Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing

HS 2014
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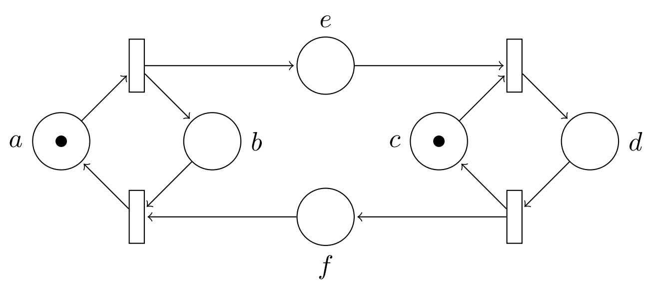
Discrete Event Systems

Exercise Sheet 14

1 Token Game

In this exercise you are asked to study the dynamics of the following petri net with the given initial token distribution:



a) Is there a reachable marking where both places a and b have a token, i.e., where $M(a) \geq 1$ and $M(b) \geq 1$ holds? Explain your decision.

b) Compute all reachable markings of the system or prove that there are infinitely many markings.

2 Calculating with Petri Nets

In this exercise you are supposed to model a function $f_i(x, y)$ on a petri net. That is, the petri net must contain two places P_x and P_y that hold x and y tokens respectively in the beginning. Additionally, the net must contain one place P_z which holds $f_i(x, y)$ tokens when the net is dead. The petri nets are supposed to work for arbitrary numbers of tokens in P_x and P_y.

a) $f_1(x, y) = 5x + y \quad \forall x, y \geq 0$

b) $f_2(x, y) = x - 2y \quad \forall y \geq 0, x \geq 2y$

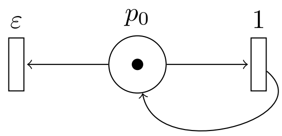
c) $f_3(x, y) = x \cdot y \quad \forall x, y \geq 0$

Hint: You may have to use inhibitor arcs for part **b)** and **c)**. An inhibitor arc between a place and a transition prevents the transition from firing as long as there is at least one token in the place (see slide 3/52).

3 Petri Net Languages

In this exercises you are supposed to give a labeled petri net P (see slide 3/37) together with an initial token distribution that accepts exactly the words of a language $\mathcal{L}$ over the alphabet $\Sigma = \{0, 1\}$. P is said to *accept* a word w iff it corresponds to a valid firing sequence σ_w and P is dead after executing σ_w.

Example: The following petri net accepts the word $v = 11$, since there exists a corresponding firing sequence $\sigma_v = 1, 1, \varepsilon$ such that P is dead after executing σ_v. (This petri net accepts the language $\mathcal{L} = 1^*$.)



Hint: You may have to use inhibitor arcs.

a) $\mathcal{L}_1 = \{0^a 1^a \mid a \geq 0\}$

b) $\mathcal{L}_2 = \{w \in \{0, 1\}^* \mid w$ contains at least as many ones as zeros$\}$

4 A Candlelight Dinner

Alice invites Bob for dinner. Unfortunately, in Alice' apartment-sharing community there is little cutlery.

a) As a starter, Alice prepared soup. There is only one spoon, and hence only one person can eat at any time. However, Alice and Bob are more interested in each other rather than in the food and are in no hurry. Therefore, whenever one of them has eaten some soup, the spoon is put back onto the table, and Alice and Bob have a little chat. At some later time, someone picks up the spoon again and eats some more. And so on.

(i) Model the situation using a petri net.

(ii) Prove that for your petri net, it holds that there is always at most one person having the spoon.

(iii) How would you change your net if Alice and Bob strictly alternated in eating?

b) Additionally to the spoon, Alice finds a fork in the kitchen. As a second plate, they will have spaghetti and vegetables. "Of course", to eat spaghetti, one needs both a fork and a spoon. The vegetables on the other hand can be eaten either with a spoon or a fork (not both). Again, Alice and Bob are in no hurry and talk after each bite. At some time, someone takes the spoon and the fork and eats some spaghetti, or someone takes either the spoon or the fork and eats some vegetables. Model this situation using a petri net!

c) Assume there is a second fork. How could you change your petri net from the previous task to model also this situation?

d) Back to the situation with one fork and one spoon. Surprisingly, Trudy – Alice' room mate – comes back from a party. She takes a joghurt from the fridge and sits down at the table. As you mighty already expect, she also needs a spoon. Extend your petri net!