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Discrete Event Systems

Exercise Sheet 13

1 Network Calculus

Given a rate function R, functions α, β and the min-plus-convolution $\otimes$ from the lecture, prove the properties on slide 6/17:

a) If $R \leq R \otimes \alpha$ holds, α is an arrival curve.

b) If $R^* \geq R \otimes \beta$ holds, β is a service curve.

2 Power-Down Mechanisms

Since Alice has not been very happy with her internship at the processor manufacturer *Tintel*, she has applied for another internship at the company *Hewlate-Packard*, which is well-known for its energy-saving laptop models. The company's intelligence division has recently acquired a laptop from their strongest competitor *Samson* which seems to outperform their own models. Alice's first assignment is to investigate further into this matter.

The state-of-the-art laptops of both Hewlate-Packard and Samson have two power modes:

- *Active state*: Energy consumption is 1 energy unit per time unit
- *Sleep state*: Energy consumption is 0 energy unit per time unit

The transition from the active state to the sleep state and back to the active state costs D energy units altogether. Over time, there are busy periods where the user works with the device and idle periods where the laptop is not used. During busy periods, the laptop must be in the active state.

The **power-down strategy** controls the idle time after which the laptop goes to sleep. The competitiveness of a strategy S is the ratio of the energy units spent by S and the energy units spent by the optimal algorithm for a given busy sequence.

Hint: To show that a strategy is c-competitive, it suffices to show that it is c-competitive for an arbitrary idle period $I = [t_1, t_2]$. At time $t_1 - \varepsilon$, both algorithms have to be in the active state because the busy period has not ended yet. At time $t_2 + \varepsilon$, both algorithms have to be in the active state again because a busy period has just begun. Hence, it suffices to compare the energy consumption of ALG and OPT within the time span $[t_1, t_2]$ (energy costs for going to sleep state and back to active state are part of the costs for I).

a) To get familiar with power-down strategies, Alice has to find a deterministic 2-competitive power-down strategy.

b) Alice's supervisor Brian tells Alice that nobody in the company has yet found a strategy that is better than 2-competitive. Prove that no deterministic strategy can be better than 2-competitive.

c) Brian is unhappy about Alice's findings from part **b)** because he knows that the power-down strategy in Samson's laptops is better than 2-competitive. Alice has to find a strategy with a competitive ratio $c < 2$ otherwise she will be fired. Help Alice!

Hint: Alice recently heard her colleagues talking about probabilistic strategies. . .