Algorithms that Adapt to Contention

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Fast Mutex Algorithm

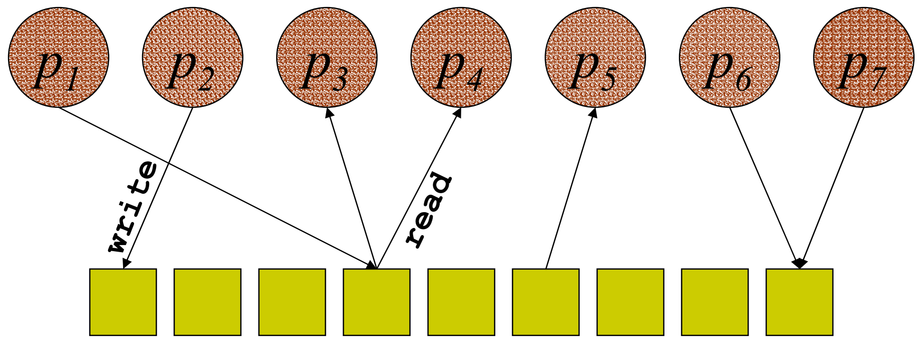
[Lamport, 1986]

- In a well-designed system, most of the time only a single process tries to get into the critical section…
- Will be able to do so in a constant number of steps.

When two processes try to get into the critical section?

O(n) steps!

Asynchronous Shared-Memory Systems



Need to collect information in order to coordinate…

When only few processes participate, reading one by one is prohibitive …

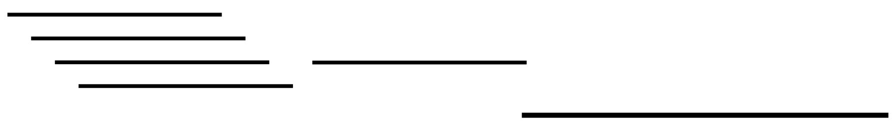
Talk Outline

- How to be adaptive in a global sense?
  - The **splitter** and its applications: renaming and collect.
- How to adapt dynamically?
  - The **sieve** and its applications : renaming and collect.
- Extensions and connections.

Adaptive Step Complexity

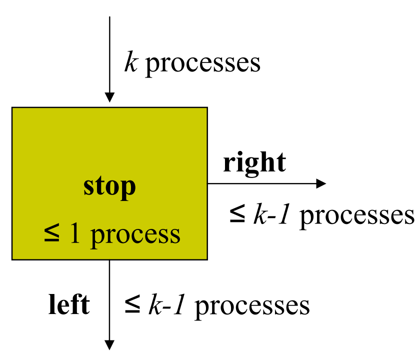
- The step complexity of the algorithm depends only on the number of active processes.

Total contention: The number of processes that (ever) take a step during the execution.



A Splitter

[Moir & Anderson, 1995]



A process stops if it is alone in the splitter.

Splitter Implementation

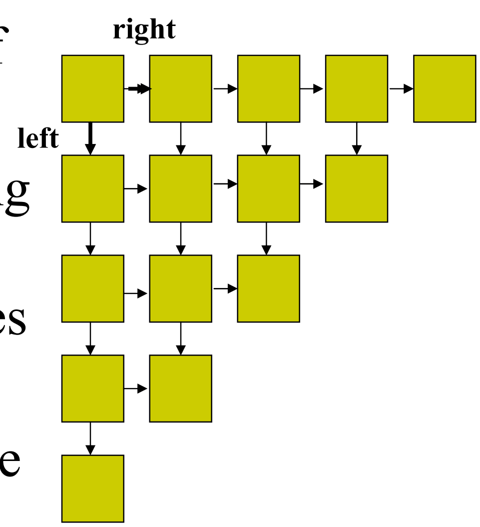
[Moir & Anderson, 1995][Lamport, 1986]

```
1. X = id_i               // write your identifier
2. if Y then return( right )
3. Y = true
4. if ( X == id_i )           // check identifier
     then return( stop )
5. else return( left )
```

Requires O(1) read / write operations, and two shared registers.

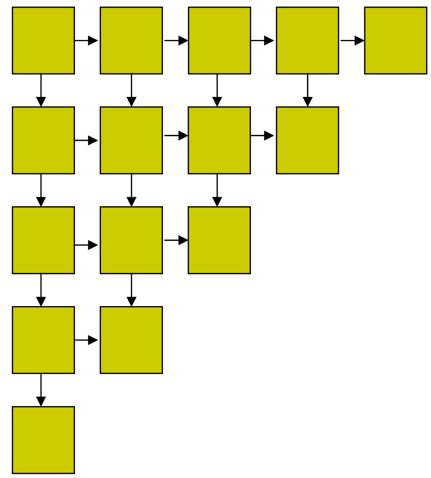
Things to do with a Splitter

- A triangular matrix of splitters.
- Traverse array, starting at the top left, according to the values returned by splitters
- Until stopping in some splitter.



Things to do with a Splitter

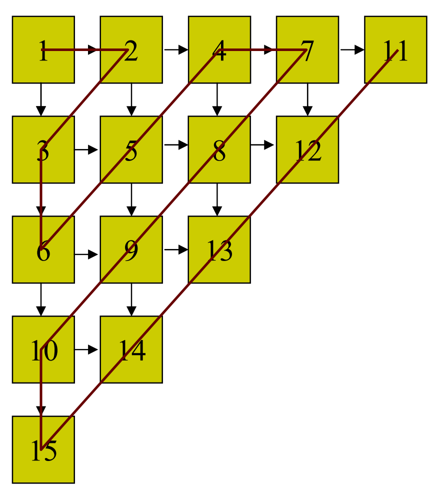
- $\geq$ one process does not follow each direction.
- ⇨ After $\leq k$ movements, a process is alone in a splitter.
- ⇨ A process stops at row, column $\leq k$
- ⇨ At most $O(k)$ steps.



Things to do with a Splitter: k^2 –Renaming

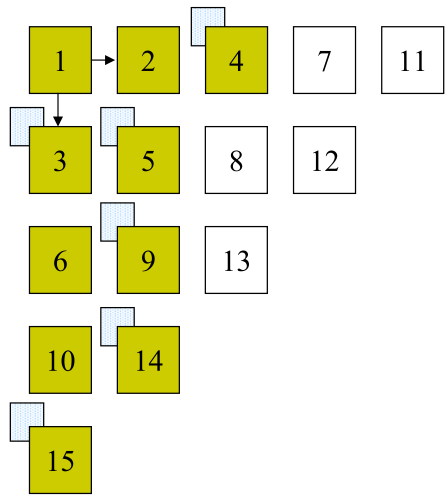
Diagonal association of names with splitters.

⇨ Take a name $\leq k^2$.



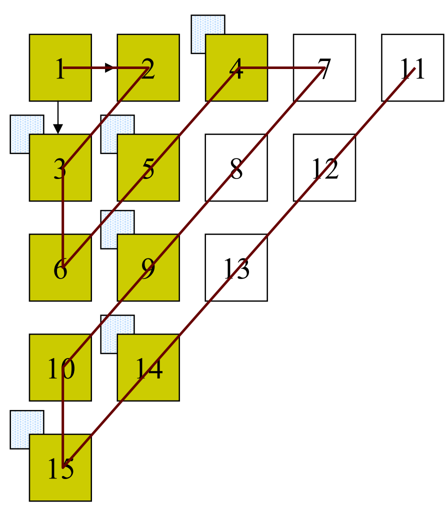
Even Better Things with a Splitter: Store

- Associate a register with each splitter.
- A process writes its value in the splitter where it stops.
- *Mark* a splitter if accessed by some process.



Even Better Things with a Splitter: Collect

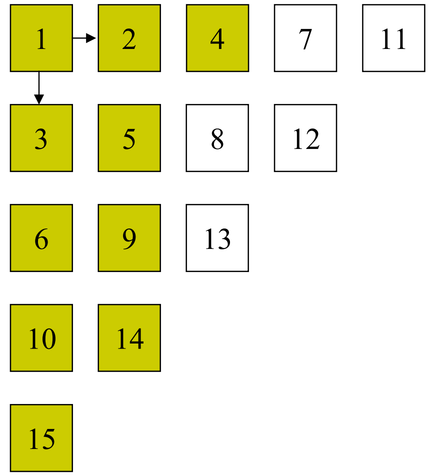
- Associate a register with each splitter.
- The current values can be collected from the associated registers.
- Going in diagonals, until reaching an unmarked diagonal.



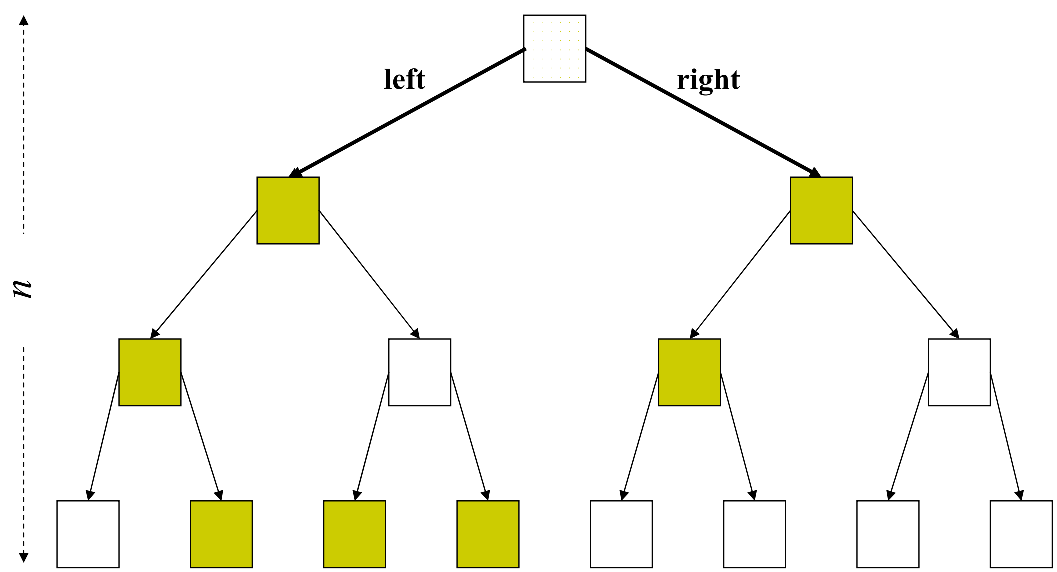
Even Better Things with a Splitter: Store and Collect

- The first store accesses $\leq k$ splitters.
- A collect may need to access k^2 splitters…

Improve?



Binary Collect Tree

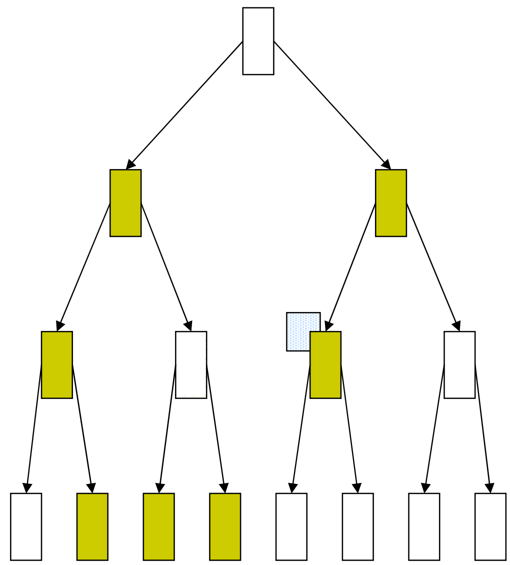


Binary Collect Tree

- To store:

  traverse the tree until stops in some splitter.

- Later, write in the register associated with this splitter.

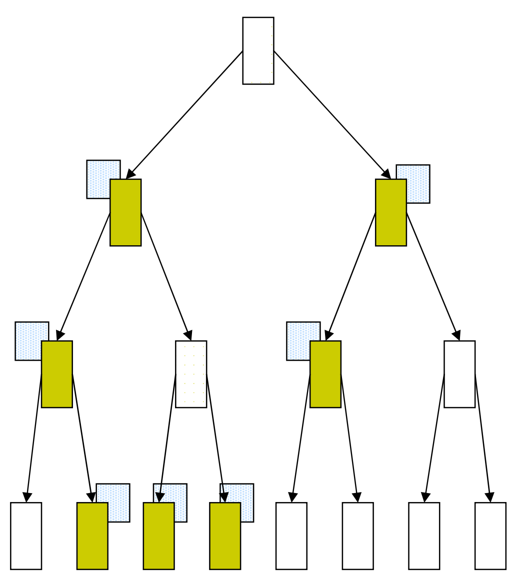


Binary Collect Tree

- To collect:

  DFS traverse the marked tree, and read the associated registers.

- Marked tree contains $\leq 2k\text{-}1$ splitters.

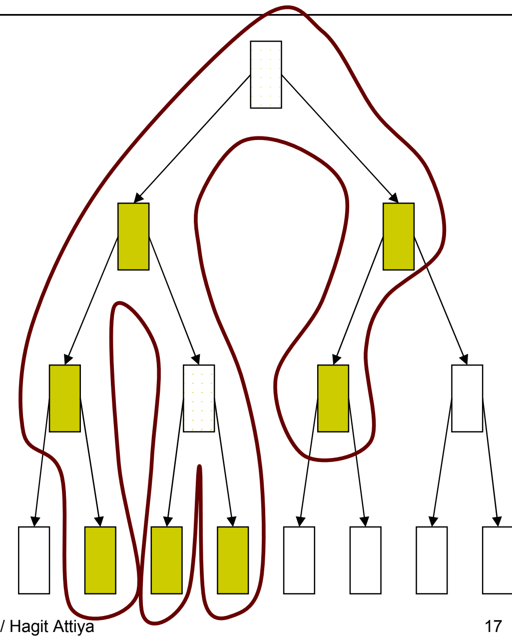


Size of Marked Sub-Tree

In a DFS ordering of the marked sub-tree,

There is ≥ one acquired node (where a process stops),

Between every pair of marked nodes.



Simple Things to Do with a Linear Collect

- Every algorithm with *f(k)* iterations of collect and store operations can be made adaptive.
  - Atomic snapshots

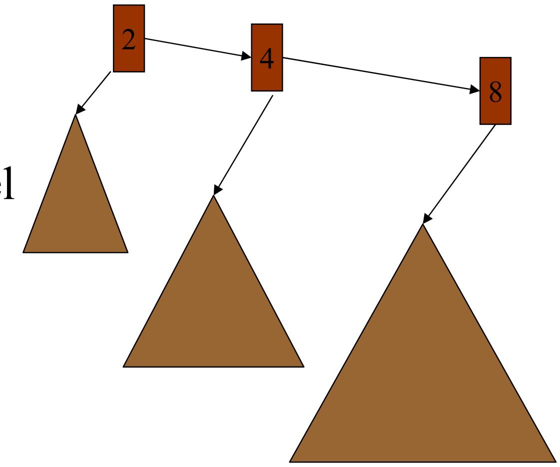
    [Afek et al. 1991]

    O(k) iterations.

    ⇨ $O(k^2)$ steps.
  - Renaming.
  - …

More Sophisticated Things to Do with a Linear Collect

- At each *spine* node:
  - Collect.
  - If # processes ≤ label
    - continue left
  - Else
    - continue right
      remember values.

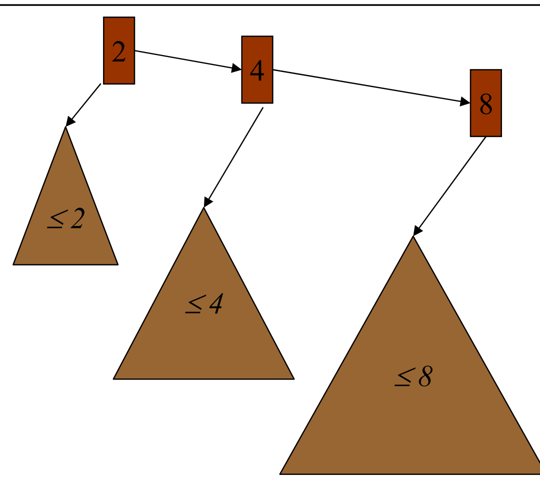


More Sophisticated Things to Do with a Linear Collect

- At most 2, 4, 8, etc. processes move to the left sub tree.

⇨ # participants in a sub-tree is bounded.

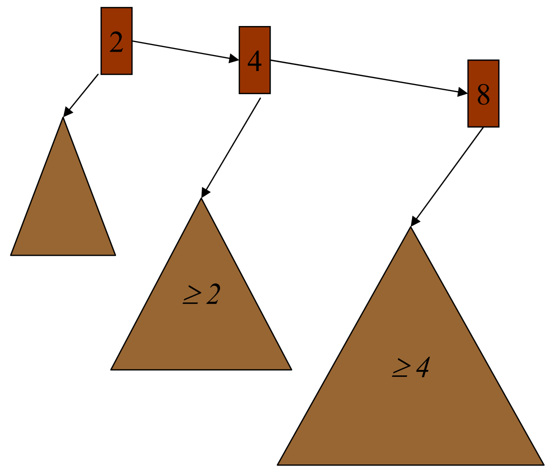
- Perform an ordinary algorithm in sub-tree.



More Sophisticated Things to Do with a Linear Collect

- If move right, at least 2, 4, 8,… participants.

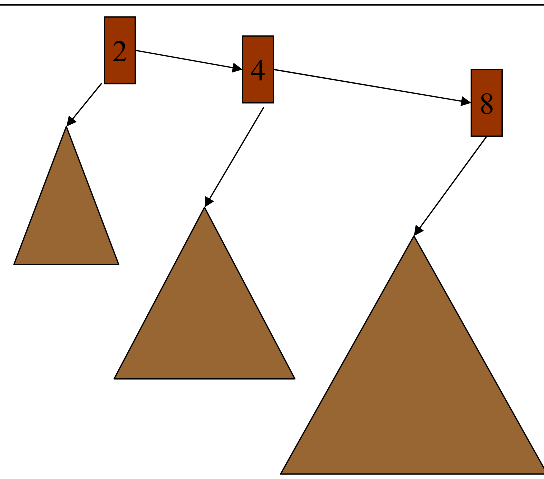
⇨ The step complexity is justified.



More Sophisticated Things to Do Efficient Atomic Snapshot

- E.g., atomic snapshot algorithm. [Attiya & Rachman, 1998]

⇨ An $O(k \log k)$ atomic snapshot algorithm.

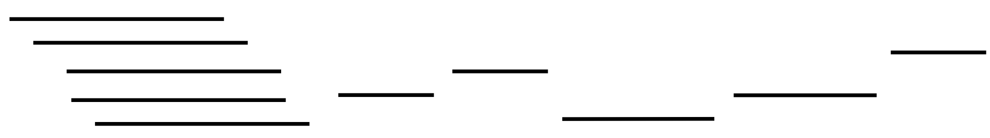


Be More Adaptive?

- In a *long-lived* algorithm…
  …processes come and go.

- What if many processes start the execution, then stop participating?
  …then starts again…
  …then stops again…

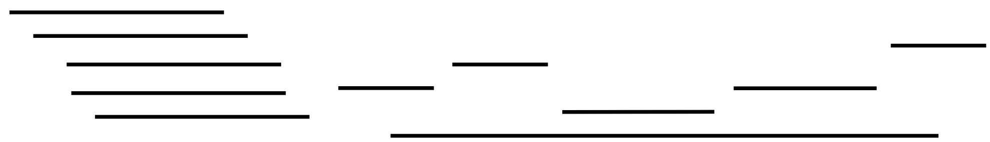
Who's Active Now?



Interval contention during an operation: The number of processes (ever) taking a step during the operation.

[Afek, Stupp &Touitou, 1999]

Who's Active Now?



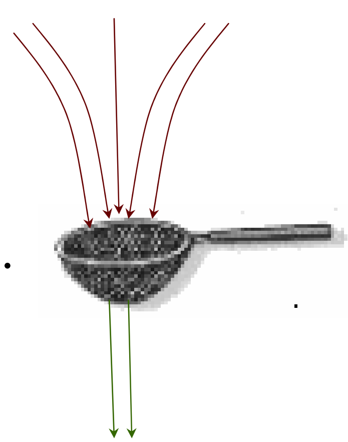
Point contention of an operation: Max number of processes taking steps together during the operation.

Clearly, point contention ≤ interval contention.

Catching Processes with a Sieve

- A dynamic object, built for repeated usage.
- When a set of processes access the sieve concurrently

  at least one is caught by the sieve.

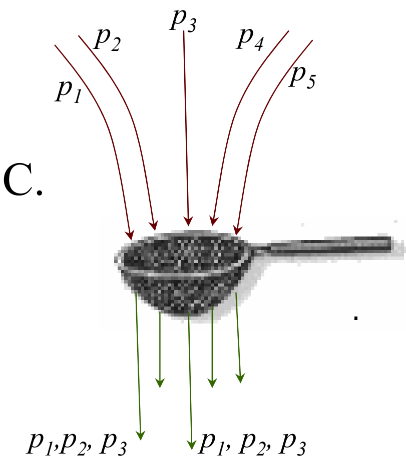


- Good synchronization properties.

Sieve: More Formally

- Returns a view of processes accessing the sieve concurrently.
  - A non-empty set of **candidates** C.
  - A process returns either ϕ or C.
  - At least one process $p_w \in C$ returns C.

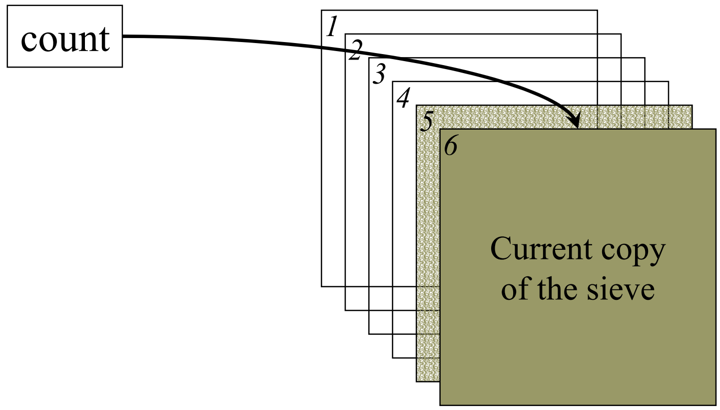
    p_w is a **winner**.



Sieve Properties

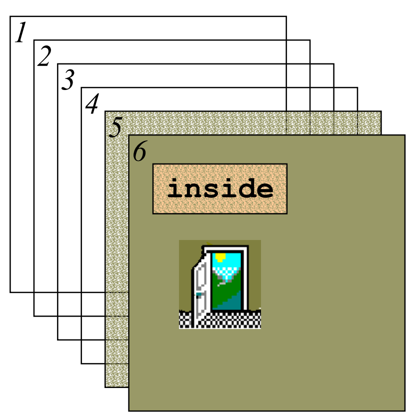
- Agreement on the set of candidates (**safety**).
  - ⇨ Synchronization.
  - ⇨ Exchange of information.
- Can be filled and emptied many times (**long-lived**).
- An empty sieve catches at least one process (**liveness**).
  - ⇨ Adapting to point contention.

Sieve Implementation: Copies



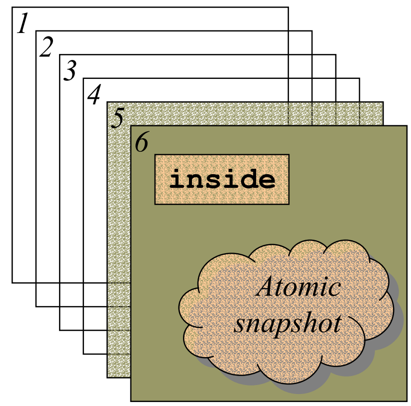
Sieve Implementation: Doorway

- Restrict access to a copy only to simultaneously active processes.
- ⇨ # processes inside the copy ≤ point contention.
- ⇨ Inside the copy, employ algorithms adaptive to total contention!



Sieve Implementation: Candidates

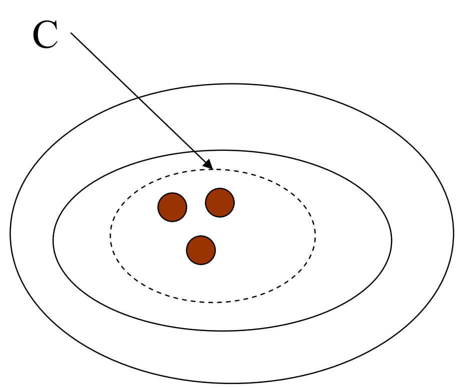
```
take an atomic snapshot
write the returned view
find minimal view C
if all processes in C
   wrote their view
   return C
else return ϕ
```



[Borowsky & Gafni]

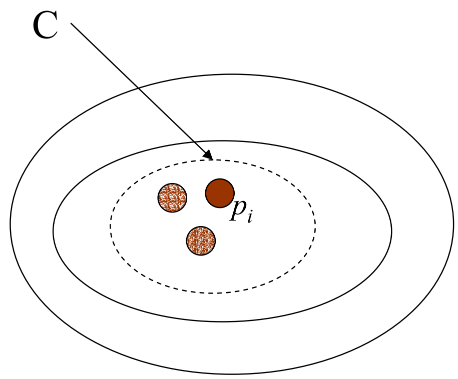
Sieve Implementation: Winners

```
take an atomic snapshot
write the returned view
find minimal view C
if all processes in C
   wrote their view
   return C
else return ϕ
```



Sieve Implementation: Winners

```
take an atomic snapshot
write the returned view
find minimal view C
if all processes in C
   wrote their view
   return C
else return ϕ
```



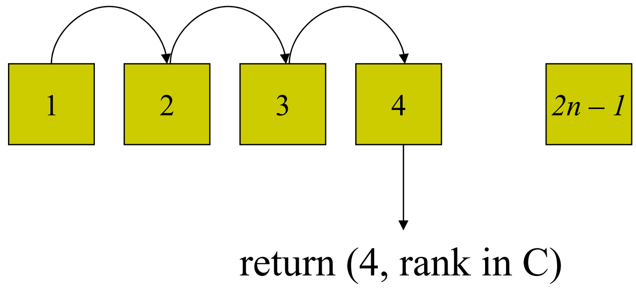
Last process in C to write a view is a **winner**.

Sieve Implementation: Managing the copies

- Candidates are synchronized (work together).
  - Increase **`count`** by 1.
  - Monotone…
- When all candidates leave a copy, open the next copy.

Things to do with a Sieve: $2k^2$–Renaming

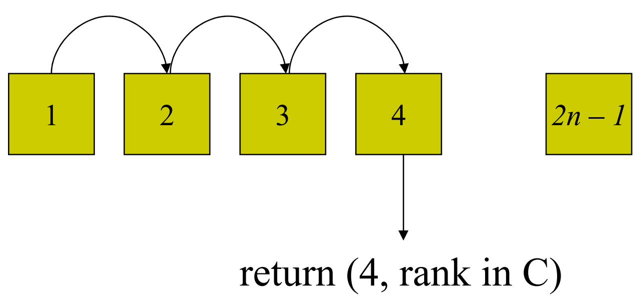
Place sieves in a row…



Agreement on set of candidates and uniqueness of copies.

⇨ **Uniqueness** of names.

Things to do with a Sieve: $2k^2$–Renaming

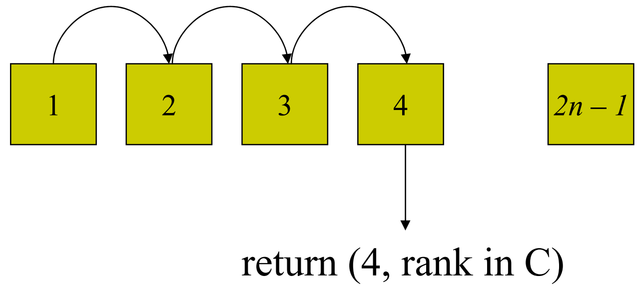


Potential method shows that a process skips $\leq 2k\text{-}2$ sieves.

- k is the point contention.

Wins in sieve $\leq 2k\text{-}1$.

Things to do with a Sieve: $2k^2$–Renaming



Wins in sieve $\leq 2k\text{-}1$.

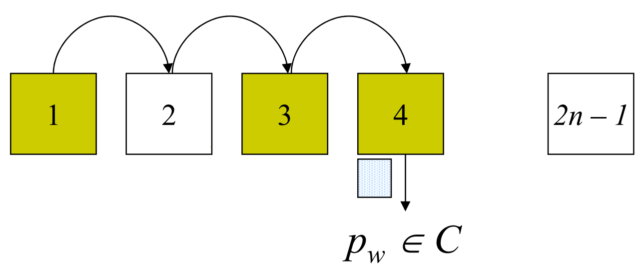
⇨ $O(f(k))$ step complexity.

C includes at most k processes.

⇨ Name $\leq 2k^2$.

Things to do with a Sieve: Store

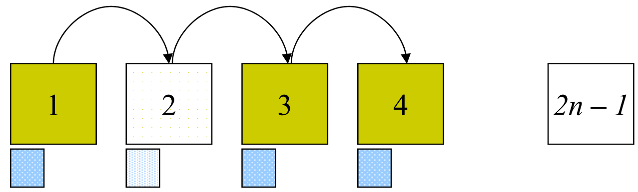
Place sieves in a row…



Agreement on set of candidates and uniqueness of copies

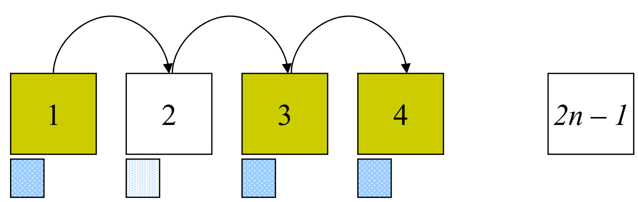
⇨ p_w writes the values of all candidates in a register associated with the sieve.

Things to do with a Sieve: Collect



- Go over the associated registers and read…

Things to do with a Sieve: Adaptive Collect?



- p_w and all other operations complete.
- A collect still has to reach the splitter in which p_w has written its value!

Bubble-Up

[Afek, Stupp & Touitou, 2000]

- Before completing an operation,
  move information from far away sieves to the first sieve.

Other Things to do with a Sieve

- Sieve-based collect can be used to implement:
  - Atomic snapshots.
  - Immediate snapshots.
- Timestamps: The vector of copy numbers.
  - ⇨ Long-lived renaming.
    Polynomial step complexity (using collect).

What About Mutex?

- Cannot have adaptive step complexity…
- Can have adaptive system response time.

  [Attiya & Bortnikov, 2002]
  - Some techniques are similar.
  - Renaming, adaptive binary tree (bottom-up!)…

What's Next?

- Other problems.
  - Snapshots, generic simulations…
- Improve and simplify.
  - Find good building blocks.
  - Local step complexity!
- Use stronger primitives (C&S, LL/SC).

  [Afek, Dauber & Touitou, 1995]

Uniform Algorithms

[Gafni, 2001]

- Adaptive algorithms can be also considered as algorithms that do not depend on the number of participants.
- Useful in the context of peer-to-peer systems, with no centralized control.
  - A huge number of potential processes.
  - Join and leave…

Space: The New Frontier

- Our results are based on a collect algorithm.
  - Either $O(K^2)$ step complexity (K is total contention),
  - Or exponential space complexity.
- A better collect algorithm?
  - $O(K)$ step complexity, and
  - Polynomial space complexity.
- A lower bound proof?

The Whole Story

This talk describes some of the results in:

- Attiya and Fouren,
  Adapting to Point Contention with a Sieve,
  Journal of the ACM, to appear.

- Attiya, Fouren and Gafni,
  An Adaptive Collect Algorithm with Applications,
  Distributed Computing, Vol. 15, No. 2 (2002).