Distributed Systems Part II

Solution to Exercise Sheet 9

1 Clock Synchronization

a) False. Assume a chain of nodes, each ones clock is equal to its left neighbors clock plus x. The average local skew is x. However, the average global skew can be much larger, depending on the size of the chain.

b) False. Assume a chain of nodes again. Nodes at even positions have clock c and nodes at odd positions $c+x$. clearly, the average global skew is smaller than x. However, the average local skew is x.

2 Time Difference of Arrival

a) The time difference of arrival ($3.3 \cdot 10^{-}6s$) leads to a range difference for satellite A and B of $3.3 \cdot 10^{-}6s \cdot 3 \cdot 10^{8} m/s \approx 1km$. The relation between our location and the range difference is $\left\|p^B - p\right\| - \left\|p^A - p\right\| \approx 1km$ with $p = (x, -x+8)$. Hence we want to minimize $r = \left\|p^B - p\right\| - \left\|p^A - p\right\| - 1km$.

b) The residual for point ($2km$,$6km$) is much smaller than the residual for point ($4km$,$4km$). Hence, point ($2km$,$6km$) is more likely to be the correct location assuming that the measurements are correct.

c) The distance between satellite B and position point ($2km$, $6km$) is 5km. Therefore, the time is $t + 16.7\mu s$ when the signal arrives at B.

3 Clock Synchronization: Spanning Tree

The grid is composed of cells and nodes. The nodes are shown as black dots in Figure 1 and the cells are the areas between four (neighboring) nodes. Now we look at an arbitrary cell in the grid. For any tree we can draw on this grid, there has to be a way to walk out of the grid without crossing the edges of the tree as shown in Figure 1 because there are no loops in a tree. This holds for every cell in the grid, especially for the cell in the middle (or adjacent to the center node) of the grid. Let us assume we leave the grid between two nodes (B and C). These two nodes are neighbors on the grid. If m is even, then the distance between node A and node B or C respectively is at least $\frac{m}{2}$. If m is odd, either of the two routes can be $\frac{m-1}{2}$ but the other is at least $\frac{m+1}{2}$.

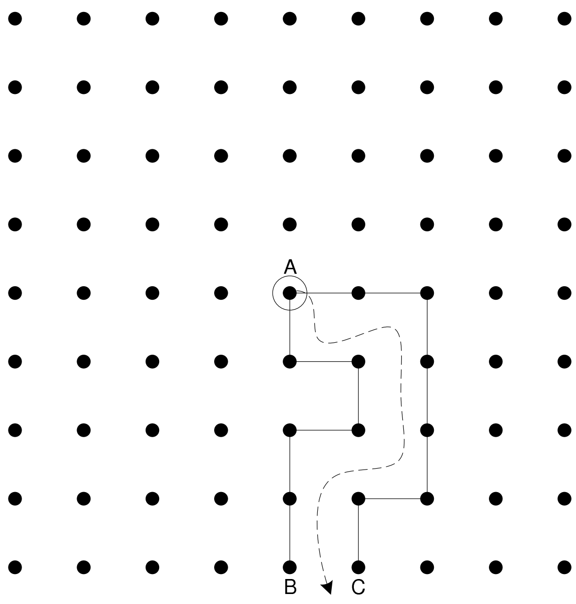

Figure 1: Node A is the center node of the spanning tree (only partially shown). The dashed line is a path through grid cells from the center node A to the outside of the grid.