On the Power Assignment Problem in Radio Networks

Luzius Meisser

Betreuung: Thomas Moscibroda

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- Problem
- Paper, Results
- Trade-Off Hops vs. Power
- Complexity
- Conclusions

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Problem

N stations in mobile network, minimize power consumption while preserving connectivity with at most h hops.

→ Min d-D h-Range Assignment

Problem

Definition:

- Nodes in d-dimensional space
- Power consumption = f(sending distance), $p = \alpha*(d^\beta)$
- Minimize Total Power Consumption
- Constraint: max h hops

→ Min d-D h-Range Assignment

If $h=\infty$, Min d-D Range Assignment

Problem

Questions:

What‛s the minimal total power consumption for a given h?

What‛s the computational complexity of finding the optimal range assignment?

Are there good approximations?

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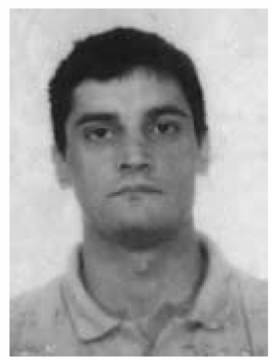
Paper

„On the power assignment problem in radio networks“


Adrea E.F. Clementi


Paolo Penna


Riccardo Silvestri

Clementi – Penna – Silvestri

University of Rome - 2000

Result: Complexity

Paper says:

Problem version	Previous results	Our results
MIN 1D RANGE ASSIGNMENT	in P [16]	–
MIN 2D RANGE ASSIGNMENT	in APX [16]	NP-complete
MIN 2D h-RANGE ASSIGNMENT, well-spread	–	in APX
MIN 2D h-RANGE ASSIGNMENT	–	in Av-APX
MIN 3D RANGE ASSIGNMENT	NP-complete, in APX [16]	APX-complete

APX: Class of NP Problems that are approximable to a constant factor in polynomial time.

[details later]

Result: Hops vs. Power

Paper proves:

Optimal 2D h-Range Assignment is in

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

What does that mean?

Result: Hops vs. Power

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

S: Set of nodes

h: maximal number of hops

δ(S): minimal distance between two nodes

Result: Hops vs. Power

Example:

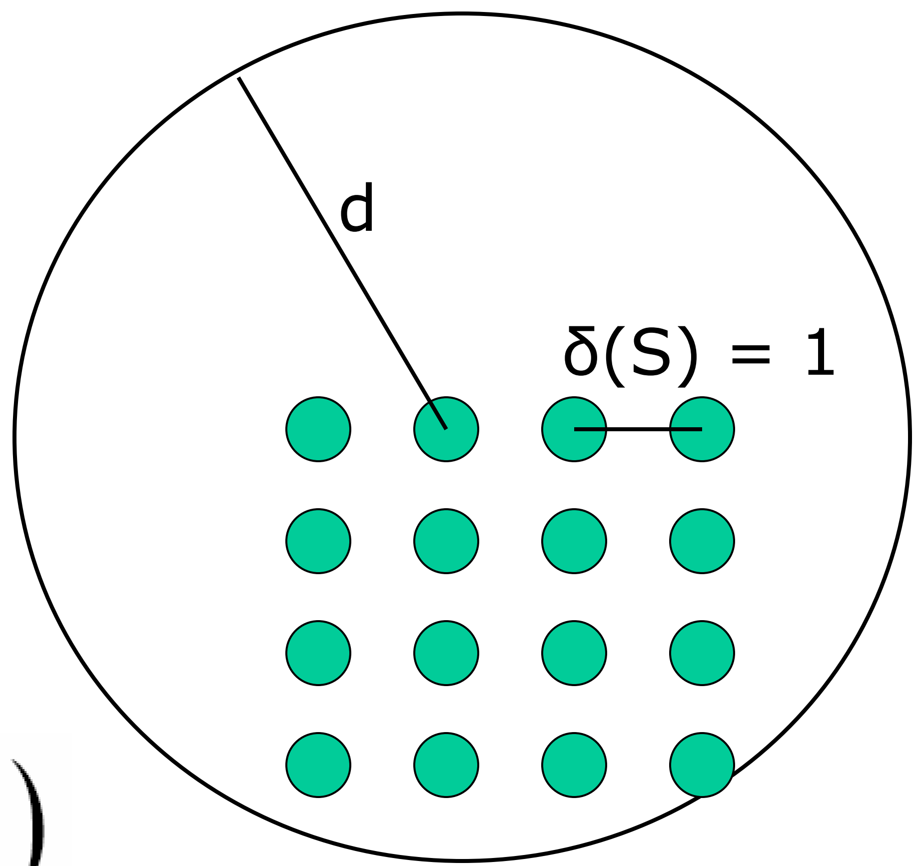
h = 1

δ(S) = 1

|S| = n

→ whiteboard

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$



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Hops vs. Power

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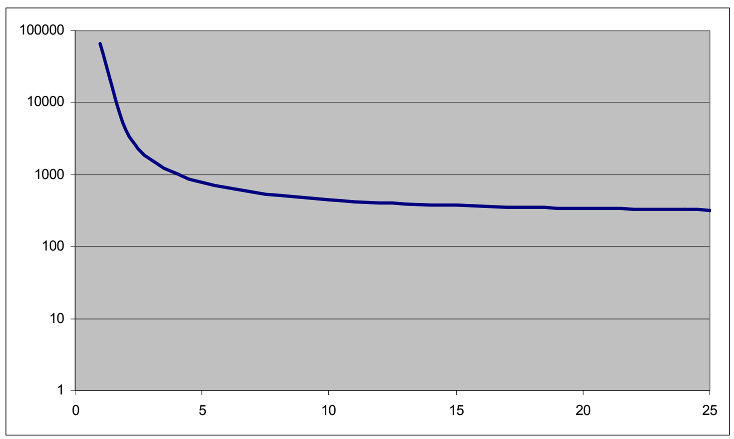
Hops vs. Power

We want to show:

Optimal 2D h-Range Assignment is in



$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

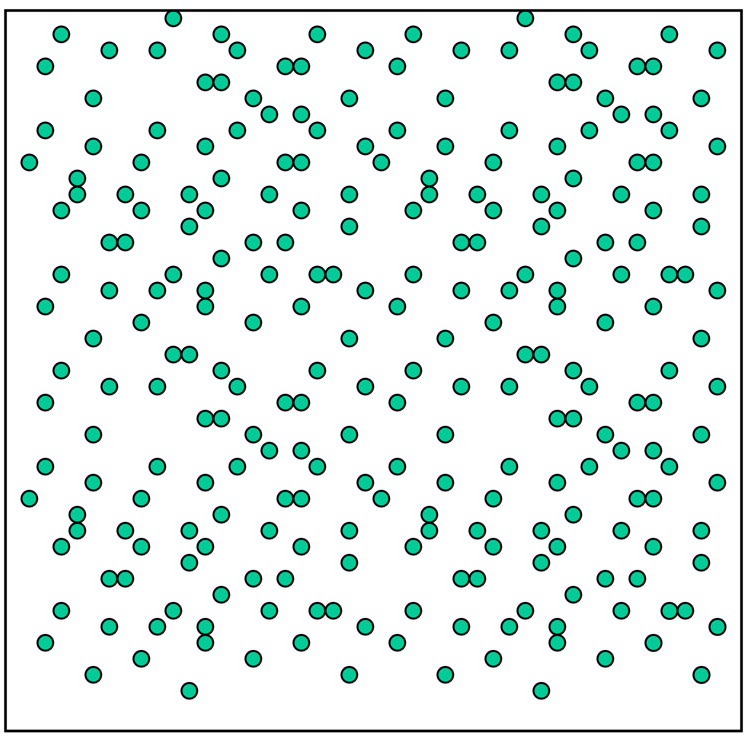
Hops vs. Power

Oh-Notation

- Lower bound: n^2 is in $\Omega(n)$
- Upper bound: 1 is in $O(n)$
- Optimum: n is in $\Theta(n)$ because n is in both, in $\Omega(n)$ and $O(n)$

Hops vs. Power

Power assignment algorithm:

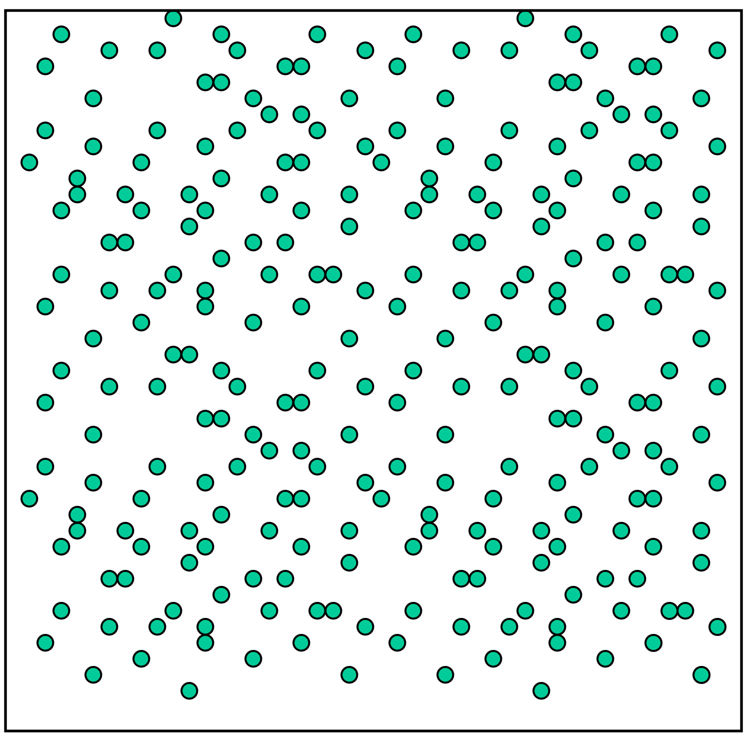


Set h=2, n=256,

Side of square: l

Hops vs. Power

Power assignment algorithm:



Set h=2, n=256,

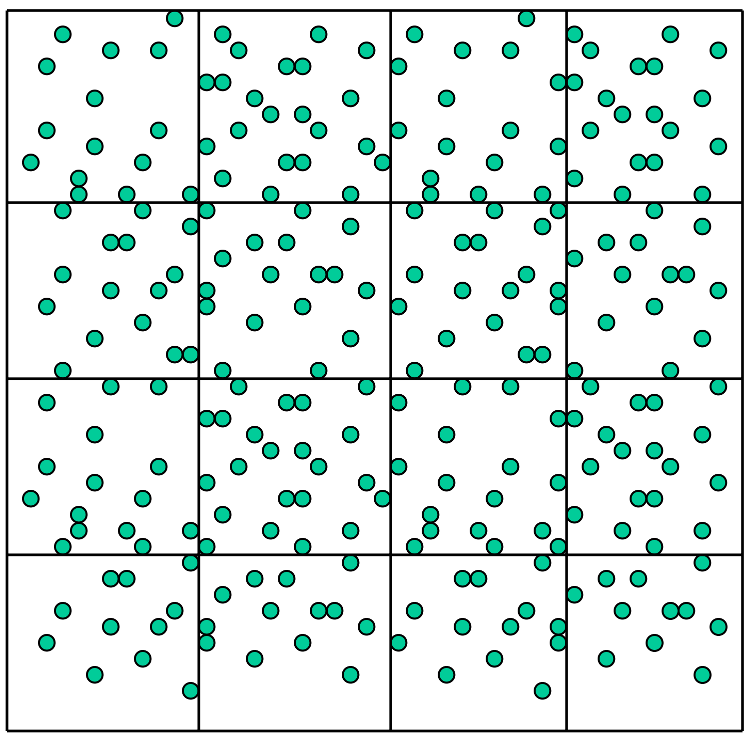
Side of square: l

Divide area into k² subsquares, with k = n^(1/2h)

→ k = 4

Hops vs. Power

Power assignment algorithm (centralized):



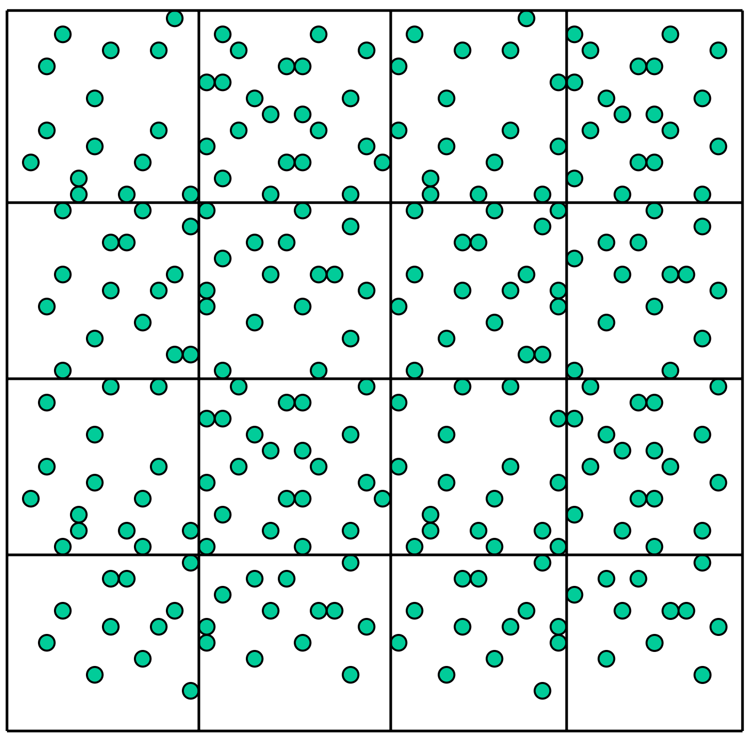
h=2, n=256

k = 4

→ 16 squares

Hops vs. Power

Power assignment algorithm (centralized):

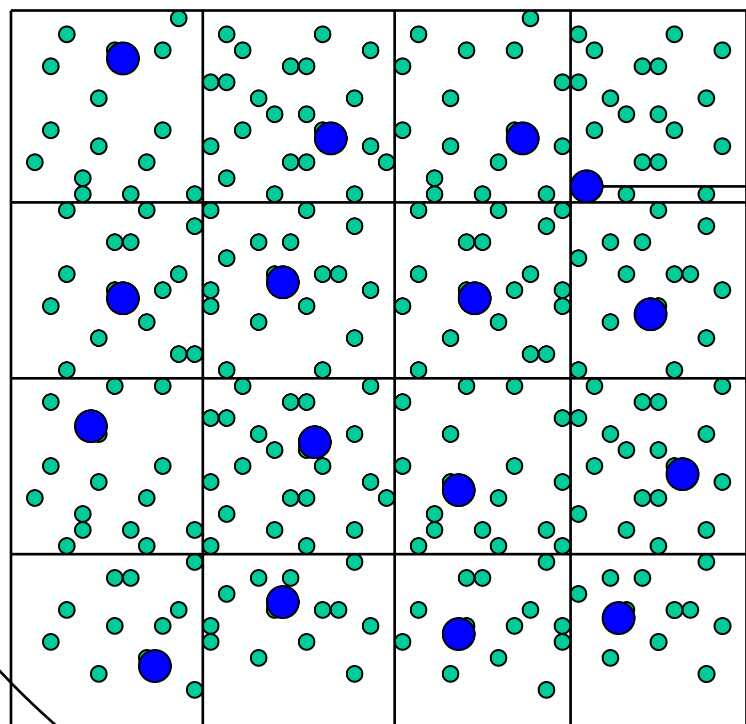


h=2, n=256

Choose 1 station in each square and give it global transmission range.

Hops vs. Power

Power assignment algorithm (centralized):



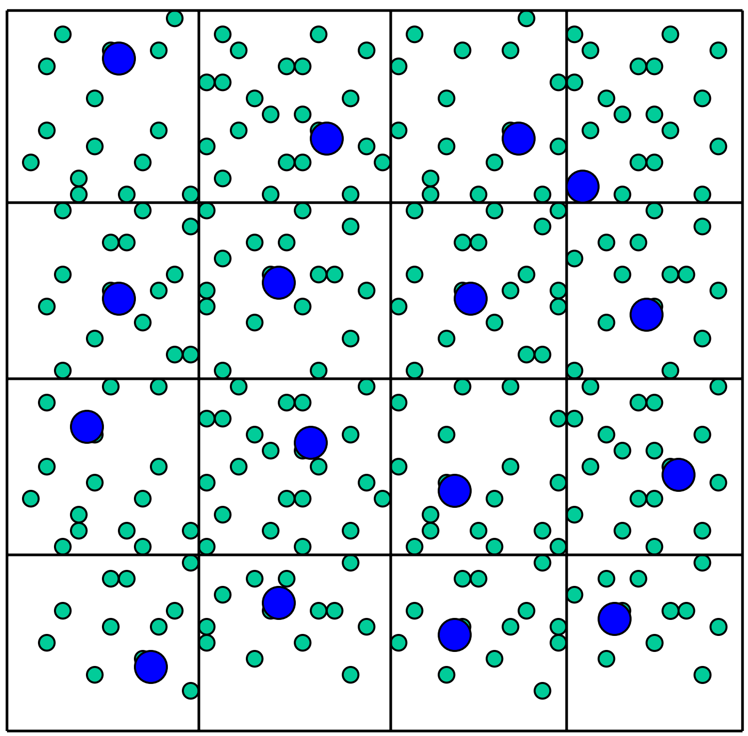
h=2, n=256

→ All the blue nodes are connected.

Cost so far: $l^2 * k^2$

Hops vs. Power

Power assignment algorithm (centralized):

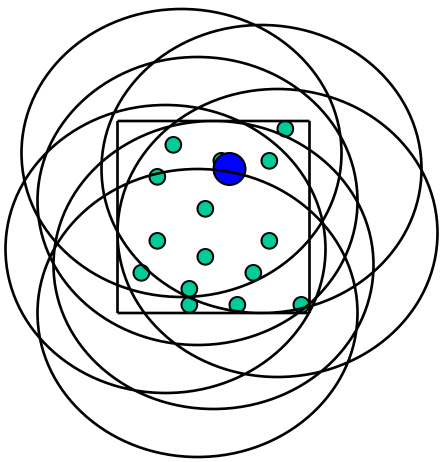


h=2, n=256

Now recursively solve the problem in each subsquare with h decreased by 1.

Hops vs. Power

Power assignment algorithm (centralized):



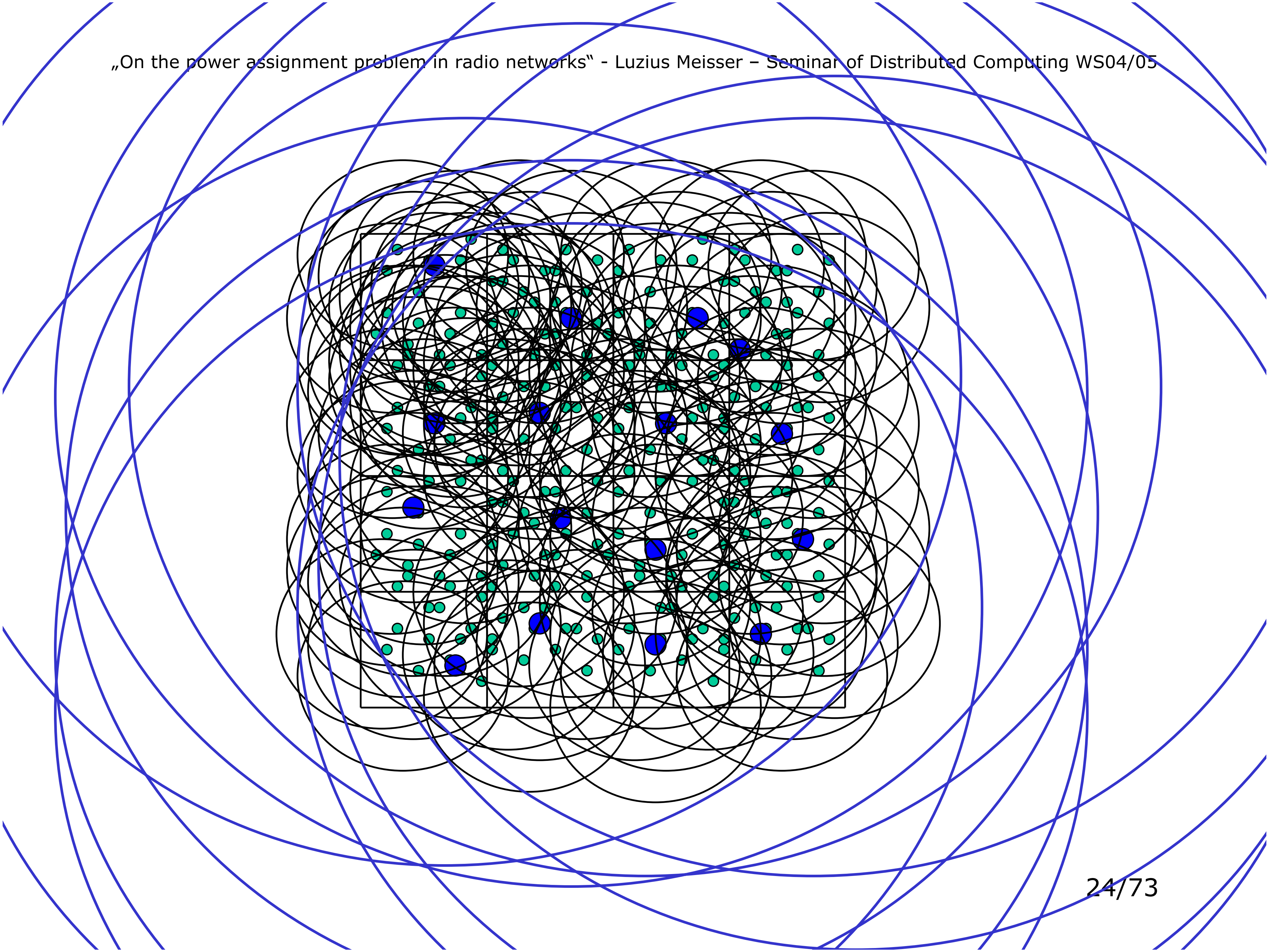
Sub-Problem:

$h=1$, $n=16$

→ all nodes get a range of l/k

→ cost of all subsquares:

$k^2 * (n/k^2) * (l/k)^2 = n * (l/k)^2$



Hops vs. Power

Power assignment algorithm (centralized):

→ All nodes are connected with at most 2 hops.

Total Cost:

$l^2 * k^2 + n*(l/k)^2$

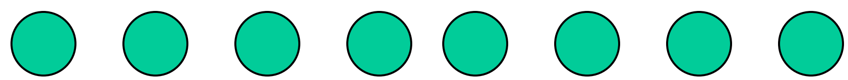
$= l^2*(n\hat{}(1/2)) + (n\hat{}(1/2))*l^2$

$= 2*l^2 *(n\hat{}(1/2))$

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

Hops vs. Power

But what about this network?



Hops vs. Power

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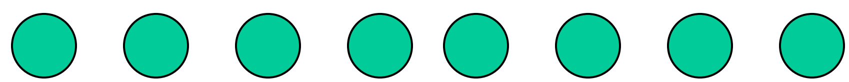
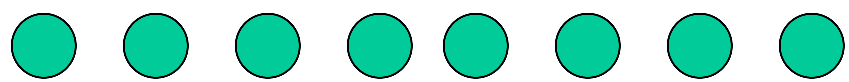


→ cost is in $O(n*(\delta(S)*n)^2) = O(n^3 * \delta(S)^2)$

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

Hops vs. Power

But what about this network?



→ cost is in $O(n*(\delta(S)*n)^2) = O(n^3* \delta(S)^2)$

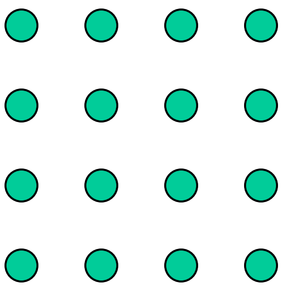
→ formula only holds for „well-spread“ instances.

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

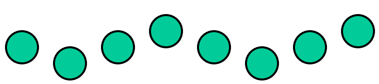
Hops vs. Power

Well Spread: D(S) = O(δ(S)*S^(1/2))

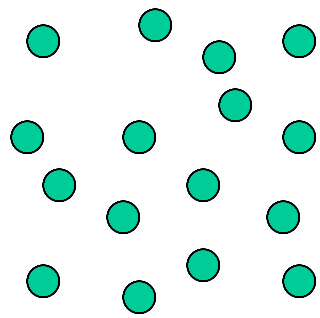
→ idea: close to grid



Perfectly „well spread“

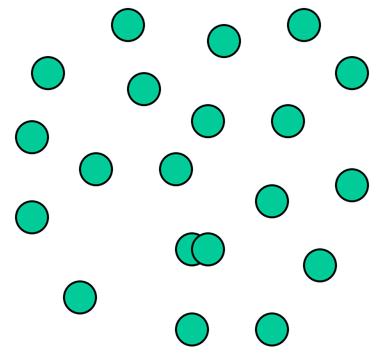


Not „well spread“



„well spread“

(obtained by moving the grid nodes a little)



Not „well spread“,

Randomly distributed on a square

Hops vs. Power

For well spread instances:

$$\Theta\left(\delta(S)^2 |S|^{1+1/h}\right)$$

For instances that are randomly distributed on a square:

$$\Theta\left(l^2 n^{1/h}\right)$$

[With high probability]

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Intermezzo

Flashback

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APX: Class of NP Problems that are approximable to a constant factor in polynomial time.

[details later]

Intermezzo

Complexity Classes

Short overview over the classes P, NP, APX, as well as over the concepts of Hardness and Completeness.

Intermezzo

Complexity Classes

P: Class of Problems that can be solved in polynomial time.

Intermezzo

Complexity Classes

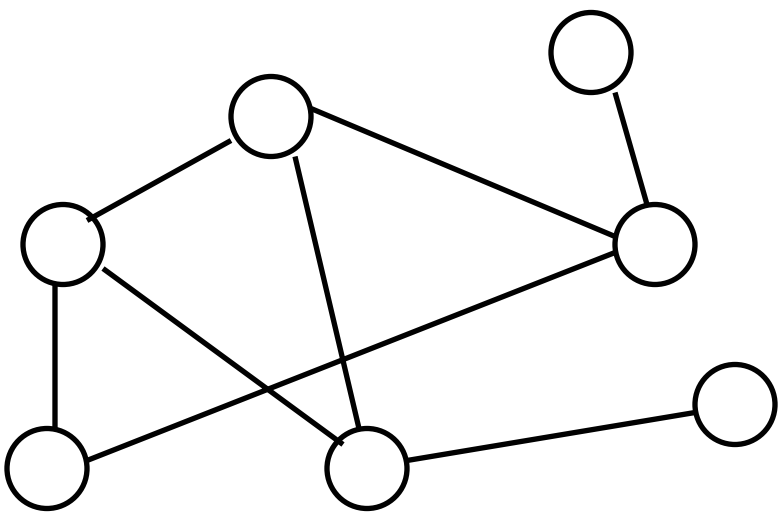
NP (or NPO): Class of Problems whose objective function can be calculated in polynomial time.

Intermezzo

Complexity Classes

Example of an NP problem:

Min Vertex Cover



Given a graph:

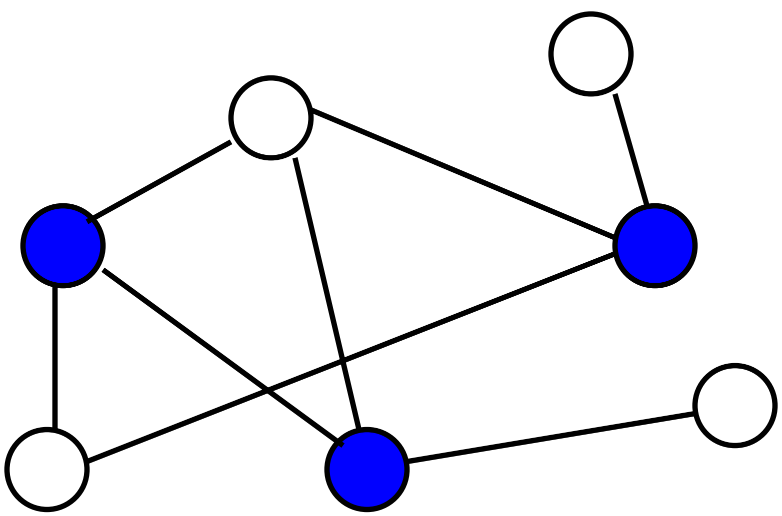
Color the minimal number of vertices blue such that every edge is connected to a blue vertex.

Intermezzo

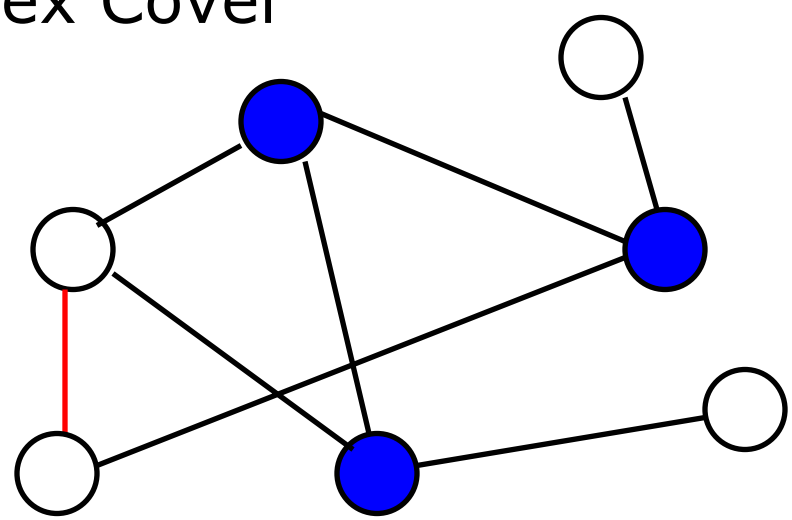
Complexity Classes

Example of an NP problem:

Min Vertex Cover



cover



no cover

Intermezzo

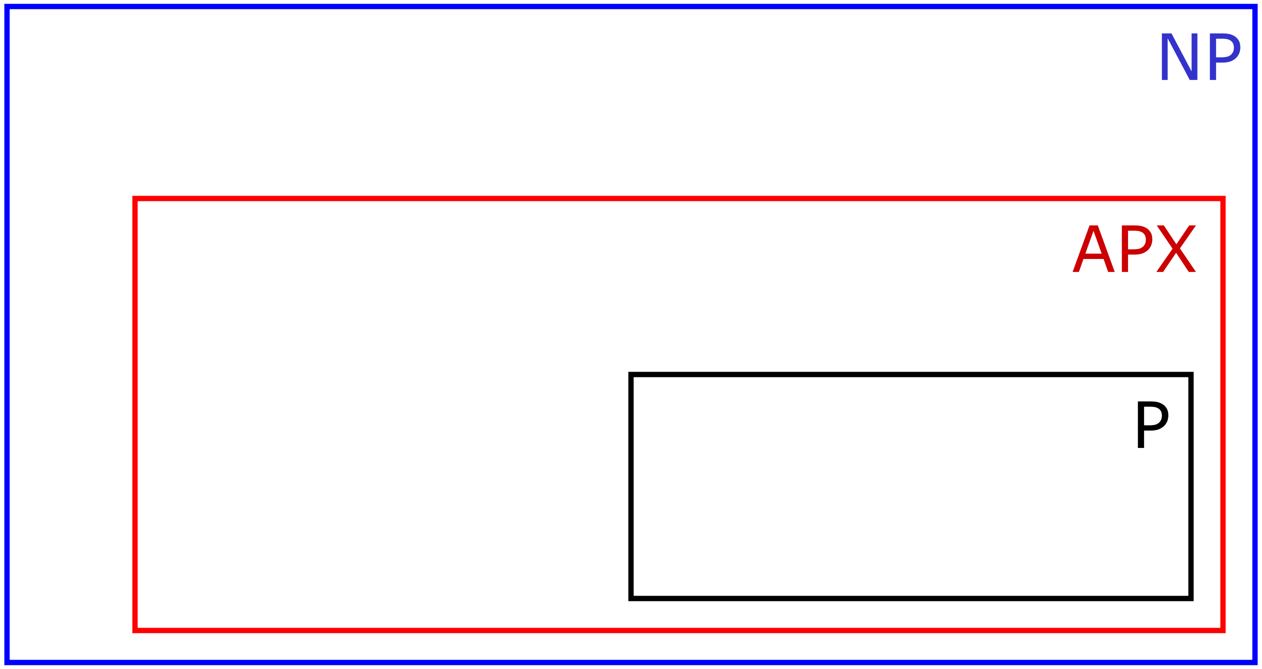
Complexity Classes

APX: Class of NP Problems that are approximable to a constant factor in polynomial time.

Examples: Min Vertex Cover restricted to cubic graphs, Travelling Salesman in Euclidean Space

Intermezzo

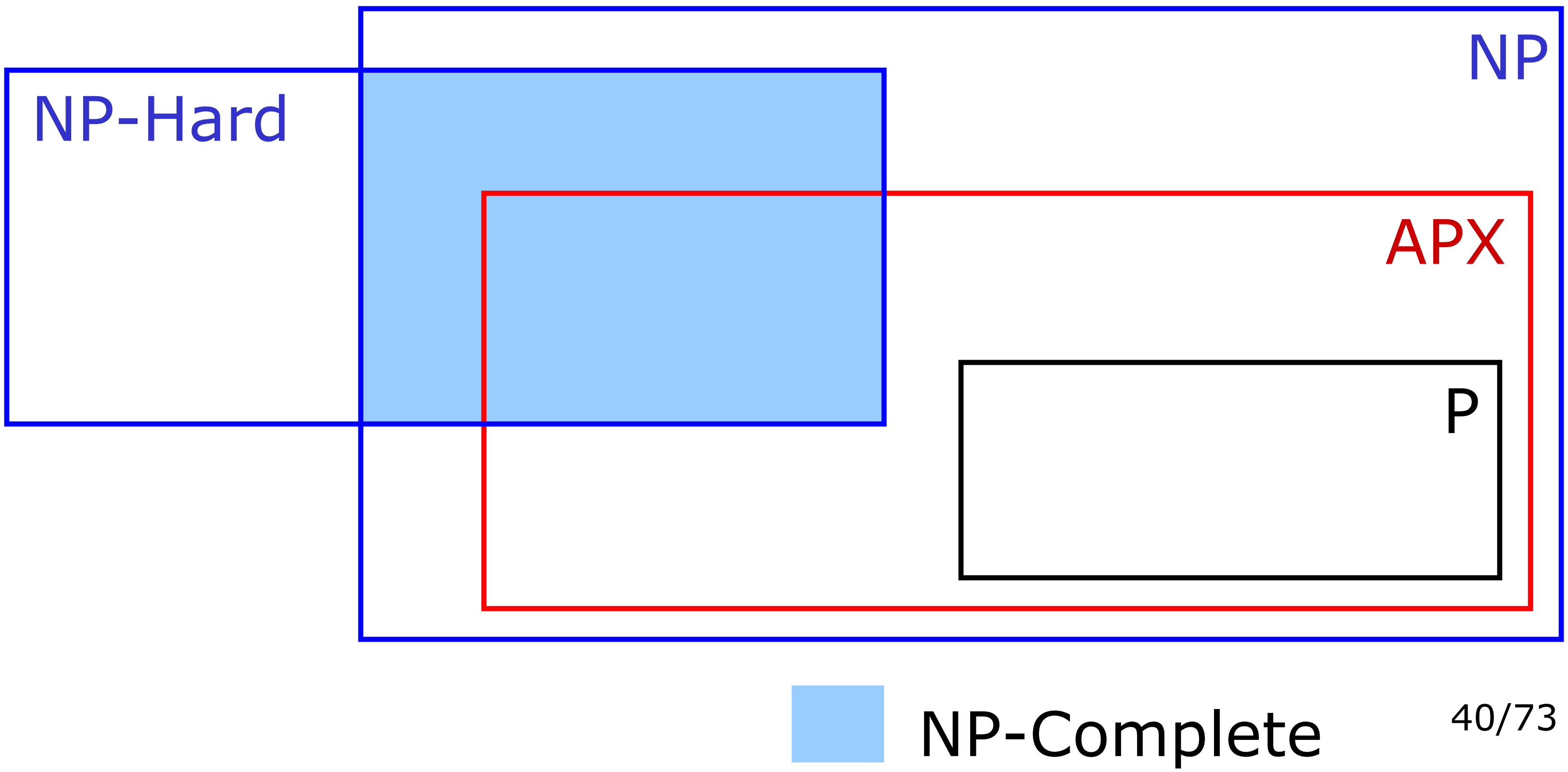
Complexity Classes



Intermezzo

Complexity Classes

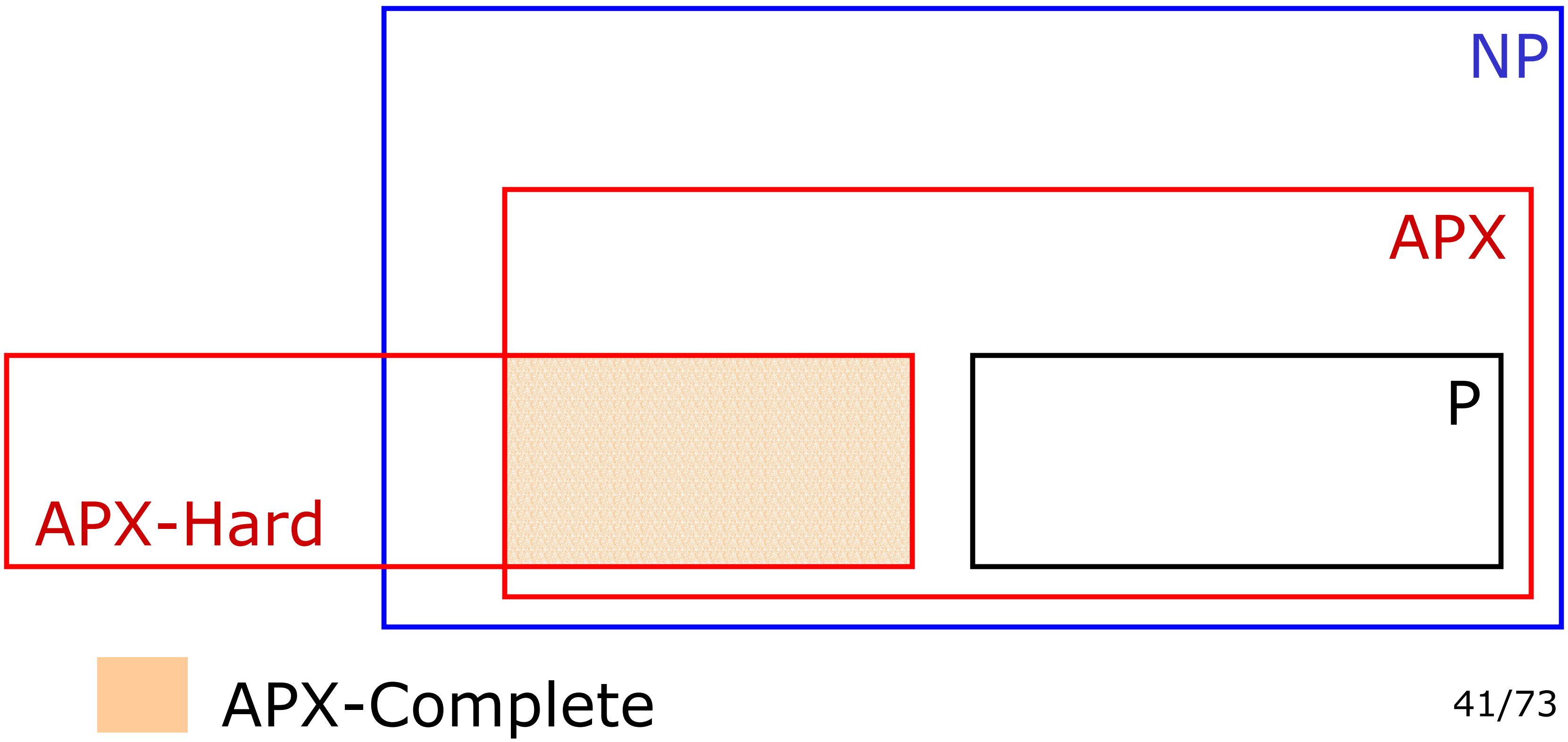
Hardness/Completeness



Intermezzo

Complexity Classes

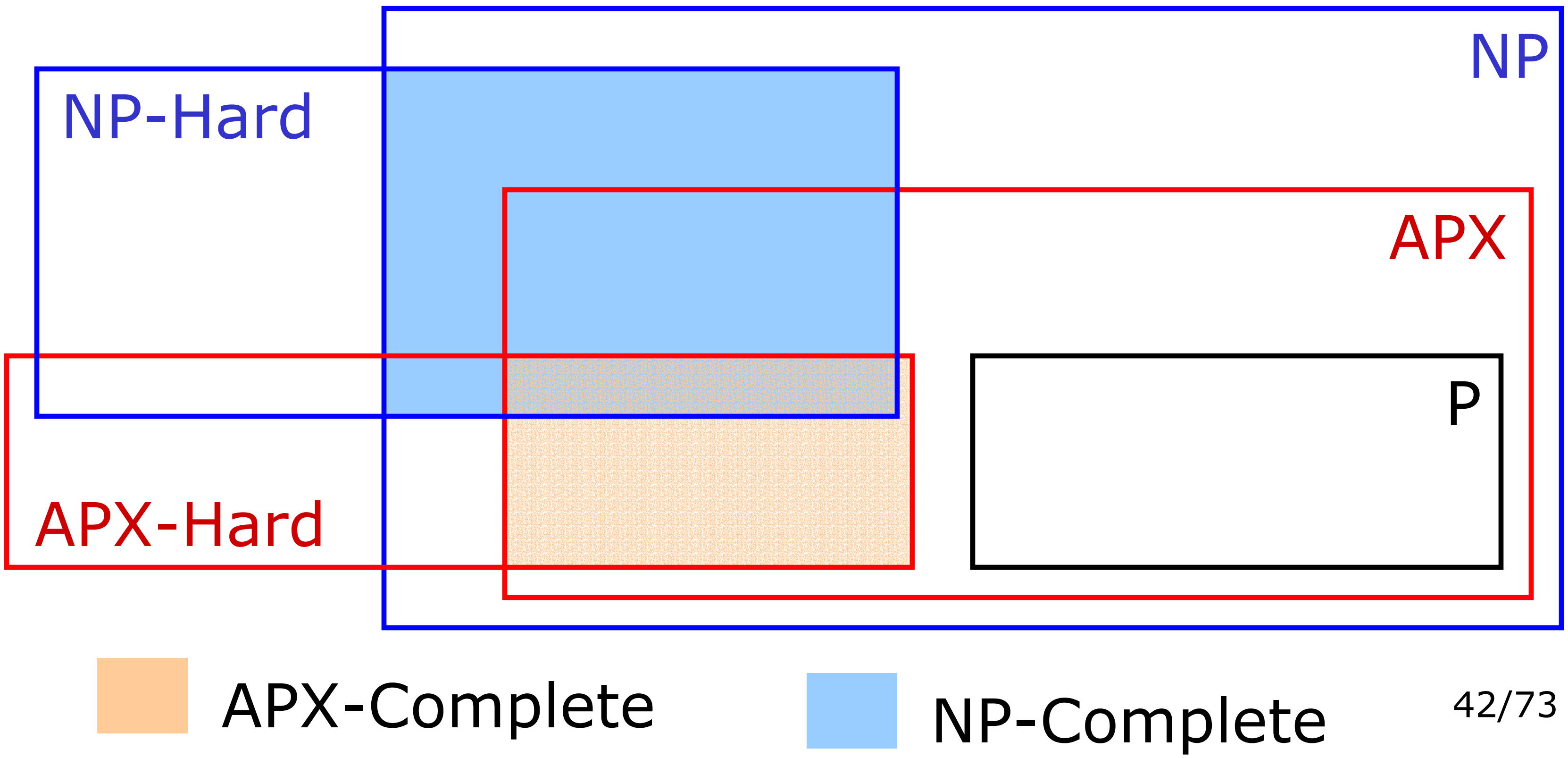
Hardness/Completeness



Intermezzo

Complexity Classes

Hardness/Completeness



Intermezzo

Complexity Classes

Recipe to prove APX-Completeness of a problem A:

- show that A is in APX by giving an approximation algorithm
- show that A is APX-Hard by reducing it to another problem B that is known to be APX-Hard
- since A is in APX and APX-Hard, it follows that A is APX-Complete

(Replace „APX“ by „NP“ for NP-Completeness)

Intermezzo

Complexity Classes

Reducibility:

A is reducible to B if: given an polynomial time algorithm that solves instances of A, we can provide a polynomial time algorithm that solves instances of B.

Additional when reducing APX problems: Show that a constant approximation factor is preserved.

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- Conclusions

Complexity Proof

APX-Completeness of Min 3D RA

1. Step according to recipe:

Show that „Min 3D Range Assignment“ is in APX.

Complexity Proof

APX-Completeness of Min 3D RA

1. Step according to recipe:

Show that „Min 3D Range Assignment“ is in APX.

This has already been done by Kirousis et al.

→ We believe them, so we can proceed to step 2, hehe. ☺

Complexity Proof

APX-Completeness of Min 3D RA

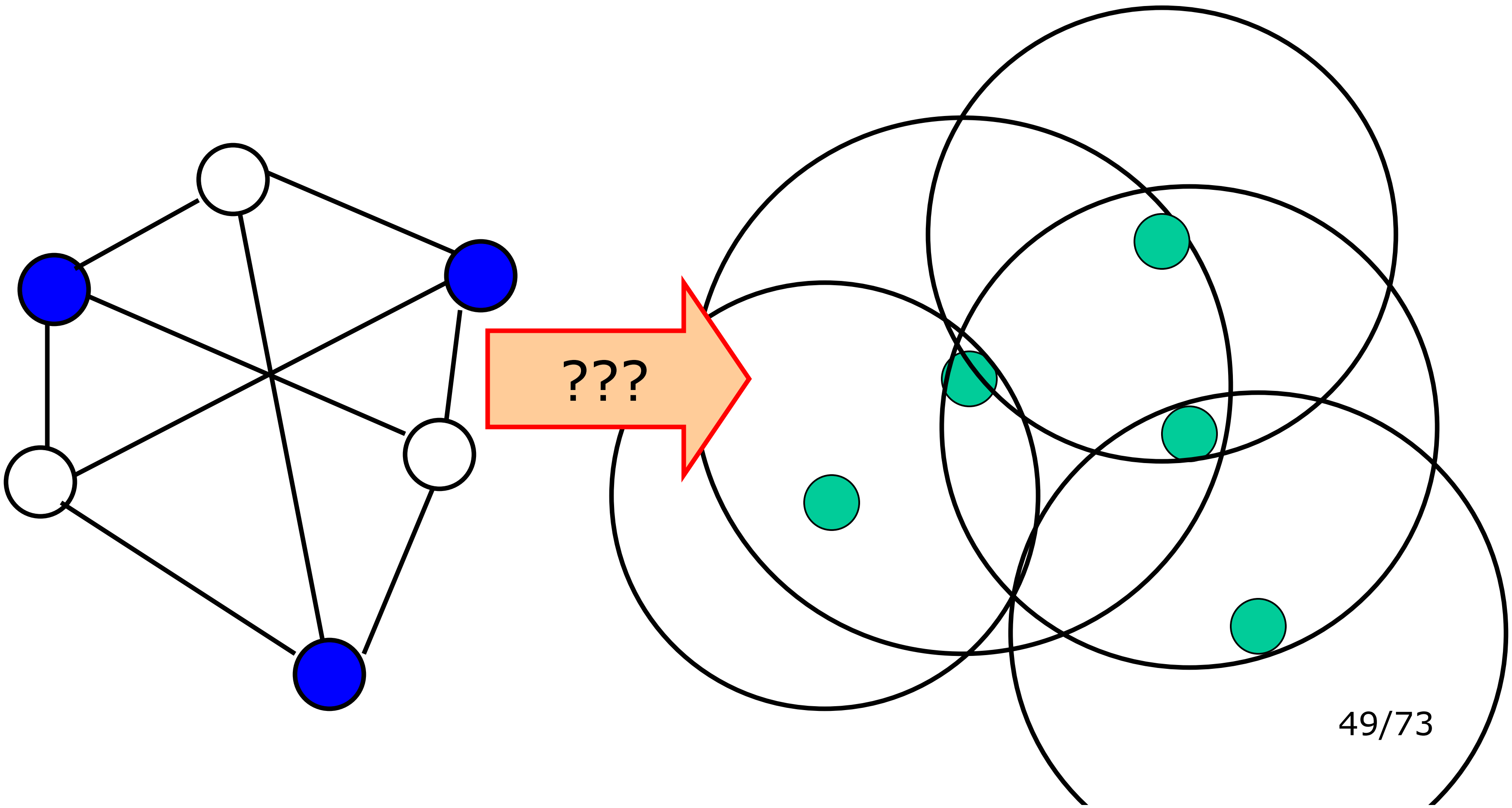
2. Step according to recipe:

Reduce „Min 3D Range Assignment" to a problem which is known to be APX-Hard.

We pick „Min Vertex Cover restricted to cubic graphs"

Complexity Proof

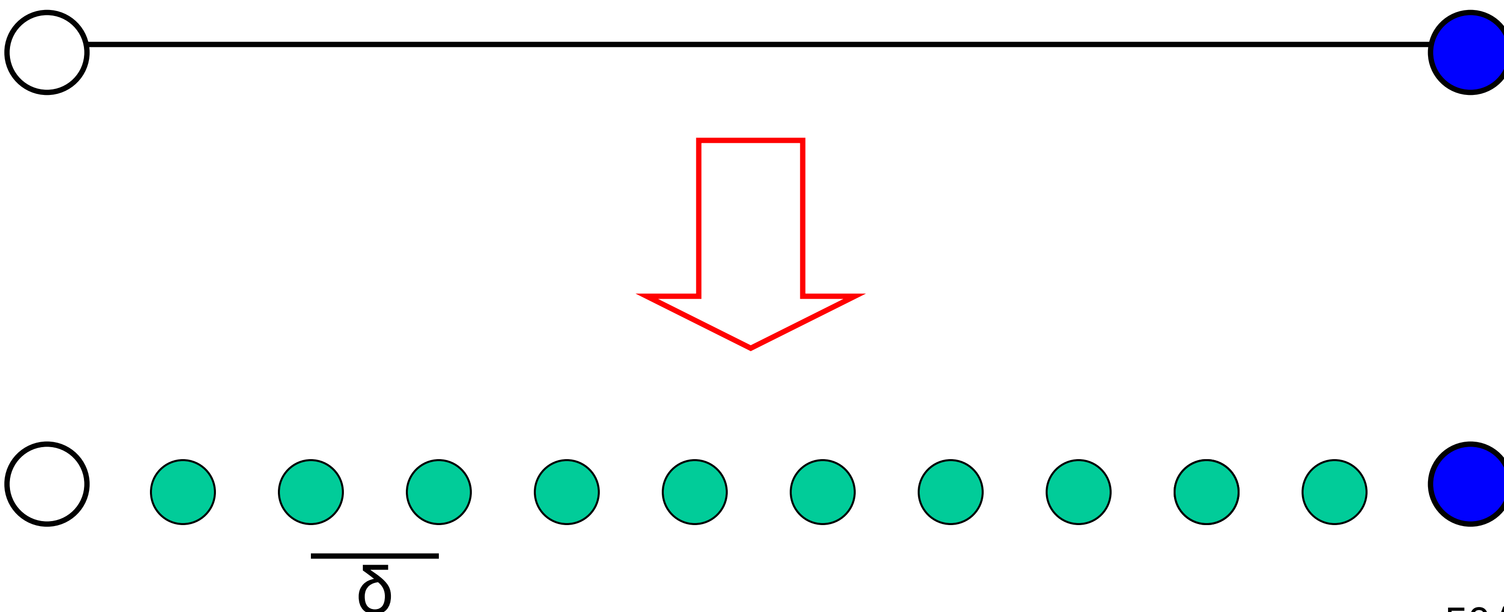
APX-Completeness of Min 3D RA



Complexity Proof

APX-Completeness of Min 3D RA

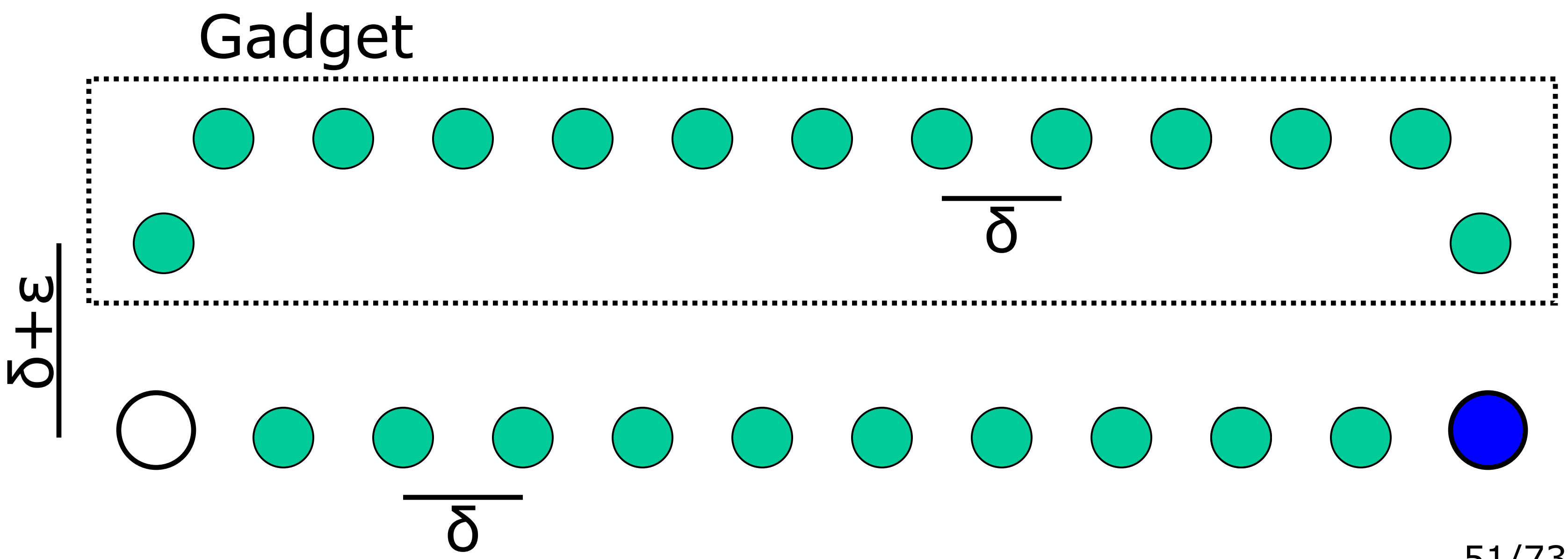
1. Each edge in Vertex Cover is replaced by a chain of radio stations



Complexity Proof

APX-Completeness of Min 3D RA

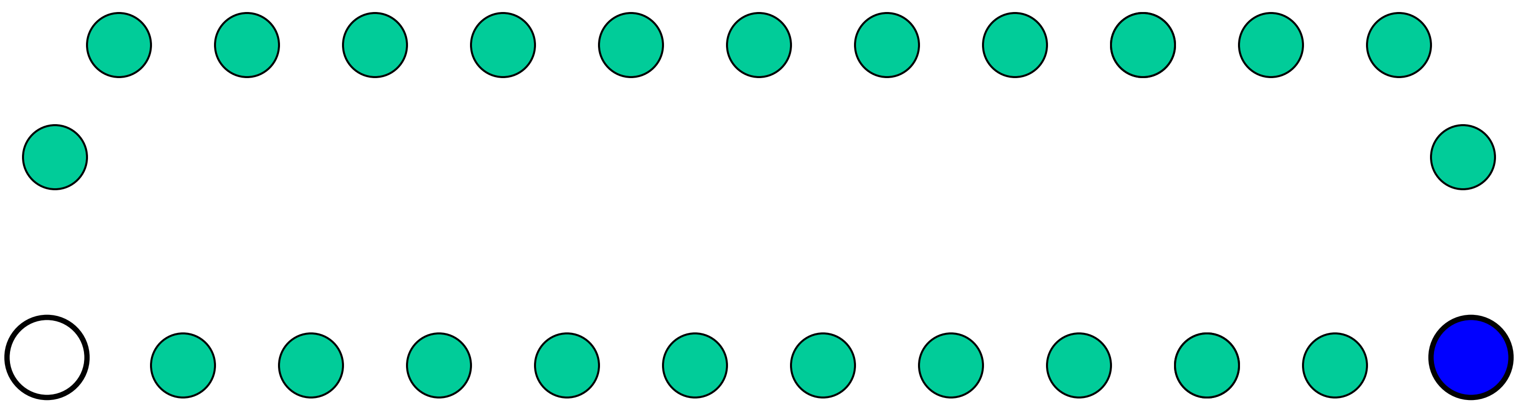
2. We add a „gadget" to each chain



Complexity Proof

APX-Completeness of Min 3D RA

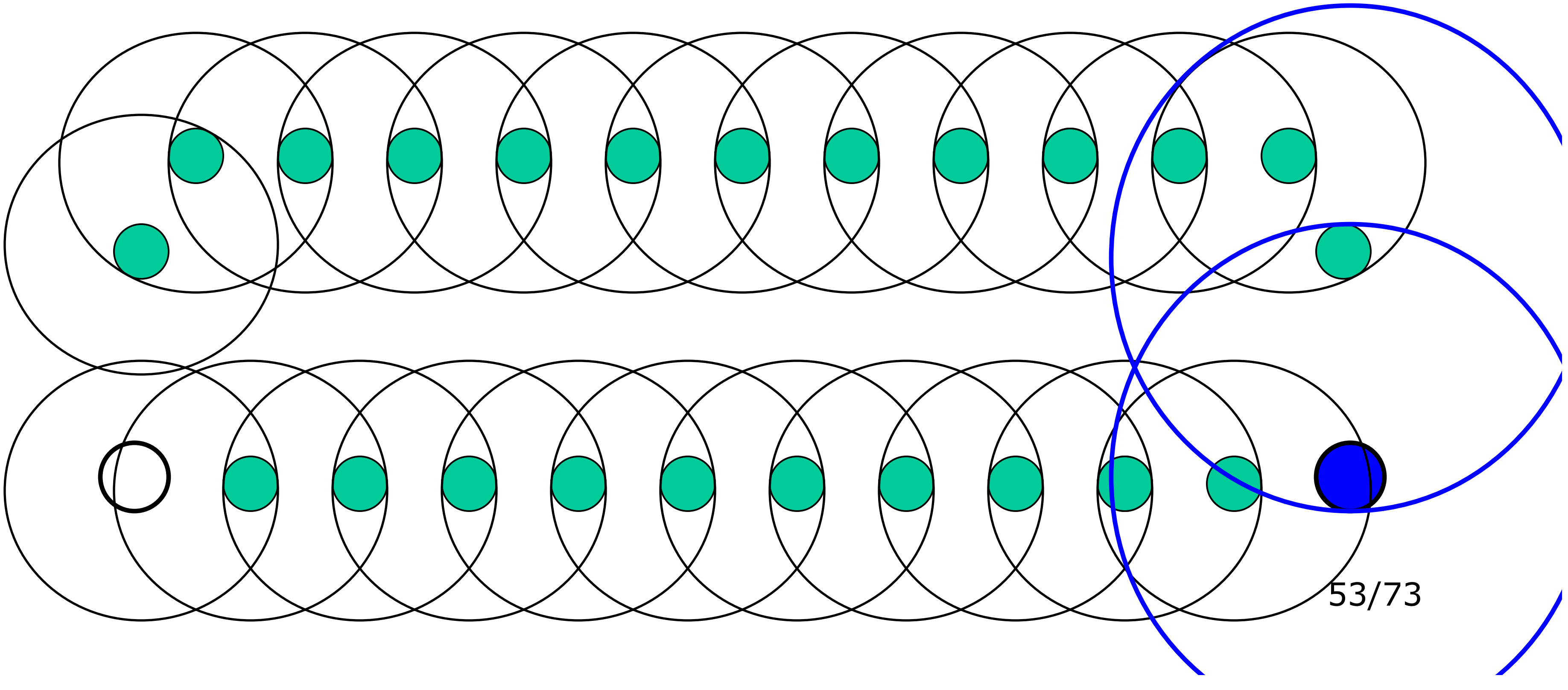
Optimal power assignment?



Complexity Proof

APX-Completeness of Min 3D RA

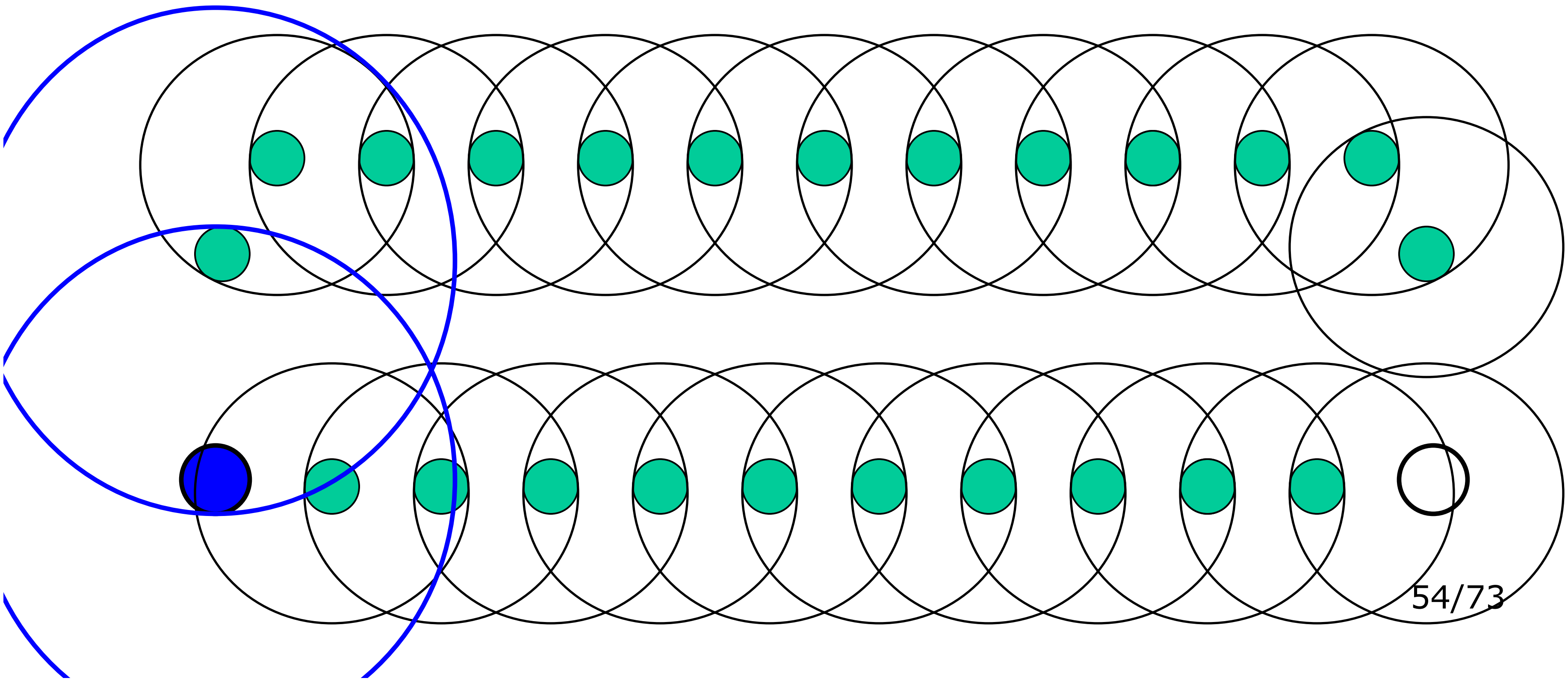
Intuitive (and correct) assignment



Complexity Proof

APX-Completeness of Min 3D RA

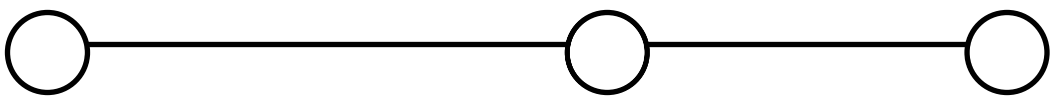
Equivalent Solution:



Complexity Proof

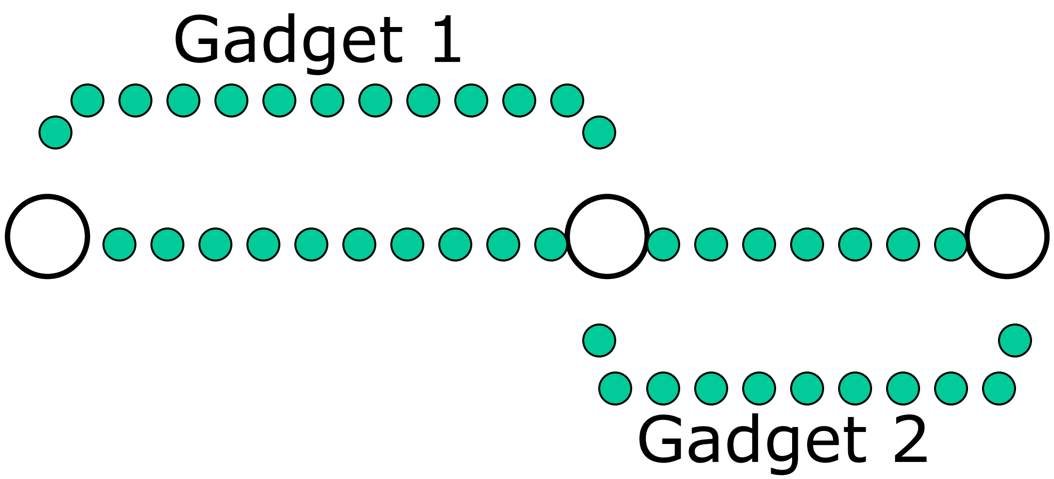
APX-Completeness of Min 3D RA

What would this graph look like when converted?



Complexity Proof

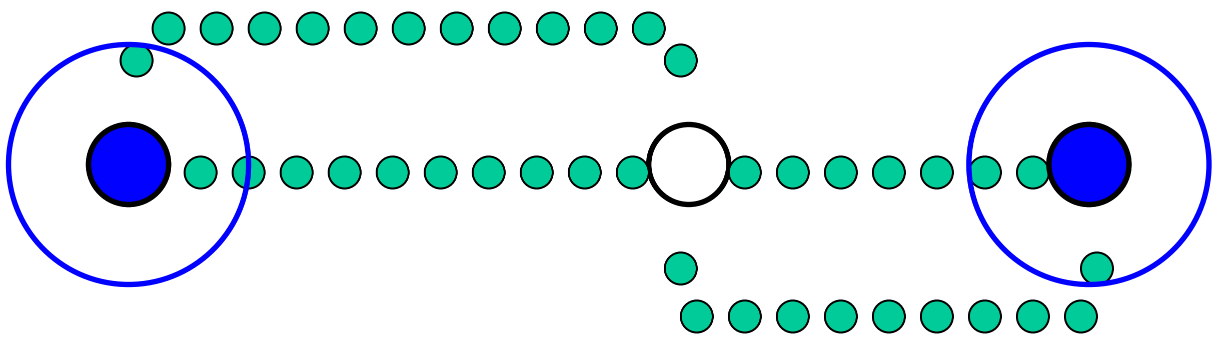
APX-Completeness of Min 3D RA



Complexity Proof

APX-Completeness of Min 3D RA

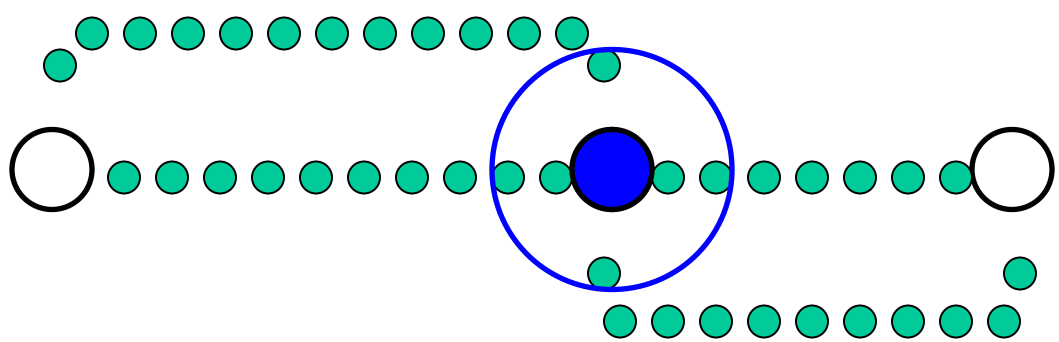
Optimal Solution: Candidate A



Complexity Proof

APX-Completeness of Min 3D RA

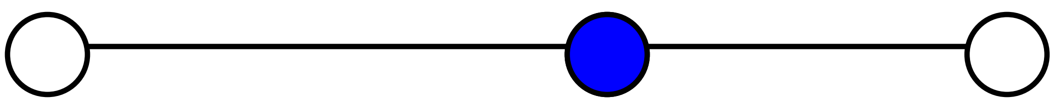
Optimal Solution: Candidate B



Complexity Proof

APX-Completeness of Min 3D RA

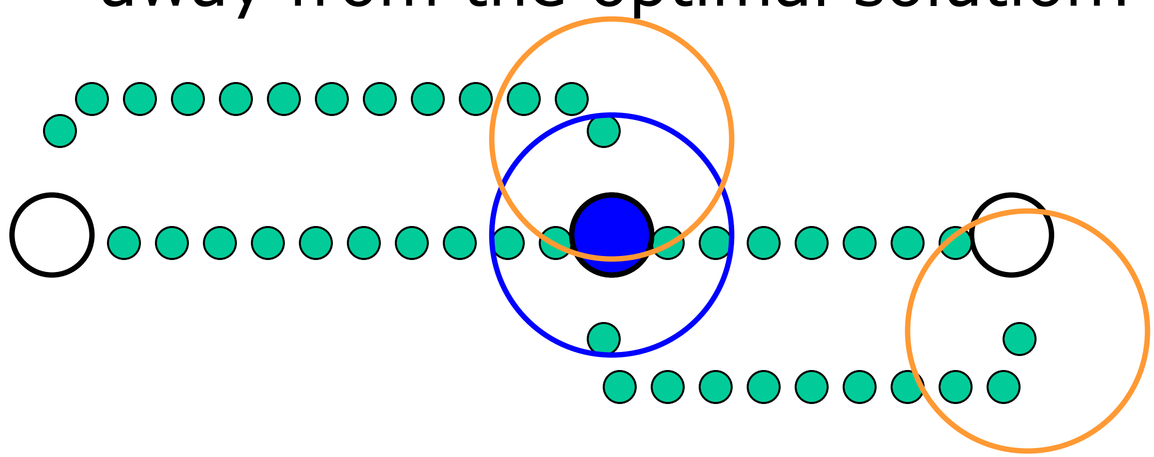
→ We have implicitly found the Min Vertex Cover by solving the Range Assignment Problem



Complexity Proof

APX-Completeness of Min 3D RA

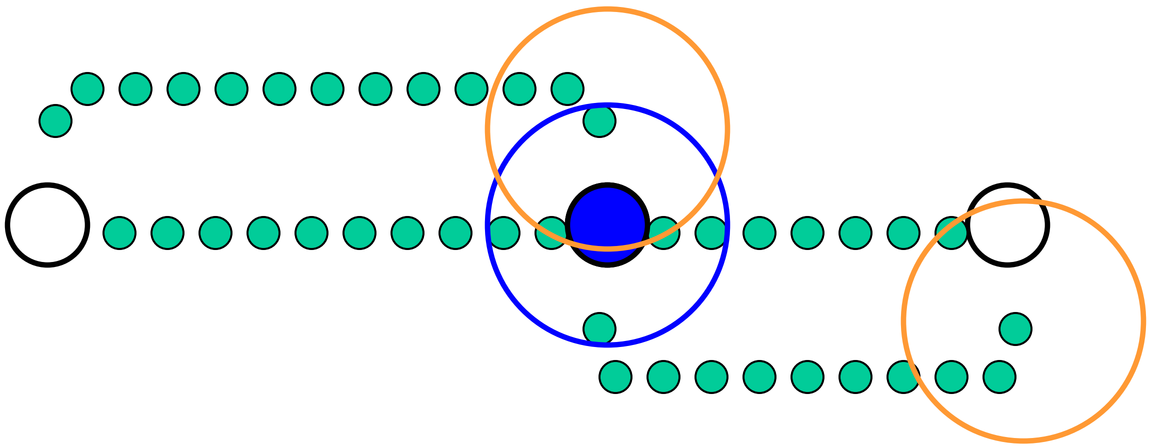
After having converted the graph, can we garantuee that we are still only a constant factor away from the optimal solution?



$$solRA = solVC*(\delta+\varepsilon)^2 + m*\varepsilon^2 + n*(\delta+\varepsilon)^2$$

Complexity Proof

APX-Completeness of Min 3D RA

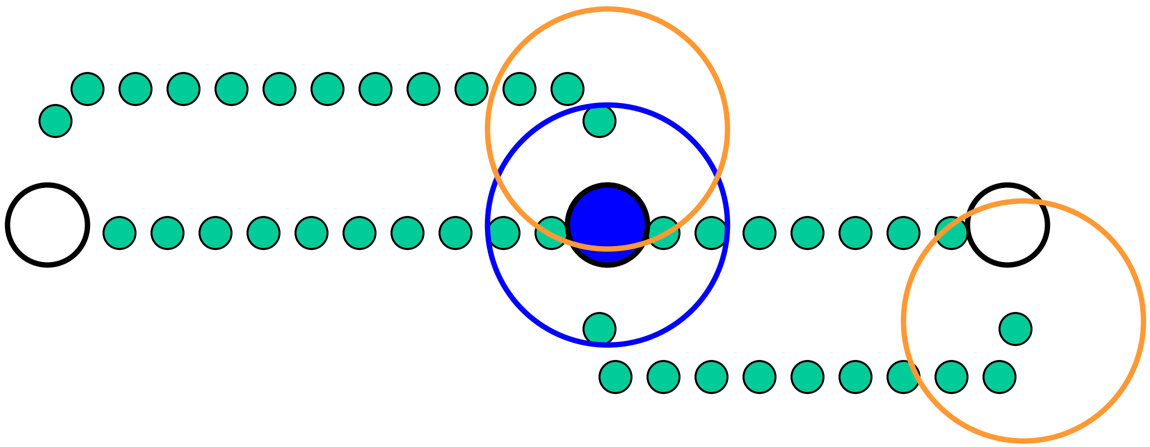


$$apxRA = apxVC*(\delta+\varepsilon)^2 + m*\varepsilon^2 + n*(\delta+\varepsilon)^2$$

Can be made smaller than any given constant.

Complexity Proof

APX-Completeness of Min 3D RA

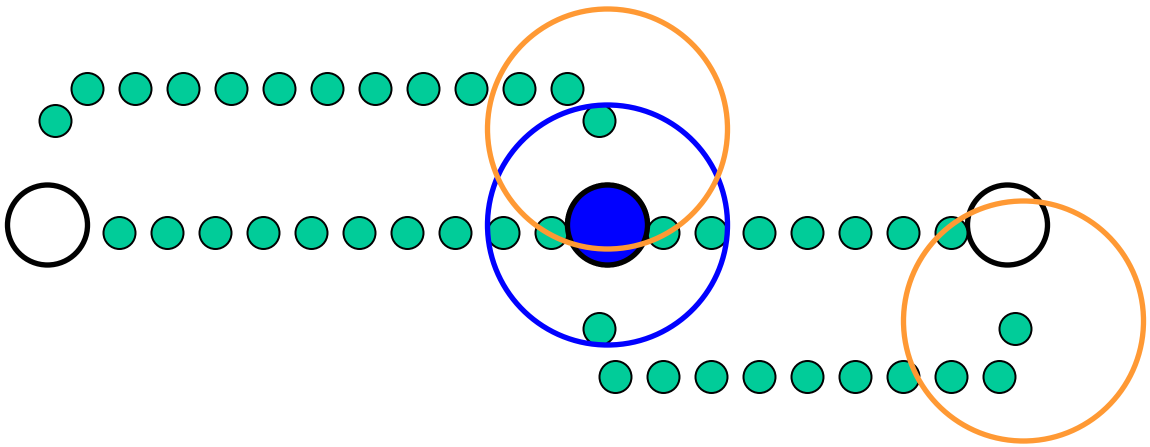


apxRA = apxVC*c2 + 1 + n*c2

Between apxVC and 3*apxVC

Complexity Proof

APX-Completeness of Min 3D RA

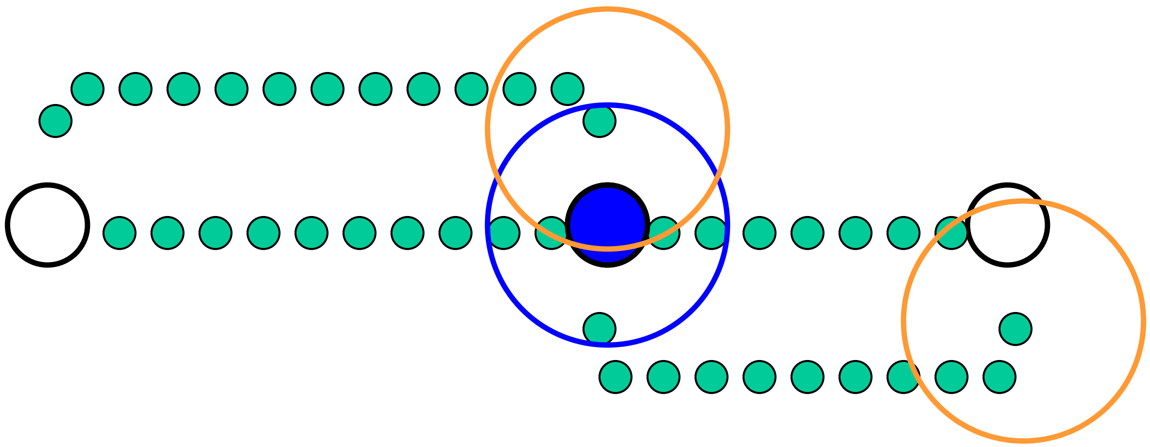


apxRA = c3*apxVC*c2 + 1

→ changing apxRA by a constant factor also changes apxVC by a constant factor.

Complexity Proof

APX-Completeness of Min 3D RA



apxRA = c3*apxVC*c2 + 1

The paper proves this in a correct way and concludes that:

fVC = 5*fRA - 4

Complexity Proof

Flashback

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Complexity Proof

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Same proof for Min 2D Range Assignment?

Complexity Proof

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Same proof for Min 2D Range Assignment?

-> only for NP-Completeness

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Conclusions

No „Min 2D h-Range Assignment“ algorithm will ever consume less energy than O(n^(1+1/h))

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MIN 2D RANGE ASSIGNMENT	in APX [16]	NP-complete
MIN 2D *h*-RANGE ASSIGNMENT, well-spread	–	in APX
MIN 2D *h*-RANGE ASSIGNMENT	–	in Av-APX
MIN 3D RANGE ASSIGNMENT	NP-complete, in APX [16]	APX-complete

Conclusions

Relevance of paper:

Superficial measurement: has been cited in 12 papers so far (all self-citations) -> low impact.

But: We can now judge the quality of distributed algos better, since we know the optimum.

Conclusions

My impression:

-A provably wrong statement

-A prove we did not understand

→ I do not entirely trust every detail in the paper (e.g. does it really work for all betas?)

Open Questions

Is „Min 2D h-Range Assignment“ APX-Complete?

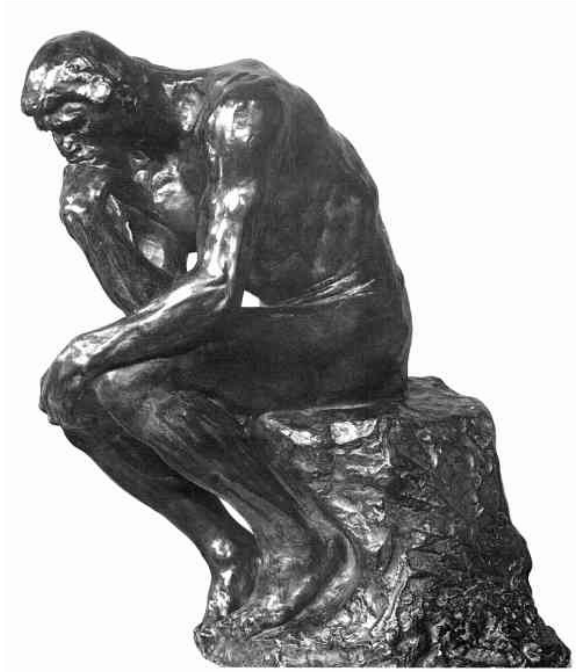
Distributed Algorithm?

Not much known about „Min d-D h-Range Assignments“ in general, even for the 1 dimensional case
(is it in P? in NP? In APX?)

→ maybe in newer papers of Clementi et al.

Questions

?



DER DENKER, 1880. Bronze, Höhe 71,9 cm. Musée Rodin, Paris