will not be planar





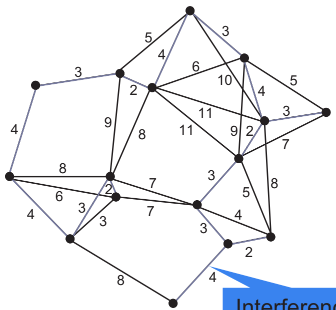
Link-based Interference Model

- LIFE (Low Interference Forest Establisher)
  - Preserves Graph Connectivity

LIFE

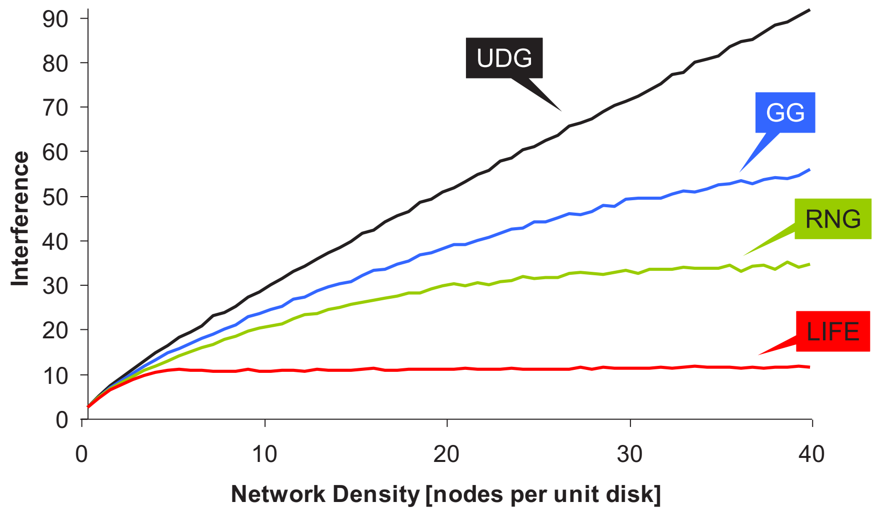
- Attribute interference values as weights to edges
- Compute minimum spanning tree/forest (Kruskal's algorithm)

LIFE constructs a minimum- interference forest



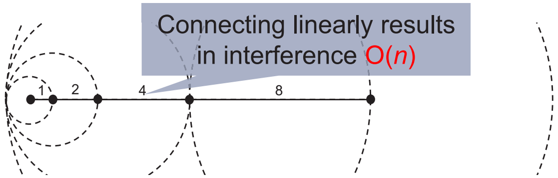
Interference 4

Average-Case Interference: Preserve Connectivity



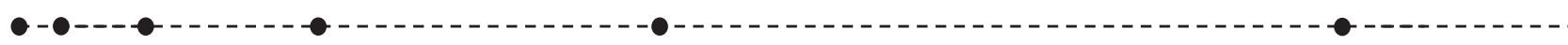
Node-based Interference Model

- Already 1-dimensional node distributions seem to yield inherently high interference...



Connecting linearly results in interference O(*n*)

- ...but the exponential node chain can be connected in a better way



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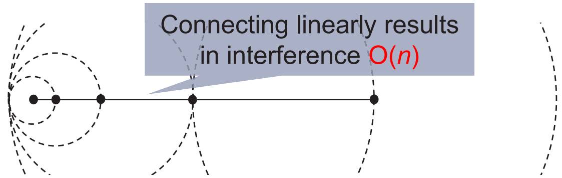


Connecting linearly results in interference O(*n*)

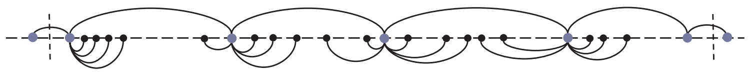
- ...but the exponential node chain can be connected in a better way

Interference $\in O(\sqrt{n})$

Matches an existing lower bound

Node-based Interference Model

- Arbitrary distributed nodes in one dimension
  - Approximation algorithm with approximation ratio in O($\sqrt[4]{n}$)



- Two-dimensional node distributions
  - Simple randomized algorithm resulting in interference O($\sqrt{n \log n}$)
  - Can be improved to O(√n)

Open problem

- On the theory side there are quite a few open problems. Even the simplest questions of the node-based interference model are open:

- We are given n nodes (points) in the plane, in arbitrary (worst-case) position. You must connect the nodes by a spanning tree. The neighbors of a node are the direct neighbors in the spanning tree. Now draw a circle around each node, centered at the node, with the radius being the minimal radius such that all the nodes' neighbors are included in the circle. The interference of a node u is defined as the number of circles that include the node u. The interference of the graph is the maximum node interference. We are interested to construct the spanning tree in a way that minimizes the interference. Many questions are open: Is this problem in P, or is it NP-complete? Is there a good approximation algorithm? Etc.