Chapter 7
NETWORK CALCULUS

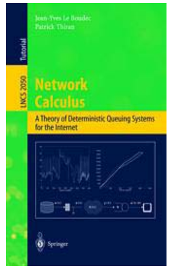
Distributed
Computing
Group

Discrete Event Systems
Winter 2004 / 2005

Overview

- Motivation / Introduction
- Preliminary concepts — Sections 1.2, 1.3, 1.4.1
- Min-Plus linear system theory — Section 3.1
- The composition theorem — Section 1.4.2

in Book "Network Calculus" by
Le Boudec and Thiran

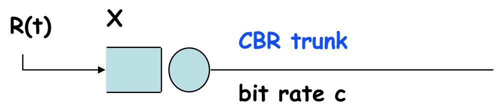


ica1www.epfl.ch/PS_files/NetCal.htm

What is Network Calculus?

- Problem:
  - Queuing theory (Markov/Jackson assumptions) too optimistic.
  - Online theory too pessimistic.
- Worst-case analysis (with bounded adversary) of queuing / flow systems arising in communication networks
- Abstraction of schedulers
- uses min, max as binary operators and integrals
  - min-plus and max-plus algebra

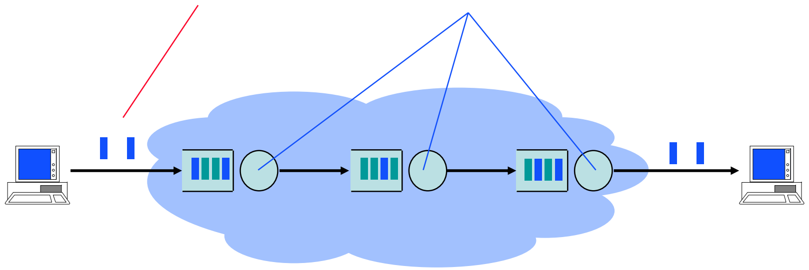
An example



- assume R(t) = sum of arrived traffic in [0, t] is known
- required **buffer** for a bit rate c is
  $\sup_{s \le t} \{R(t) - R(s) - c\,(t-s)\}$

Arrival and Service Curves

- Similarly to queuing thoery, Internet integrated services use the concepts of *arrival curve* and *service curves*

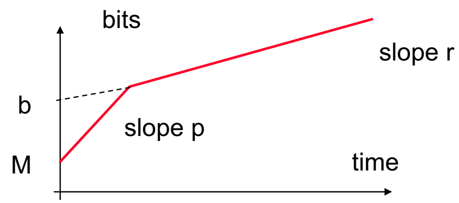


Arrival Curves

- Arrival curve α: $R(t) - R(s) \leq \alpha(t-s)$

Examples:

- leaky bucket $\alpha(u) = ru+b$
- reasonable arrival curve in the Internet
  $\alpha(u) = \min(pu + M, ru + b)$



Arrival Curves can be assumed sub-additive

- Theorem (without proof):

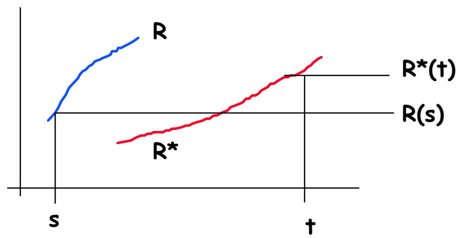
  α can be replaced by a *sub-additive* function

- sub-additive means: $\alpha(s+t) \leq \alpha(s) + \alpha(t)$
- concave $\Rightarrow$ subadditive

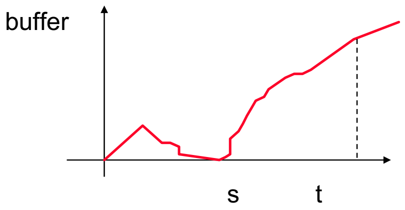
Service Curve

- System S offers a service curve β to a flow iff for all *t* there exists some *s* such that

$$R^*(t) - R(s) \geq \beta(t-s)$$



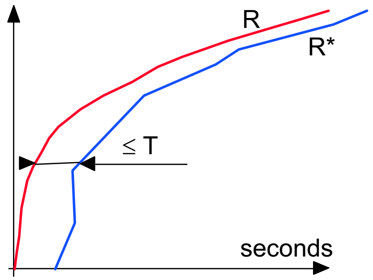
Theorem: The constant rate server has service curve $\beta(t)=ct$

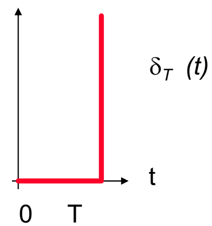


Proof: take s = beginning of busy period. Then,

$$R^*(t) - R^*(s) = c\ (t-s)$$
$$R^*(t) - R(s) = c\ (t-s)$$

The guaranteed-delay node has service curve δ_T

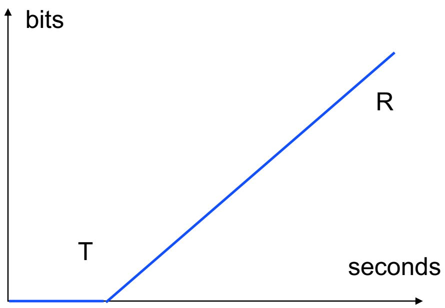




Function δ_T

A reasonable model for an Internet router

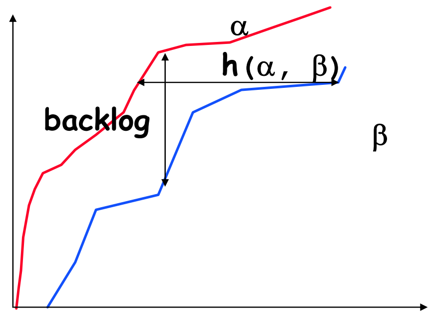
- rate-latency service curve



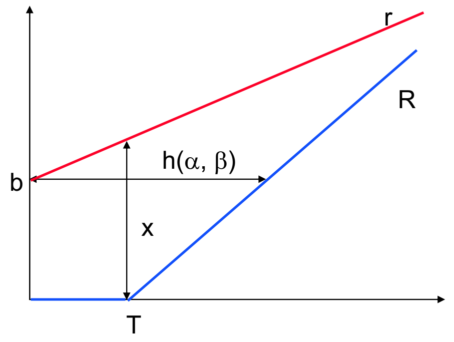
Tight Bounds on delay and backlog

If flow has arrival curve α and node offers service curve β then

- backlog $\leq \sup(\alpha(s) - \beta(s))$
- delay $\leq h(\alpha, \beta)$



For reasonable arrival and service curves



- delay bound: $b/R + T$
- backlog bound: $b + rT$

Another linear system theory: Min-Plus

- Standard algebra: R, +, ×
  $a \times (b + c) = (a \times b) + (a \times c)$
- Min-Plus algebra: R, min, +
  $a + (b \wedge c) = (a + b) \wedge (a + c)$

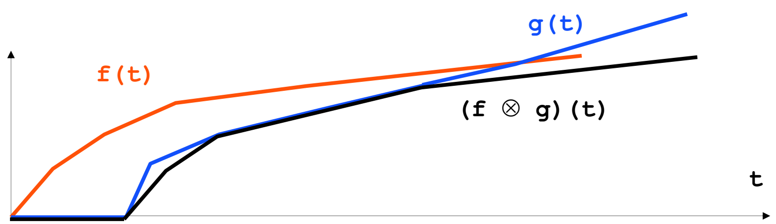
Min-plus convolution

- Standard convolution:

$$(f * g)(t) = \int f(t-u) g(u) du$$

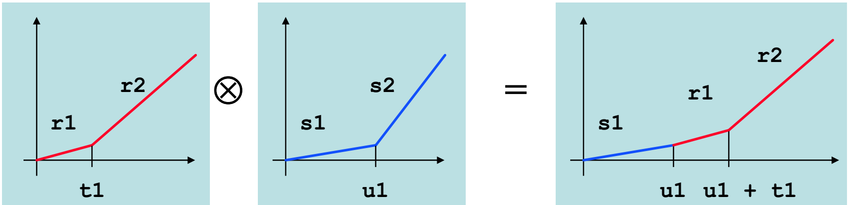
- Min-plus convolution

$$f \otimes g\ (t) = \inf_u \{ f(t-u) + g(u) \}$$



Examples of Min-Plus convolution

- $f \otimes \delta_T\ (t) = f\ (t-T)$
- convex piecewise linear curves, put segments end to end with increasing slope



Arrival and Service Curves vs. Min-Plus

- We can express arrival and service curves with min-plus
- Arrival Curve property means

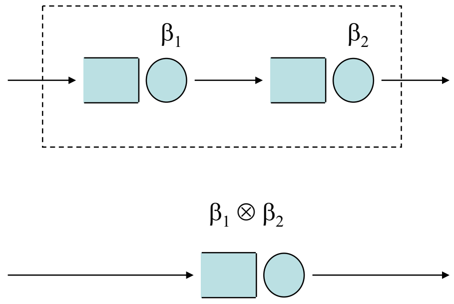
$$R \le R \otimes \alpha$$

- Service Curve guarantee means

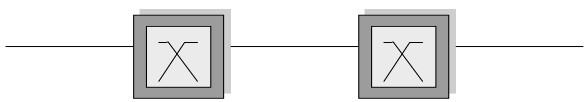
$$R^* \ge R \otimes \beta$$

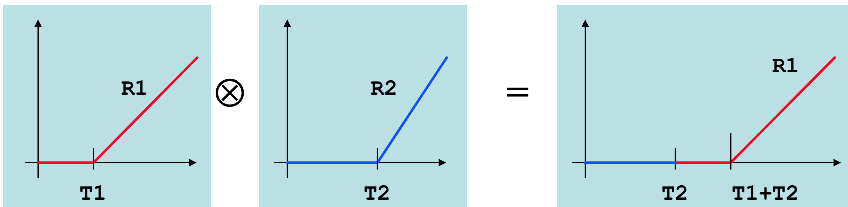
The composition theorem

- **Theorem**: the concatenation of two network elements offering service curves β_i and β_2 respectively, offers the service curve $\beta_1 \otimes \beta_2$

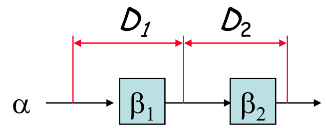


Example: Tandem of Routers

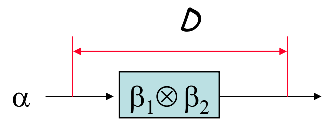




Pay Bursts Only Once



$$D_1 + D_2 \le (2b + RT_1)/ R + T_1 + T_2$$



$$D \le b /R + T_1 + T_2$$

end to end delay bound is less