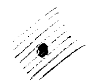
Theorem: $n - m + f = 2$

Proof [Sketch]

$m = 0$:  $\Rightarrow n - m + f = 1 - 0 + 1 = 2$ ✓

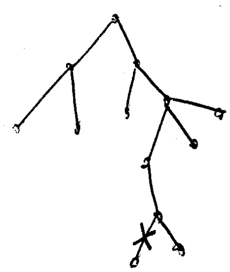
$m > 0$: (assume formula correct for $m-1$)

Tree

remove leaf

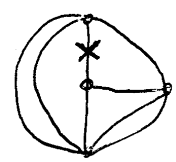
$\Rightarrow n' = n - 1$

$m' = m - 1$ ✓



Not tree

remove edge of cycle



$\Rightarrow m' = m - 1$

$f' = f - 1$ ✓

Theorem Simple, connected, planar graph with n nodes has at most $3n-6$ edges ($n \geq 3$)

Proof

- each edge bounds at most 2 faces
- each face bounded by at least 3 edges

$\Rightarrow 3f \leq 2m$

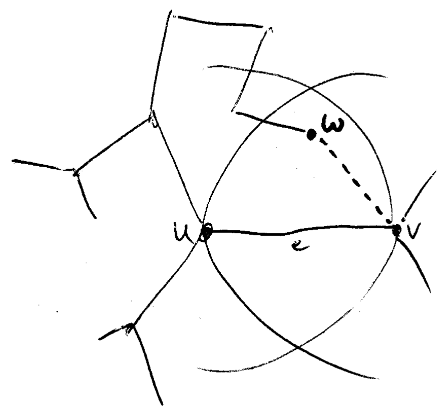
$$n - m + f = 2$$

$$3n - 3m + 3f = 6 \Leftrightarrow \underline{\underline{3n-6}} = 3m - 3f \geq 3m - 2m = \underline{\underline{m}}$$

MST ⊆ RNG

Assume Contradiction : $e \in MST$

$e \notin RNG \Rightarrow$ there is a point w in the interior (strictly)



Remove e from MST
$\Rightarrow$ Two trees T_u, T_v

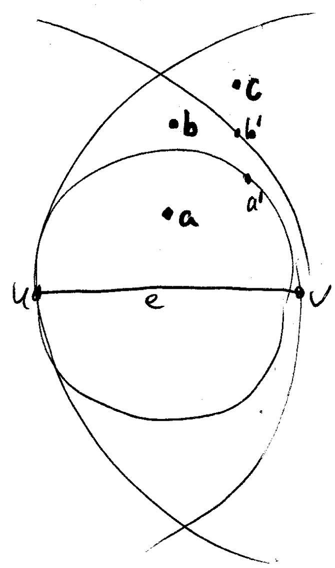
w is $\in$ of $\{T_u, T_v\}$

w.l.o.g $w \in T_u$

We can reconnect T_u with T_v with the edge (v,w)
better MST! ↯ ∎

RNG ⊆ GG

by Definition



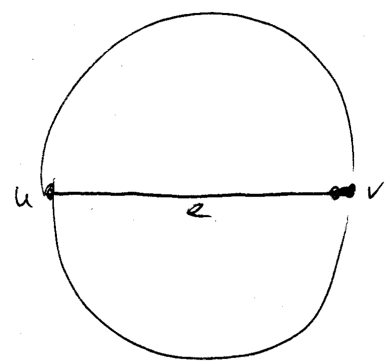
	RNG	GG
a)	$\notin$	$\notin$
b)	$\notin$	$\in$
c)	$\in$	$\in$

Comment:

a')	$\notin$	$\notin$
b')	$\in$	$\in$

GG ⊆ DT

by Definition

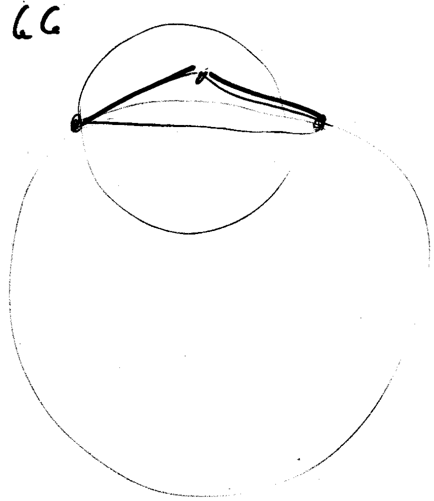


e in GG if disk(u,v) contains no other node

DT: e in DT if <u>any</u> disk with u,v on boundary contains no other node

Example:

DT, GG



MST connected

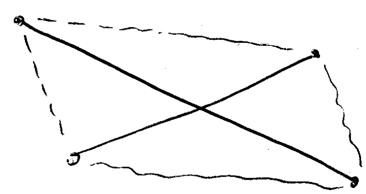
by Definition

DT planar

by definition ... however, not quite so easy..

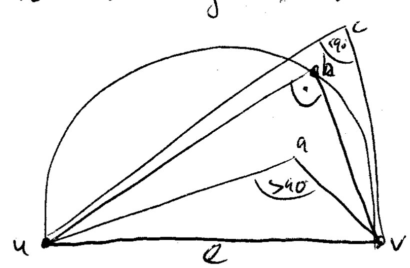
GG planar

Assume not



= 360°

there is an angle ≥ 90°

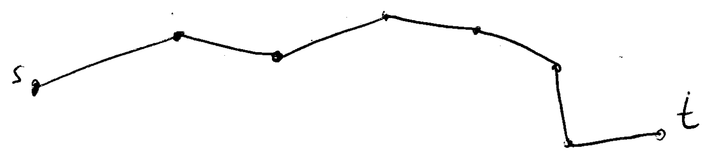


edge e exists ⇒ angle < 90° ↯

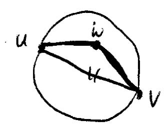
GG contains Minimum Energy Path

Proof:

Let this be MEP



Assume two nodes are not neighbor in GG.
Then there is a node w in the circle by u,v.



If uw or wv are not neighbors then you do the same again (recursively)

Otherwise $E(u,w) + E(w,v) = \overline{uw}^{\alpha} + \overline{wv}^{\alpha} \leq \overline{uv}^{\alpha}$ (for $\alpha \geq 2$)

GG ∩ UDG

Def: UDG

$e \in E$ of UDG $\iff |e| \leq 1$.

GG ∩ UDG contains Minimum Energy Path

Proof as above, except the first path is MEP in UDG