# MST Construction in O(log log n) Communication Rounds


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## Content of the Talk





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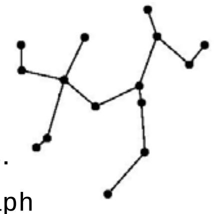
## I. Introduction of the Problem

### Definitions



- A tree is a connected graph without cycles.
- A subgraph that spans all vertices of a graph is called a spanning subgraph.
- Among all the spanning trees of a weighted and connected graph, the one (possibly more) with the least total weight is called a minimum spanning tree (MST).

# I. Introduction of the Problem

## Definitions

• In a MST algorithm, |V| - 1 edges have to be chosen in total. In each phase of the algorithm, probably only a fraction of those edges are chosen.

• Nodes that are directly or indirectly connected using chosen edges only belong to the same cluster.

•The minimum weight outgoing edge (MWOE) is the edge with the lowest weight among all incident edges leading to other clusters.



# I. Introduction of the Problem

## Usage of MST

Minimize the cost associated with global operations such as broadcasts!

→ Minimize the message complexity:

Avoid traffic explosion by using a spanning tree (no cycles!)

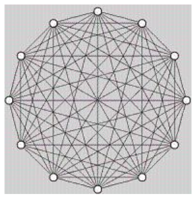
→ Minimize the time complexity:

If the edge weights represent the delay on the link, then MST minimizes the execution time.



# I. Introduction of the Problem

## System Model



• The system is represented by a complete weighted undirected graph.

G=(V,E,w) where w(e) denotes the weight of edge $e \in E$ and |V|=n. All edge weights are different (w.l.o.g.).

• Each node has a distinct ID of O(log n) bits.

• Each node knows all the edges it is incident to and their weights.

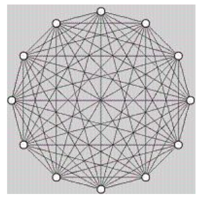
• Each node knows about all the other nodes.

• The synchronous model is used.



# I. Introduction of the Problem

## Synchronous Model



Communication advances in global rounds.

In each round, processes send messages, receive messages and do some local computation.

The time complexity is the number of rounds until the computation terminates in the worst case.

The message complexity is the number of messages exchanged in the worst case.

### Getting a Feeling for the Problem...

How hard is it to compute the MST in a distributed system (assuming a fully connected graph)?

All nodes know the weights of all incident edges. If all nodes send this information to all other nodes, then all nodes suddenly have the entire picture!

→ A simple algorithm that requires only one round!

However, that is not really interesting...

Therefore, the message size is limited to $O(\log n)$ bits!

Note that the previous algorithm requires messages of size $O(n \log n)$!





### Getting a Feeling for the Problem...

Since each node ID (and edge weight) requires $O(\log n)$ bits, this implies that only a constant number of node IDs (and edge weights) can be packed into a single message!

We demand that all nodes know the MST at the end of the computation!

How can the MST be constructed now?

→ Let's look at a simple algorithm first...

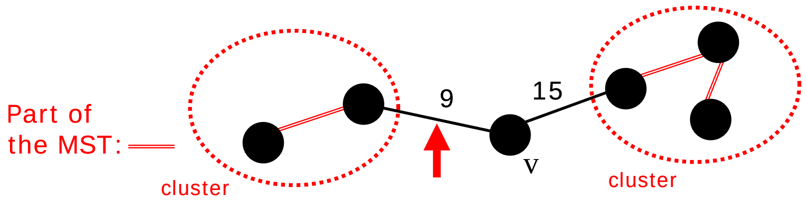




### A simple Algorithm

All MST algorithms (local or distributed) are based on the following lemma:

**Lemma 1**: It is always safe to add an edge to the spanning tree, if this edge is the MWOE of a node v.








### A simple Algorithm

**Phase k**: Code for node v in cluster F

**Input**: Set of chosen edges that build node clusters

1. Compute the MWOE
2. Send the MWOE to all nodes in the same cluster
3. Receive messages from the other nodes
4. If own MWOE is the lightest, then broadcast it to all other nodes and add this edge (→ All edges have to know all clusters after each round)
5. Receive other broadcast messages and add those edges as well

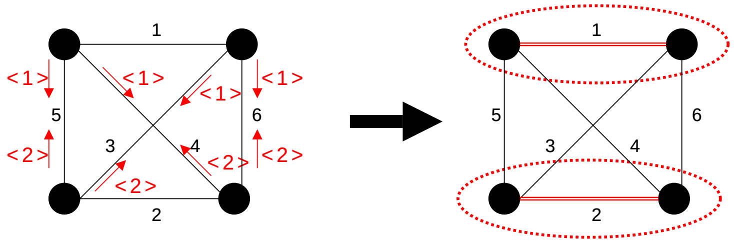
# I. Introduction of the Problem

## A simple Algorithm

Example

<w({v,u})> = <v,u,w({v,u})>

**Round 1:**




Broadcast the lightest edge to the other nodes

Add edges and update clusters



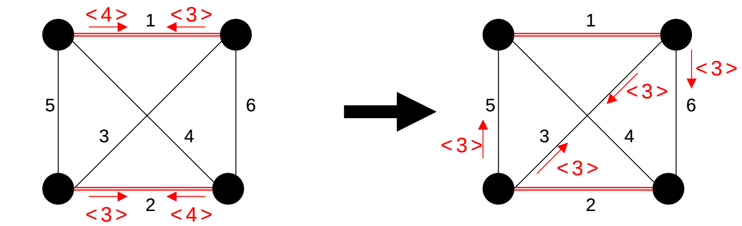
# I. Introduction of the Problem

## A simple Algorithm

Example

<w({v,u})> = <v,u,w({v,u})>

**Round 2:**




Send MWOE to all nodes in the same cluster

Broadcast the lightest edge to the other nodes



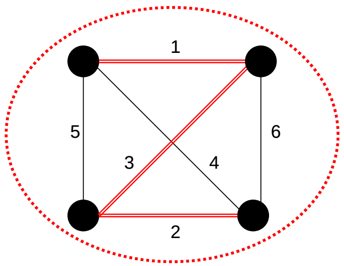
# I. Introduction of the Problem

## A simple Algorithm

Example

<w({v,u})> = <v,u,w({v,u})>

**Round 2:**




Add edges and update clusters

The algorithm is obviously correct. Since the minimum cluster size <u>doubles</u> in each round, the algorithm computes the MST in

<u>**O(log n)**</u> rounds!

**Can it be improved???**

**Lower bound???**



# Content of the Talk

I. Introduction of the Problem

II. Previous Results

- Lower and Upper Bounds
- Open Questions

III. The New Algorithm

IV. Analysis of the Algorithm

V. Summary

VI. Extensive Example

## II. Related Work

### Previous Results

Δ denotes the constant diameter of the graph, i.e. the maximum distance between any two nodes of the graph.

| | Known Algorithms | Known Lower Bounds |
|---|---|---|
| $\Delta = 1$ | O(log n) | ??? |
| $\Delta = 2$ | O(log n) | ??? |
| $\Delta \geq 3$ | $O(\sqrt{n})$ | $\Omega(n^{1/4})$ |



## II. Related Work

### Previous Results

Even our simple algorithm has this complexity!!

Interesting „jump“!

| | Known Algorithms | Known Lower Bounds |
|---|---|---|
| $\Delta = 1$ | O(log n) | ??? |
| $\Delta = 2$ | O(log n) | ??? |
| $\Delta \geq 3$ | $O(\sqrt{n})$ | $\Omega(n^{1/4})$ |



## II. Related Work

### Previous Results

We will now derive an algorithm with time complexity O(log log n)!

| | Known Algorithms | Known Lower Bounds |
|---|---|---|
| $\Delta = 1$ | O(log log n) | ??? |
| $\Delta = 2$ | O(log n) | ??? |
| $\Delta \geq 3$ | $O(\sqrt{n})$ | $\Omega(n^{1/4})$ |



## Content of the Talk

I. Introduction of the Problem

II. Previous Results

III. The New Algorithm

- General Idea & Problems
- The Algorithm Step by Step

IV. Analysis of the Algorithm

V. Summary

VI. Extensive Example

# III. The New Algorithm

## General Idea

In order to reduce the number of rounds, obviously clusters have to grow faster!

In our simple algorithm, we used the MWOE of each cluster to merge clusters.

With this approach, the minimum cluster size doubled in each phase.

It would certainly be faster, if the t lightest outgoing edges of each cluster were used, where t is in the order of the number of nodes in the cluster!

→ This is exactly what our new algorithm will do!



# III. The New Algorithm

## General Idea

In order to reduce the number of rounds, obviously clusters have to grow faster!

Let $\beta_k$ denote the minimum cluster size in phase k, then it holds for our simple algorithms that

$$\beta_{k+1} \geq 2 \cdot \beta_k$$

and

$$\beta_0 := 1$$

thus

$$2^k \leq \beta_k \leq n \rightarrow k = O(\log n)$$



# III. The New Algorithm

## General Idea

Our goal is to improve the following inequality:

$$\beta_{k+1} \geq 2 \cdot \beta_k$$

We will derive an algorithm for which it holds that:

$$\beta_{k+1} \geq \beta_k (\beta_k + 1)$$

Thus the cluster sizes grow quadratically as opposed to merely double in each phase! In order to achieve such a rate, information has to be spread faster!

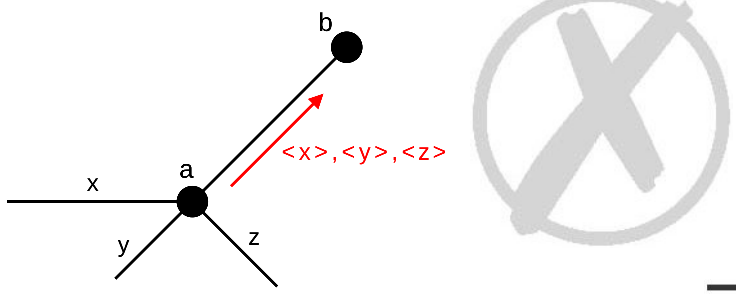
We will use a simple trick for that…



# III. The New Algorithm

## General Idea

Unfortunately, we cannot send a lot of information over a single link…



# III. The New Algorithm

## General Idea

However, we can send a lot of information from different nodes to a particular node $v_0$!

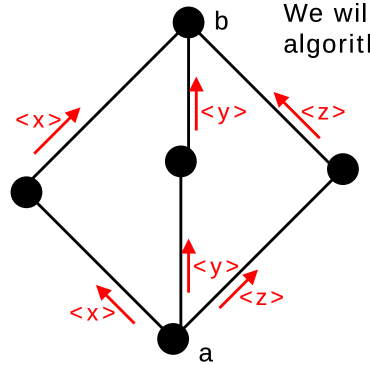
A node can simply send parts of the information, that it wants to transmit to a specific node, to other nodes. These nodes can send all parts to the specific node in one step!!

This can be used to <u>share workload</u>!!!



# III. The New Algorithm

## General Idea




We will use this trick twice in our algorithm!



# III. The New Algorithm

## General Idea

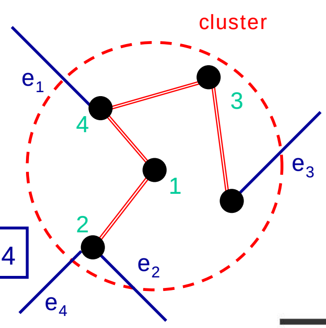
Our new algorithm will execute the following steps in each phase.

Let $\beta$ be the minimal cluster size.

1. Each cluster computes the $\beta$ lightest edges $e_1,\ldots,e_\beta$ to other distinct clusters

2. Assign at most one of those lightest edges to the members of the cluster!

$\beta$ = 4






# III. The New Algorithm

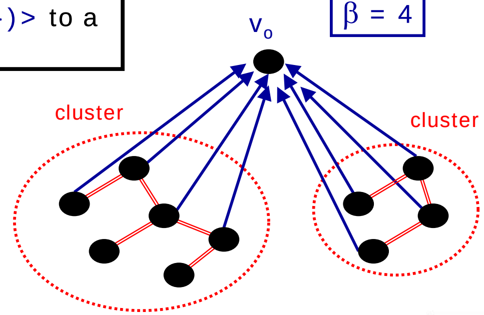
## General Idea

Step 2 and 3 together is exactly our trick!

3. Each node with an edge <v,u,w({v,u})> assigned to it, sends <v,u,w({v,u})> to a specific node $v_0$

4. Node $v_0$ computes the lightest edges that can be safely added to the spanning tree

$\beta$ = 4

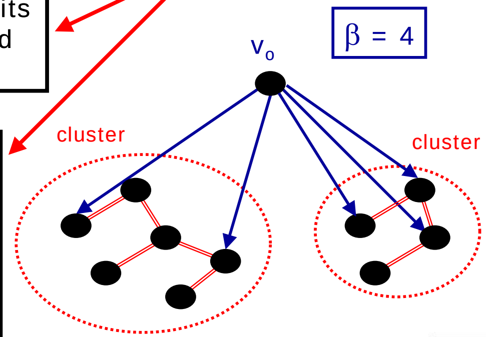


## General Idea

Our trick again!

5. Node $v_0$ sends a message to a node, if its assigned edge is added to the spanning tree

6. Each node, that received a message, broadcasts it to all other nodes (→ All nodes have to now about all added edges!)








## General Idea

This way, more edges can be added in one phase!

However, it is not clear yet how fast it really is...

Furthermore, we do not know yet how these steps work in detail!!!

→ There are a few obvious problems...





## Problems

First problem:

How can the $\beta$ lightest outgoing edges of a specific cluster be computed?

→ This is actually not so difficult. The procedure Cheap_Out in the actual algorithm copes with this problem. We will treat it in the following section.





## Problems

Second problem:

How can the designated node $v_0$ know which edges can be added without creating a cycle in the constructed graph?

Let's illustrate the problem with an example graph!

# III. The New Algorithm

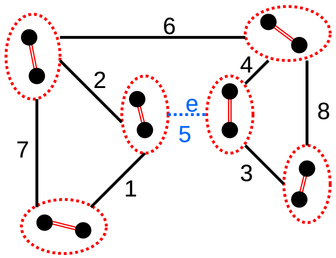
## Problems

In our example:

|V| = n = 12

β = 2 (minimum cluster size)

This is the picture of the designated node $v_0$ after receiving the β = 2 lightest outgoing edges of each cluster



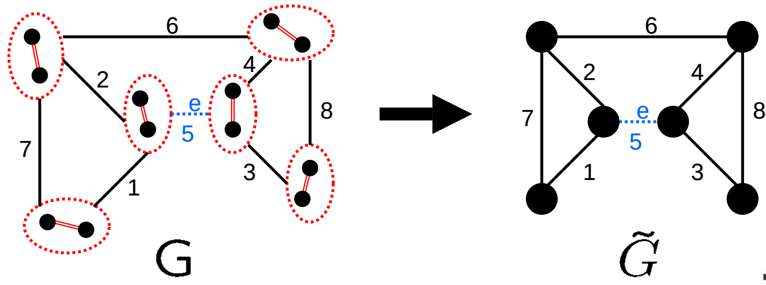

$v_0$ does not know about the edge e! It is the 3rd lightest edge of both adjacent nodes!



# III. The New Algorithm

## Problems

$v_0$ can construct a logical graph. Its nodes are the clusters and its edges are the β = 2 lightest outgoing edges.

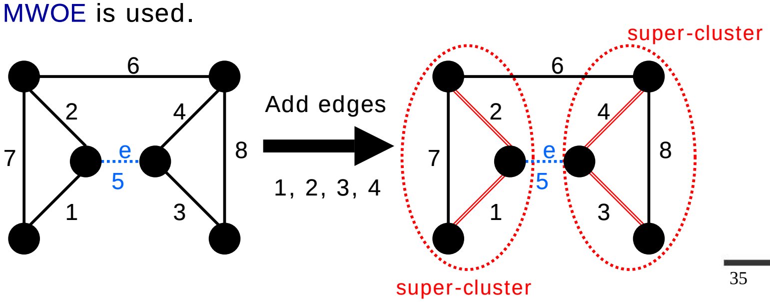





# III. The New Algorithm

## Problems

Based on the knowledge of the β = 2 lightest outgoing edges, $v_0$ can locally merge nodes of the logical graph into clusters. The 4 edges with weights 1, 2, 3 and 4 can be chosen safely, since always the MWOE is used.

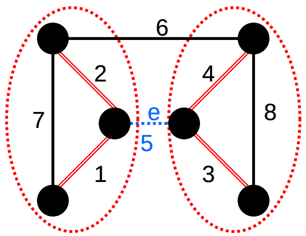





# III. The New Algorithm

## Problems

If the edge with weight 6 is used to finish the construction of the spanning tree, then the resulting tree is not the MST!!!




The problem is that in <u>both</u> (super-)clusters, at least one of the nodes has already used up all of its β outgoing edges. The (β+1)th outgoing edge might be lighter than other edges!!!

**So, when is it safe to add an edge???**

## III. The New Algorithm

### The Algorithm Step by Step

Let‘s put everything together and solve the open problems!

Initally, each node is itself a cluster of size 1 and no edges are selected.

The algorithm consists of 6 steps. Each step can be performed in constant time.

All 6 steps together build one phase of the algorithm, thus the time complexity of one phase is O(1).

A specific node in each cluster F, e.g. the node with minimum ID, is considered the leader $\ell(F)$ of the cluster F.



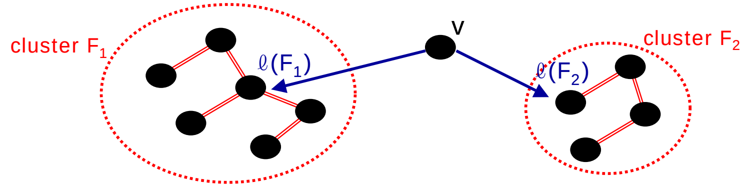
## III. The New Algorithm

### The Algorithm Step by Step

**Step 1**

(a) Each node v computes the minimum-weight edge e(v,F) that connects v to any node of cluster F for all clusters other than the own cluster

(b) Each node v sends e(v,F) to the leader $\ell(F)$ for all clusters other than the own cluster





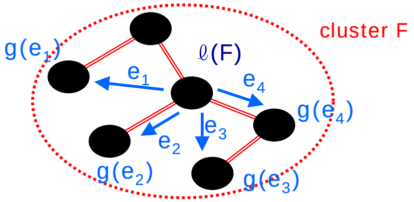

## III. The New Algorithm

### The Algorithm Step by Step

**Step 2**

(a) Each leader v of a cluster F (i.e. $\ell(F)$ =v) computes the lightest edge between F and every other cluster

(b) Each leader v performs procedure Cheap_Out → Selects the β lightest outgoing edges and appoints them to its nodes




If edge e is appointed to v, then v is denoted e‘s guardian g(e)



## III. The New Algorithm

### The Algorithm Step by Step

**Procedure Cheap_Out**

Code for the leader of cluster F

Input: Lightest edge e(F,F‘) for every other cluster F‘

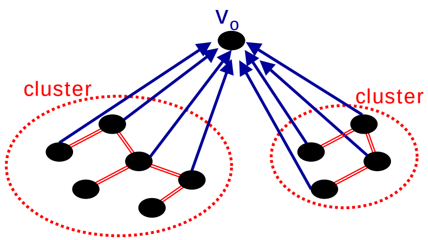
1. Sort the input edges in increasing order of weight
2. Define β = min{|F|, <# of clusters>}
3. Choose the first β edges of the sorted list
4. Appoint the node with the ith largest ID as the guardian of the ith edge, i = 1,...,β
5. Send a message about the edge to the node it is appointed to

# III. The New Algorithm

## The Algorithm Step by Step

**Step 3**

All nodes, that are guardians for a specific edge, send a message to the designated node $v_0$, e.g. the node with the minimal ID in the graph




$v_0$ knows the $\beta$ lightest outgoing edges of each cluster!



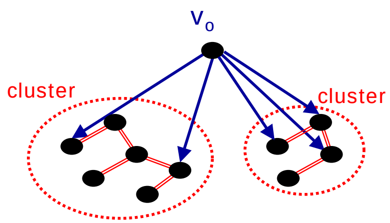
# III. The New Algorithm

## The Algorithm Step by Step

**Step 4**

(a) $v_0$ locally performs procedure Const_Frags → Computes the edges to be added

(b) For all added edges, $v_0$ sends a message to g(e)





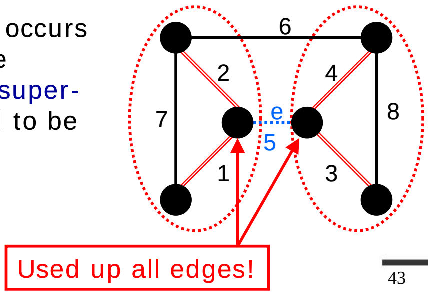

# III. The New Algorithm

## The Algorithm Step by Step

**How does Const_Frags work?**

As we've seen before, a problem occurs when all $\beta$ outgoing edges of a cluster are used up!

More precisely, a problem occurs <u>only if</u> there is at least one cluster <u>in each</u> of the two super-clusters that are supposed to be merged!!





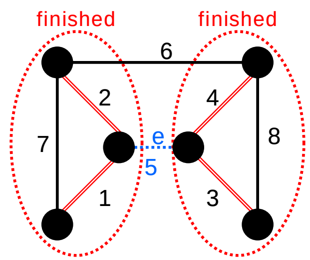

# III. The New Algorithm

## The Algorithm Step by Step

**How does Const_Frags work?**

We call a super-cluster containing a cluster that used up all of its $\beta$ edges finished.

If an edge is the lightest outgoing edge of one super-cluster that is <u>not finished</u>, then it is still safe to add it, no matter if the other super-cluster is finished, since we are sure that there is no better edge to connect the unfinished super-cluster to other clusters.



## III. The New Algorithm

### The Algorithm Step by Step

**Procedure Const_Frags**

Code for the designated node $v_0$

Input: the $\beta$ lightest outgoing edges of each cluster

1. Construct the logical graph
2. Sort the input edges in increasing order of weight
3. Go through the list, starting with the lightest edge:

   If the edge can be added without creating a cycle then

      add it

   else

      drop it



## III. The New Algorithm

### The Algorithm Step by Step

**Procedure Const_Frags**

If the two super-clusters were merged then the new super-cluster is declared <u>finished</u> if

• the edge is the heaviest edge of a cluster in any of the two super-clusters or

• any of the two super-clusters is already finished

If the edge is dropped (→ both clusters already belong to the same super-cluster) then the super-cluster containing the clusters the edge leads to is declared <u>finished</u> if

• the edge is the heaviest edge of any of the two clusters



## III. The New Algorithm

### The Algorithm Step by Step

**Procedure Const_Frags**

Note: If a super-cluster is declared finished then it will remain finished

All edges between finished super-clusters are deleted (before looking at the next lightest edge)

Those are the dangerous edges!



## III. The New Algorithm

### The Algorithm Step by Step

**Step 5**

All nodes, that received a message from $v_0$, broadcast their edge to all other nodes

**Step 6**

Each node adds all edges and computes the new clusters

If the number of clusters is greater than 1, then the next phase starts

# III. The New Algorithm

## The Algorithm Step by Step

**The entire algorithm for node v in cluster F**

1. Compute the minimum-weight edge e(v,F') that connects v to cluster F' and send it to $\ell$(F') for all clusters F' $\neq$ F
2. if v = $\ell$(F): Compute lightest edge between F and every other cluster. Perform Cheap_Out
3. if v = g(e) for some edge e: Send <e> to $v_0$
4. if v = $v_0$: Perform Const_Frags. Send message to g(e) for each added edge e
5. if v received a message from $v_0$: Broadcast it
6. Add all received edges and compute the new clusters



# Content of the Talk



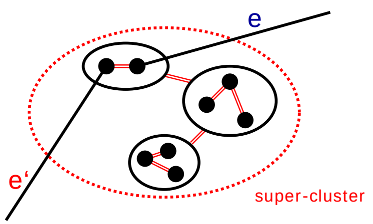


# IV. Analysis of the Algorithm

## Correctness

It suffices to show that whenever an edge is added, it is part of the MST → We only have to analyze Const_Frags!

Proof [Sketch]: We always add the lightest outgoing edge of each super-cluster. Because of Lemma 1, this is always the right choice! Therefore, we only have to watch out that we choose the right edges from the various clusters (since we only know some of the outgoing edges)!!!



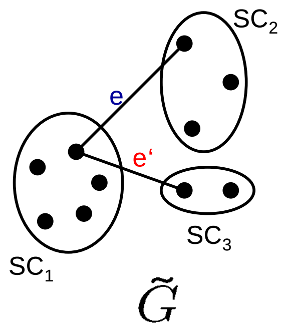



# IV. Analysis of the Algorithm

## Correctness

Assume edge e is used to merge super-cluster $SC_1$ and $SC_2$. Further assume there is a lighter outgoing edge e' (w(e') < w(e)) that connects super-cluster $SC_1$ with another super-cluster, say, $SC_3$.

e' is incident to the same cluster in $SC_1$ as e.




● is a cluster

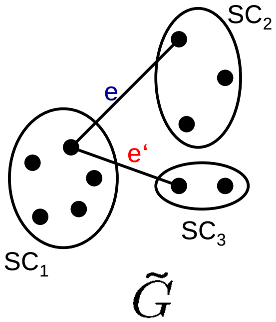
# IV. Analysis of the Algorithm

## Correctness

Case 1: e‘ is among the $\beta$ lightest outgoing edges

Since w(e‘) < w(e), e‘ must have been considered before e, thus either $SC_1$ and $SC_3$ have been merged before or e‘ was dropped because $SC_1 = SC_3$. Eitherway e‘ cannot be an outgoing edge when the algorithm adds e.

→ Contradiction!






# IV. Analysis of the Algorithm

## Correctness

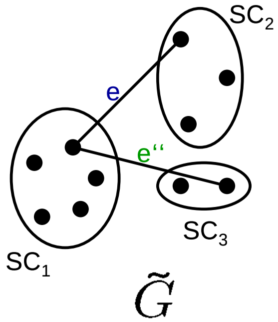
Case 2: e‘ is <u>not</u> among the $\beta$ lightest outgoing edges

Case 2.1: There is a lighter edge e‘‘ from the cluster to the super-cluster $SC_3$:

It follows that w(e‘‘) < w(e‘). Since $SC_1 \neq SC_3$, e‘‘ has not been considered yet, thus w(e) < w(e‘‘). It follows that

w(e) < w(e‘).

→ Contradiction!






# IV. Analysis of the Algorithm

## Correctness

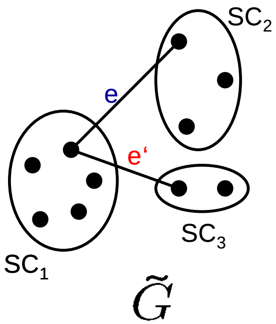
Case 2: e‘ is <u>not</u> incident to the same cluster as e

Case 2.2: There is <u>no</u> lighter edge from the cluster to the super-cluster $SC_3$:

Thus all $\beta$ outgoing edges have lower weights than e‘. In particular, this holds also for e, thus w(e) < w(e‘).

→ Contradiction!

■






# IV. Analysis of the Algorithm

## Time Complexity

Each phase requires O(1) rounds, but how many phases are required until termination?

Reminder: $\beta_k$ denotes the minimum cluster size in phase k.

**Lemma 2**: It holds that

$$\beta_{k+1} \geq \beta_k (\beta_k + 1).$$

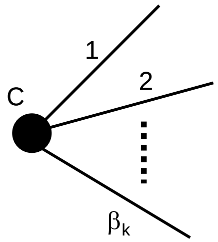
# IV. Analysis of the Algorithm

## Time Complexity

Proof [Sketch]:

We prove a stronger claim: Whenever a super-cluster is declared finished in phase k+1, it contains at least $\beta_k$ + 1 clusters.

Each cluster has (at least) $\beta_k$ outgoing edges in phase k + 1, since the $\beta_k$ is the minimal cluster size after phase k.





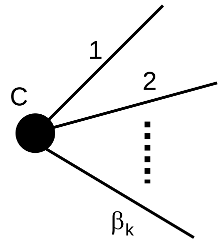

# IV. Analysis of the Algorithm

## Time Complexity

**Case 1**: The super-cluster is declared finished after one of its clusters has used up all of its $\beta_k$ outgoing edges. Let C be that cluster.

Let‘s call those edges edge 1, 2, ..., $\beta_k$ and the clusters they are leading to $C_1$, $C_2$,..., $C_\beta$.

If the inspection of an edge <u>does not</u> result in a merge, then the clusters already belong to the same super-cluster! If there is a merge, then they belong to the same super-cluster afterwards.





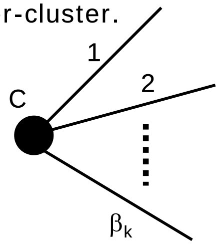

# IV. Analysis of the Algorithm

## Time Complexity

Thus, at the end, the super-cluster contains at least C, $C_1$, $C_2$,..., $C_\beta$!! → The super-cluster contains at least $\beta_k$ + 1 clusters.

**Case 2**: The super-cluster is declared finished after merging with an already finished super-cluster.

Using an inductive argument, the finished super-cluster must already contain at least $\beta_k$ + 1 clusters, since one of its clusters has used up all of its $\beta_k$ edges. ■






# IV. Analysis of the Algorithm

## Time Complexity

**Theorem 1:** The time complexity is O(log log n) rounds.

Proof: By Lemma 2, it holds that $\beta_{k+1} \geq \beta_k (\beta_k + 1)$. Furthermore it holds that $\beta_0$ := 1. Hence it follows that

$$\beta_k \geq 2^{2^{k-1}}$$

for every k ≥ 1. Since $\beta_k \leq n$, it follows that k ≤ log(log n) +1. Since each phase requires O(1) rounds, the time complexity is O(log log n). ■

# IV. Analysis of the Algorithm

## Message Complexity

**Theorem 2:** The message complexity is $O(n^2 \log n)$.

The proof is simple: Count the messages exchanged in steps 1, 3, 4 and 5. We will not do that here.

Adler et al. showed that the minimal number of bits required to solve the MST problem in this model is $\Omega(n^2 \log n)$, thus this algorithm is optimal!!!



# Content of the Talk

I. Introduction of the Problem

II. Previous Results

III. The New Algorithm

IV. Analysis of the Algorithm

V. Summary

- Results & Conclusions

VI. Extensive Example



# V. Summary

## Results

The presented algorithm solves the MST problem in the given model in $O(\log \log n)$ rounds.

The algorithm sends $O(n^2 \log n)$ bits in total, which is asymptotically optimal.



# V. Summary

## Additional Results

The algorithm also yields nice results, when the number of bits is not limited to $O(\log n)$:

**Theorem (Larger Messages):** If the message size is $O(n^\varepsilon \log n)$ for some $\varepsilon > 0$, then the number of rounds is $O(\log 1/\varepsilon)$ and the total number of bits communicated is $\Theta(n^2 \log n)$.

# V. Summary

## Conclusions

O(log log n) is a nice result!

Much faster than O(log n)...

The algorithm consists of a few well defined steps and is easy to understand. ☺

There are (many) open questions, though...



# V. Summary

## Conclusions

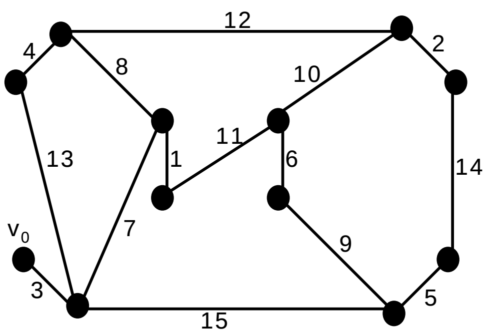
*„An obvious question we leave open is whether the algorithm can be improved, or is there an inherent lower bound of $\Omega$(log log n) on the number of communication rounds required to construct an MST in this model.“*

- Is there a faster algorithm?
- Is $\Omega$(log log n) a lower bound?
- Is there an algorithm with time complexity O(log log n) for graphs of diameter 2?



# VI. Extensive Example

## The graph

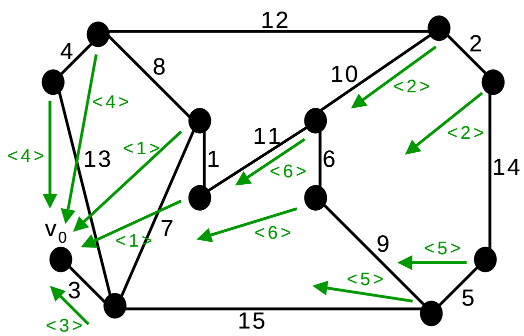



**All other edges are heavier!!!**



# VI. Extensive Example

## Phase 1

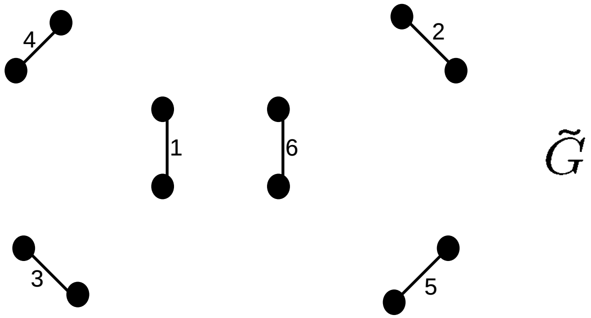



1. Not necessary
2. Not necessary
3. Send MWOE to $v_0$
4. Const_Frags!

# VI. Extensive Example

## Phase 1 – Const_Frags

1. Construct logical graph

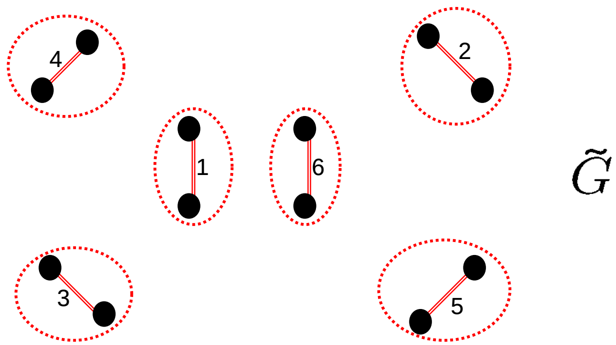





# VI. Extensive Example

## Phase 1 – Const_Frags

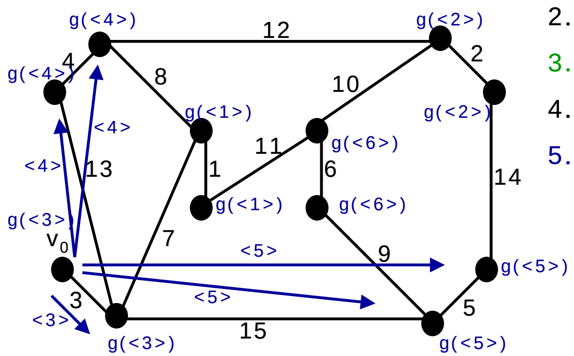
2. Add edges






# VI. Extensive Example

## Phase 1



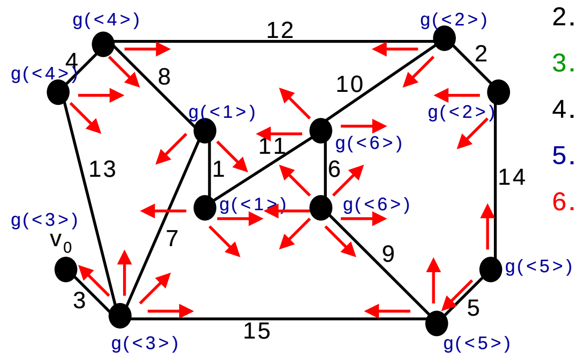

Only some messages are displayed

1. Not necessary
2. Not necessary
3. Send MWOE to $v_0$
4. Const_Frags!
5. Send e to g(e)



# VI. Extensive Example

## Phase 1



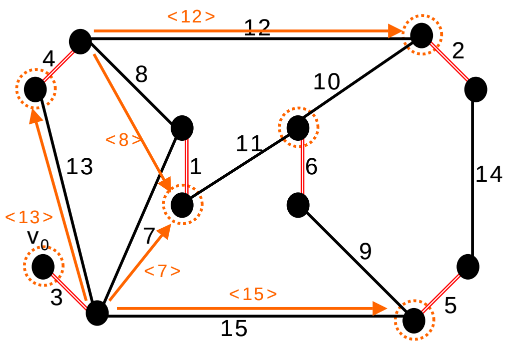

Only some messages are displayed

1. Not necessary
2. Not necessary
3. Send MWOE to $v_0$
4. Const_Frags!
5. Send e to g(e)
6. Broadcast e and update the clusters

# VI. Extensive Example

## Phase 2

1. Compute e(v,F‘) and send it to ℓ(F‘)



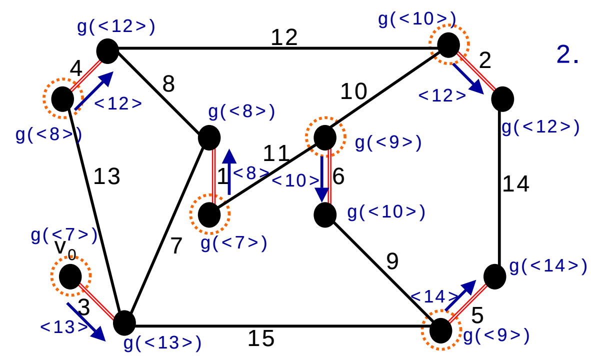

Only some messages are displayed



# VI. Extensive Example

## Phase 2

1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select $\beta$ = 2 lightest outgoing edges and appoint guardians

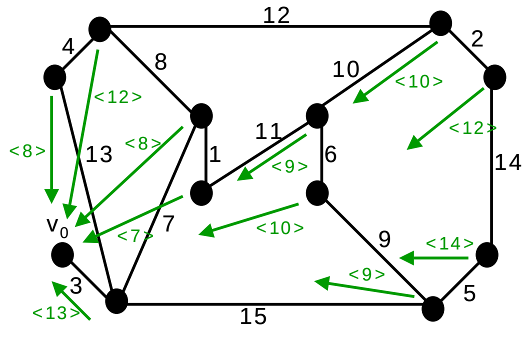





# VI. Extensive Example

## Phase 2

1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select $\beta$ = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to $v_0$
4. Const_Frags!

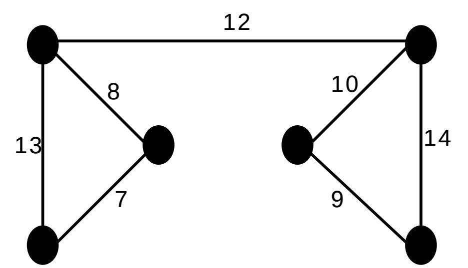





# VI. Extensive Example

## Phase 2 – Const_Frags

1. Construct logical graph

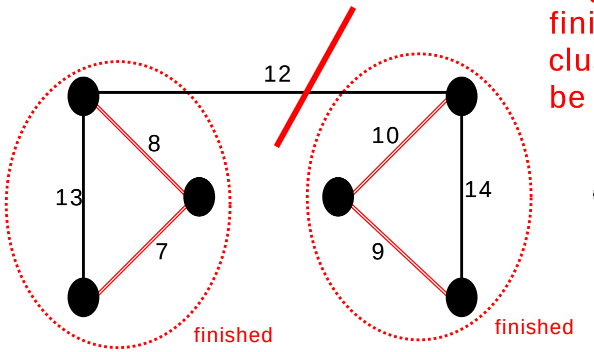



$\tilde{G}$

# VI. Extensive Example

## Phase 2 – Const_Frags

2. Add edges



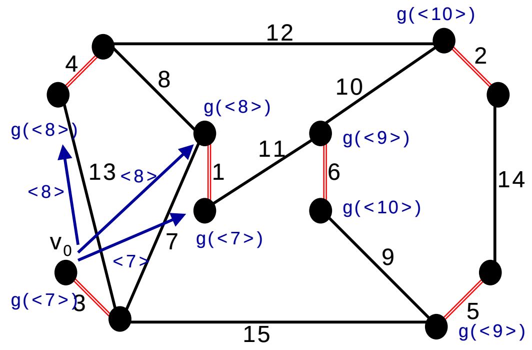

$\tilde{G}$

Edges between finished super-clusters must not be added!



# VI. Extensive Example

## Phase 2



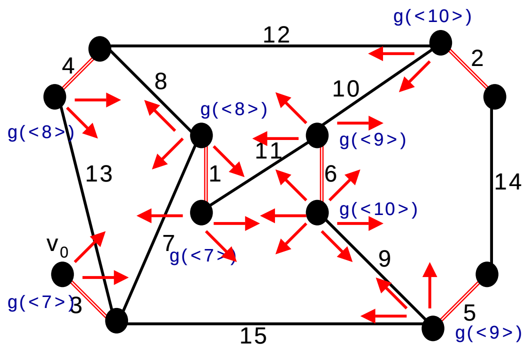

1. Compute e(v,F') and send it to ℓ(F')
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to $v_0$
4. Const_Frags!
5. Send e to g(e)

Only some messages are displayed



# VI. Extensive Example

## Phase 2



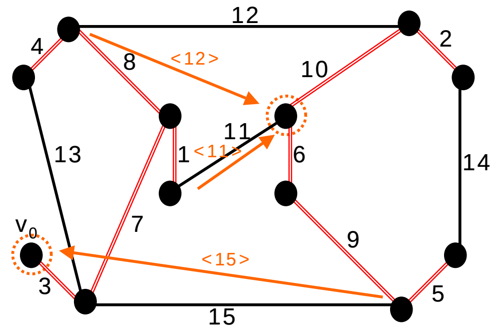

1. Compute e(v,F') and send it to ℓ(F')
2. Select β = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to $v_0$
4. Const_Frags!
5. Send e to g(e)
6. Broadcast e and update the clusters

Only some messages are displayed



# VI. Extensive Example

## Phase 3

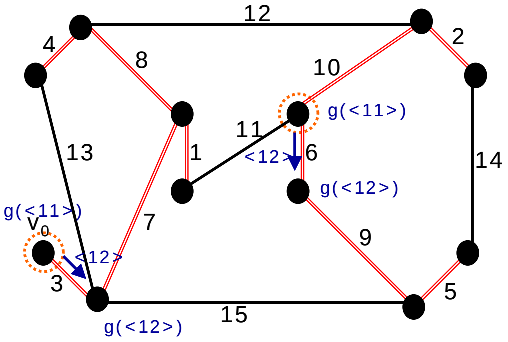



1. Compute e(v,F') and send it to ℓ(F')

Only some messages are displayed

## VI. Extensive Example

### Phase 3

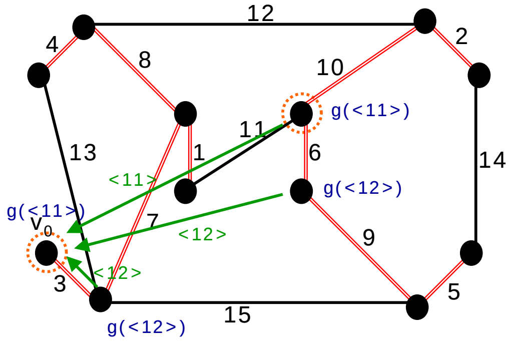



1. Compute e(v,F') and send it to $\ell$(F')
2. Select $\beta$ = 2 lightest outgoing edges and appoint guardians



## VI. Extensive Example

### Phase 3



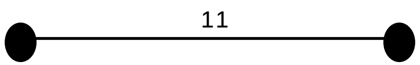

1. Compute e(v,F') and send it to $\ell$(F')
2. Select $\beta$ = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to $v_0$
4. Const_Frags!



## VI. Extensive Example

### Phase 3 – Const_Frags

1. Construct logical graph



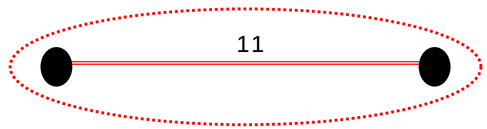

$\tilde{G}$



## VI. Extensive Example

### Phase 3 – Const_Frags

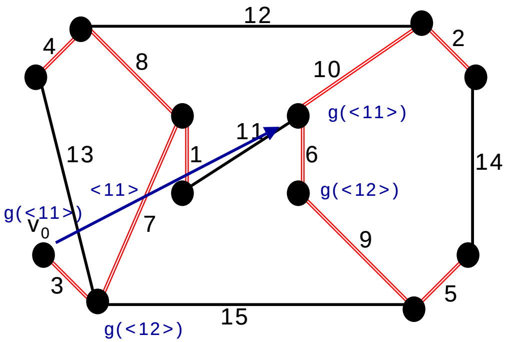
2. Add edges




$\tilde{G}$

# VI. Extensive Example

## Phase 3

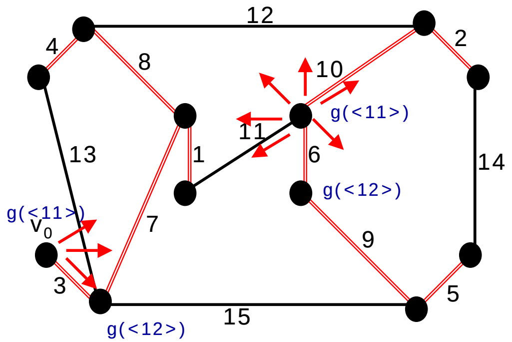



1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select $\beta$ = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to $v_0$
4. Const_Frags!
5. Send e to g(e)



# VI. Extensive Example

## Phase 3



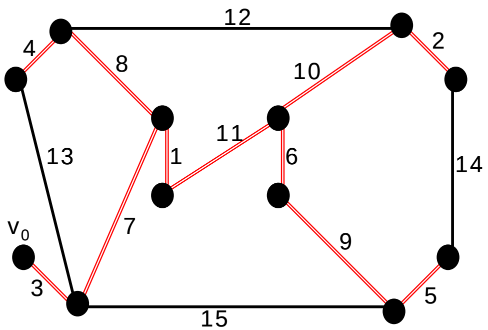

1. Compute e(v,F‘) and send it to ℓ(F‘)
2. Select $\beta$ = 2 lightest outgoing edges and appoint guardians
3. Send appointed edge to $v_0$
4. Const_Frags!
5. Send e to g(e)
6. Broadcast e and update the clusters

Only some messages are displayed



# VI. Extensive Example

## After phase 3




**Done!** ☺



# That‘s all Folks!