
Network Layer

Graphs

Definition 2.1 (Graph). *A graph G is a pair (V, E) , where V is a set of nodes and $E \subseteq V \times V$ is a set of edges between the nodes. The number of nodes is denoted by n and the number of edges by m .*

- **Directed graph:** each edge has a direction.
- **Undirected graph:** all the edges have no direction.
- **Weighted graph:** each edge has a weight $w(e)$.
 - The weight of the graph G is $w(G) = \sum w(e)$.

In the internet computers, smartphones, routers etc. are nodes and the wired and wireless connections are edges.

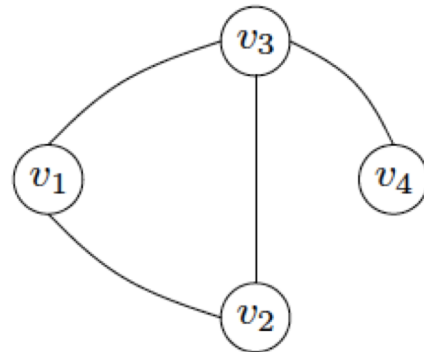
Graphs Representation

- **Adjacency matrix:**

- $n \times n$ matrix with 1 in location (i,j) iff nodes i and j are connected and 0 otherwise.
- If the graph is weighted, the 1s are replaced with the weights.

- **Adjacency list:**

- Every element corresponds to an edge of the graph identified by its endpoints.
- Better representation for sparse graphs.



	v_1	v_2	v_3	v_4
v_1	0	1	1	0
v_2	1	0	1	0
v_3	1	1	0	1
v_4	0	0	1	0

Figure 2.2: A graph $G = (V, E)$ with node set $V = \{v_1, v_2, v_3, v_4\}$ and edge set $E = \{\{v_1, v_2\}, \{v_1, v_3\}, \{v_2, v_3\}, \{v_3, v_4\}\}$, and the adjacency matrix of G .

Paths and trees

Definition 2.4 (Path). *Let $G = (V, E)$ be a graph. A path between nodes v_1 and v_k is a sequence of nodes (v_1, v_2, \dots, v_k) , where $\{v_i, v_{i+1}\} \in E$ for all $1 \leq i < k$. The path has $k - 1$ hops.*

- **Connected graph:** exists a path between any two nodes.
- **Cycle:** a sequence of connected nodes such that the first and last node of the sequence are the same and no other node appears twice.
- **Tree:** a connected graph that contains no cycles. Has $n-1$ edges.
- **Spanning tree:** a tree that connects all nodes in a graph.
- **Rooted tree:** tree with a special root node r . Every other node v has a parent, i.e., the node adjacent to v and closer to r .

Spanning trees

Subgraph:

Definition 2.8 (Subgraph). *Let $G = (V, E)$ be a graph. A subgraph $G' = (V', E')$ of G is a graph such that $V' \subseteq V$ and $E' \subseteq E$.*

Spanning tree:

Definition 2.9 (Spanning tree). *Given a graph $G = (V, E)$, a spanning tree $T = (V, E')$ is a subgraph of G that is a tree.*

Minimum spanning tree:

Definition 2.10 (MST). *Given a weighted graph $G = (V, E, \omega)$, a minimum spanning tree (MST) T is a spanning tree that minimizes the total weight $\omega(T)$.*

Minimum spanning tree algorithm

Algorithm 2.11 MST Algorithm

- 1: Given a weighted graph $G = (V, E, \omega)$
 - 2: Let $S = \{u\}$ be a set of visited nodes, initialized with any node $u \in V$
 - 3: Let T be a tree just consisting of the single node $u \in S$, no edges
 - 4: **while** $S \neq V$ **do**
 - 5: Find minimum weight edge $e = \{v, w\}$ with $v \in S$ and $w \in V \setminus S$
 - 6: Add node w to S
 - 7: Add edge e to T
 - 8: **end while**
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- Greedily adds edges with the lowest weight at each iteration.
- Outputs a minimum spanning tree.
- The time complexity is $O(m \log n)$.

Shortest path

- **Shortest path** between two nodes: path with the minimum total weight.
- **Distance** between two nodes $d(u,v)$: total weight of the minimum path.
- **Shortest path tree (SPT)**: a spanning tree T , rooted at r , of graph G , where the distance from any node to r in T equals the distance $d(r,v)$ in G .

Shortest path algorithm

Algorithm 2.16 SPT Algorithm

- 1: Given a weighted graph $G = (V, E, \omega)$ and a node $r \in V$
 - 2: Set a parent node $p_v = \text{null}$ for every node $v \in V$
 - 3: Set $d_r = 0$ and $d_v = \infty$ for every node $r \neq v \in V$
 - 4: Let $S = \{r\}$ be the set of visited nodes
 - 5: **while** $S \neq V$ **do**
 - 6: Find edge $e = \{v, w\}$ with $v \in S$ and $w \in V \setminus S$ with minimum $d_v + \omega(e)$
 - 7: Set $p_w = v$
 - 8: Set $d_w = d_v + \omega(e)$
 - 9: $S = S \cup \{w\}$
 - 10: **end while**
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- Greedily adds the node with the minimum distance to the root at each iteration.
- The time complexity is $O(m \log n)$

Addressing

Every node in a graph has an **address**. In the internet, an IP address.

- **IPv4**: 32-bit address written in 4 chunks of 8 bits separated by dots.
- **IPv6**: 128-bit address written in 8 chunks of 16 bits separated by colons. Each chunk is written as 4 hexadecimal digits.

Prefix: A prefix of k bits corresponds to the first k bits of the address.

An address block or **subnet** is a set of addresses that share the same prefix.

Addressing: IPv6

IPv6 is conceived to enlarge the address space given the fast increase of devices connected to the internet.

IPv6 address notation can be compressed:

- Leave out leading zeros in every chunk.
- Consecutive section of zeros replaced with double colon (only once).

IPv4 addresses are included in the IPv6 domain. Usually written in hexadecimal as: `::ffff` + IPv4

- e.g: IPv4 -> 8.8.4.4 | IPv6 -> `::ffff:8:8:4:4`

Packets

Definition 2.22 (Packet). *Every network packet contains a header and a payload. The payload of a packet corresponds to the actual data of the packet. The header contains information for delivering the payload.*

- Size in IPv4 limited to 65,535 bytes -> many packets needed
- **Header:** source and destination addresses and other options.
 - IPv4: 20 to 60 bytes
 - IPv6: 40 bytes
- **Time-To-Live (TTL):** number of hops a packet is allowed to travel before it is dropped.

Routing

A routing protocol decides along which path a packet travels from its source to its destination.

- **Routing table of node v :** maps every destination address to a neighbour of v .
- **Forwarding:** process of an intermediate node receiving a packet and sending it to the next node.
- **Hierarchical addressing:** match a destination address to longest prefix in the routing table.
- **Default route:** where to forward packets when no specific entry is available in the routing table.

Routing table example

Routing table of v_1	
Destination	Next node
v_1	deliver
v_2	v_2
v_3	v_3
v_4	v_3

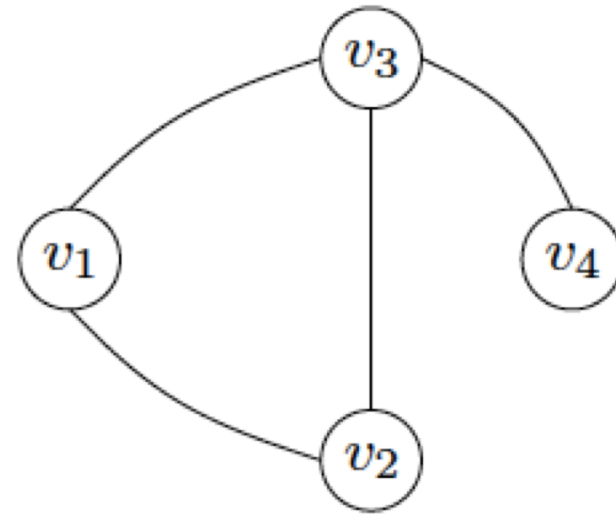


Table 2.24: A simplified routing table.

Link state (LS) routing

Algorithm 2.25 Link-State (LS) Routing Algorithm.

- 1: Given a weighted graph $G = (V, E, \omega)$
 - 2: Learn $\omega(e)$ for every edge $e \in E$
 - 3: Compute shortest paths to all nodes, e.g., by using Algorithm 2.16
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LS routing allows:

- Nodes can discover changes in the network and update their routing tables.
- Other advanced features like multiple path routing.

Drawback: The nodes need to know the whole network. LS routing is feasible only at small scale within Autonomous Systems (AS)

- **Autonomous system:** a collection of nodes owned by a company.

Distance Vector (DV)

Algorithm 2.27 Distance-Vector (DV) Routing Algorithm.

- 1: Given a weighted graph $G = (V, E, \omega)$ and a node $u \in V$
 - 2: Initialize a distance estimate $D(u \rightarrow v) = \omega(\{u, v\})$ for all neighbors $N(u)$ and $D(u \rightarrow w) = \infty$ for all other nodes
 - 3: Send distance vector $\mathcal{D}(u) = \{D(u \rightarrow v) \mid v \in N(u)\}$ to all neighbors $N(u)$
 - 4: **while true do**
 - 5: Upon receiving a distance vector $\mathcal{D}(v)$ from a neighbor v , update the distance estimate to all destinations accordingly
 - 6: **if** $D(u \rightarrow w)$ changed for any w **then**
 - 7: Send the updated distance vector $\mathcal{D}(u)$ to all neighbors
 - 8: **end if**
 - 9: **end while**
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Keeps and updates the distance between all nodes

- **Dsitributed:** nodes do not need the knowledge of the whole network.
- *Count to infinity problem.*

Intra-domain vs inter-domain

- **Intra-domain routing:** routing within an autonomous system.
 - E.g: RIP routing protocol
- **Inter-domain routing:** routing between different autonomous systems -> Border Gateway Protocol (**BGP**):
 - BGP solves count-to-infinity problem.
 - Allows **outbound policies:** a node can decide which traffic to attract
 - Allows **inbound policies:** a node can decide through which neighbour to route.

BGP algorithm

Algorithm 2.29 Border Gateway Protocol (BGP)

- 1: Basically, BGP is a DV Routing Protocol, see Algorithm 2.27
 - 2: BGP nodes send out announcements about every 30 seconds
 - 3: BGP nodes send reachability information: every node announces which address blocks (prefixes) it can reach
 - 4: Instead of just distance, nodes announce the whole AS path to each prefix
 - 5: The network is not weighted, an edge between two AS nodes costs 1.
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Tunnels

Definition 2.30 (Tunnel). *The payload of a packet is a complete packet, with header and payload. In other words, we have two headers.*

- A tunnel embeds a packet inside another packet. Use cases:
 - Virtual Private Network (VPN).
 - Cross Firewalls.
 - Translate between IPv4 and IPv6.
 - Virtual circuit routing.