



## Distributed Systems Part II

### Exercise Sheet 8

**Quiz** \_\_\_\_\_

### 1 Selling a Franc

Form groups of three to five people. One person is the auctioneer who has to provide one (imaginary) franc. Every other member of the group is a bidder. The franc is allocated to the highest bidder (for his/her last bid). Bids must be a multiple of CHF 0.05. This auction has a crux. Every bidder has to pay the amount of money he/she bid (last bid) – it does not matter if he/she gets the good. Play the game!

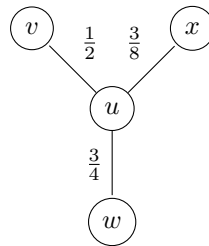
- a) Where did it all go wrong?
- b) What could the bidders have done differently?

**Basic** \_\_\_\_\_

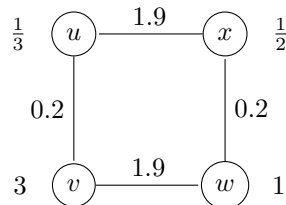
### 2 Selfish Caching

- a) For each of the following caching networks, compute the social optimum, the pure Nash equilibria, the price of anarchy (*PoA*) as well as the optimistic price of anarchy (*OPoA*):

i.  $d_u = d_v = d_w = d_x = 1$



ii. The demand is written next to a node.

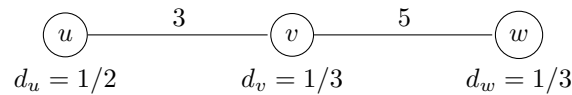


### 3 Selfish Caching with variable caching cost

The selfish caching model introduced in the lecture assumed that every peer incurs the same caching cost. However, this is a simplification of the reality. A peer with little storage space could experience a much higher caching cost than a peer who has terabytes of free disc space available. In this exercise, we omit the simplifying assumption and allow variable caching costs  $\alpha_i$  for node  $i$ .

What are the Nash Equilibria in the following caching networks given that

- i.  $\alpha_u = 1, \alpha_v = 2, \alpha_w = 2,$
- ii.  $\alpha_u = 3, \alpha_v = 3/2, \alpha_w = 3 ?$



Does any of the above instances have a dominant strategy profile? What is the PoA of each instance?

**Advanced** \_\_\_\_\_

### 4 Matching Pennies

Tobias and Stephan like to gamble, and came up with the following game: Each of them secretly turns a penny to heads or tails. Then they reveal their choices simultaneously. If the pennies match Tobias gets both pennies, otherwise Stephan gets them.

Write down this 2-player game as a bi-matrix, and compute its (mixed) Nash equilibria!

**Mastery** \_\_\_\_\_

### 5 PoA Classes

The  $PoA$  of a class  $\mathcal{C}$  is defined as the maximum  $PoA$  over all instances in  $\mathcal{C}$ . Let

- $\mathcal{A}_{[a,b]}^n$  be the class of caching networks with  $n$  peers,  $a \leq \alpha_i \leq b, d_i = 1,$  and each edge has weight 1,
- $\mathcal{W}_{[a,b]}^n$  be the class of networks with  $n$  peers,  $a \leq d_i \leq b, \alpha_i = 1,$  and each edge has weight 1.

Show that  $PoA(\mathcal{A}_{[a,b]}^n) \leq \frac{b}{a} \cdot PoA(\mathcal{W}_{[\frac{1}{b}, \frac{1}{a}]}^n)$  for all  $n > 0$ .