

Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich



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Principles of Distributed Computing Exercise 1

1 Vertex Coloring

- a) In the lecture, a simple distributed algorithm which colors an arbitrary graph with $\Delta+1$ colors in n synchronous rounds was presented (Δ denotes the greatest degree, n the number of nodes of the graph). To run in n rounds, the nodes of the graph had to be numbered from 1 to n. Devise a synchronous distributed algorithm for the case the IDs are unique but unbounded numbers (i.e. the nodes have arbitrary IDs instead of being numbered from 1 to n). Your algorithm should also use at most $\Delta+1$ colors and terminate in a linear number of synchronous rounds.
- **b)** What is the total number of messages your algorithm sends?
- c) Does your algorithm also work in an asynchronous environment? If yes, formulate the asynchronous equivalent to your algorithm, if no, describe why.

2 Counting the Nodes of a Tree

In this exercise, we assume that the communication graph T is a tree. We consider different aspects of the problem of counting the number of nodes of T.

- a) Suppose that a node $v \in T$ wants to know the total number of nodes. Develop a distributed algorithm \mathcal{A} for this task. \mathcal{A} can be started by every node v of T, it should determine the number of nodes of T and report it to v. How long does your algorithm need until v knows the result?
- b) Suppose now that all nodes would like to know the number of nodes in T. Devise an algorithm with which all nodes of a tree T concurrently calculate the number of nodes.
- c) For the last question of this exercise, we assume that the tree T has an odd number of nodes. In such a tree, there is a unique node v which allows to divide T into two parts whose sizes are as equal as possible. Can you use your results of Question 2b) to develop a distributed algorithm to find v? What can you say about the sizes of both parts of the achieved partition of T?