



# Discrete Event Systems

## Exercise Sheet 3

### 1 Pumping Lemma [Exam]

Are the following languages regular? Prove your claims!

- a)  $L_1 = \{0^a 1^b 0^c 1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2 \text{ and } c = d\}$
- b)  $L_2 = \{0^a 1^b 0^c 1^d \mid a, b, c, d \geq 0 \text{ and if } a = 1 \text{ and } b = 2 \text{ then } c = d\}$

### 2 Deterministic Finite Automata [Exam]

Transform the NFA  $A$  in Figure 1 into an equivalent DFA using the powerset construction presented in the lecture, while assuming  $\Sigma = \{0, 1\}$ . (*Hint: Only construct states which are necessary!*)

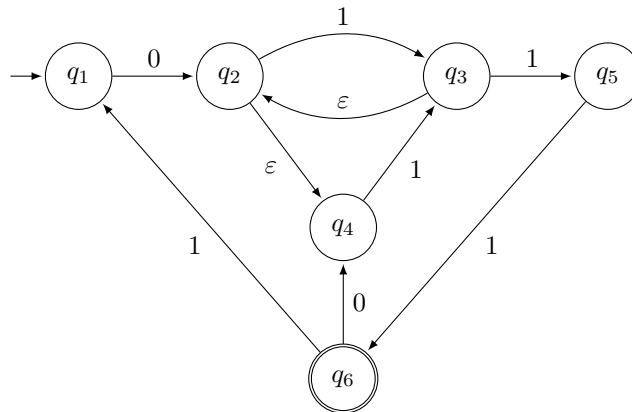


Figure 1: NFA  $A$ .

### 3 Transforming Automata [Exam]

Consider the DFA  $B$  in Figure 2 over the alphabet  $\Sigma = \{0, 1\}$ . Give a regular expression for the language  $L$  accepted by the automaton  $B$ . If you like, you can do this by ripping out states as presented in the lecture (slide 1/83 ff.).

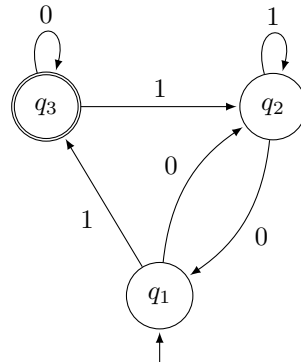


Figure 2: DFA  $B$ .

### 4 Regular and Context-Free Languages

- Consider the context-free grammar  $G$  with the production  $S \rightarrow SS \mid 1S2 \mid 0$ . Describe the language  $L(G)$  in words, and prove that  $L(G)$  is not regular.
- The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language  $L$  that is regular.

### 5 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet  $\Sigma = \{0, 1\}$ :

- $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$
- $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

### 6 Pushdown Automata

Consider the following context-free grammar  $G$  with non-terminals  $S$  and  $A$ , start symbol  $S$ , and terminals “(”, “)”, and “0”:

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

- What are the eight shortest words produced by  $G$ ?
- Context-free grammars can be ambiguous. Prove or disprove that  $G$  is unambiguous.
- Design a push-down automaton  $M$  that accepts the language  $L(G)$ . If possible, make  $M$  deterministic.