Random Walks on a Graph

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1. Introduction

Given a graph and a starting point, we select a neighbour of it at random, and move to this neighbour; then we select a neighbour of this point at random, and move to it etc. The (random) sequence of points selected this way is a random walk on the graph.
The paper [1] gives a survey of the theory of random walks and points out different examples of application. Random walks can be represented by Markov chains. This allows the use of linear algebra techniques in the analysis of random walks.

2. Summary

After giving a good introduction to the field of random walks and pointing out some interesting applications the author introduces the reader to basic notations and facts. He then explains the measures of a random walk that play the most important role in the quantitative theory of random walks; these are the access time (or hitting time), the commute time, the cover time and the mixing rate. He then gives the known bounds to this main parameters.
The second part deals with the analysis and proof of presented theorems. The author exploits the mentioned connection to algebra in depth by making use of eigenvalue and spectra analysis techniques. He then explains and males use of the connection to electrical networks.
The last section presents examples of algorithm in which design random walk has been used and examples of combinatorial problems which were solved by using random walk techniques. One of the most famous ones is the fundamental problem of finding the volume of a convex body.

3. Related Work

Sampling in algorithm design was used to compute the volume of polytope [2] and also for a perfect matching algorithm [3].
The presented upper and lower bounds [4, 5, 6] are used to analyse the behaviour of web surfers as a random walk on the web graph [7] or to give better bounds on routing problems [8] compared to results received by multicommodity flows.
The results presented are even used in a study about group communication in ad-hoc networks [9] to get bounds on the cover time (broadcasting).

4. Discussion

The paper gives a good overview of the theory of random walks on graphs even tough it was published more than ten years ago. Random walks is a well covered topic and the bounds given in the paper are provably asymptotically best possible.
Nevertheless the presented results are proved on static graphs and it is mostly unknown how far they hold for dynamically changing graphs as for e. g. the Web or an Ad-Hoc network.
Moreover the research could proof if better bounds exist for special graphs. This could help in other sientific fields which use graphs with special properties related to there problems.
The paper doesn't mention the wide use of random walks in commercial use. Random walks are used to model the share prices and other factors at the stock market. Furthermore in Global Information Retrieval Systems such as Google Random Walks are used to improve the search and indexing algorithms.

References

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