



Discrete Event Systems

Exercise 9

1 “Hopp FCB!”

Besides its moodiness, the *FC Basel* (FCB) soccer club is confronted with yet another problem: sporadically, its players get sick. Assume that the whole team consists of n players. Assume that the time period a fit player remains fit is exponentially distributed with parameter μ (independently of the state of the other players). On the other hand, the time a sick player remains sick is exponentially distributed with parameter λ .

- Model the situation as a birth-and-death Markov process where the states denote the number of players which are fit.
- Derive a formula for the probability that exactly i players are fit.
- Assume that the FCB has 20 players, and that $\lambda^{-1} = 4$ months and $\mu^{-1} = 10$ months. Calculate the probability that the FCB can participate at a given match.

2 A Binary Game

Anna and Markus play the following game: Well hidden from the other player, they write either 0 or 1 onto a note. Then, they disclose their decision. If the sum modulo 2 is 0, Anna wins, and vice versa if the sum is 1.

- Anna’s strategy is to write both 0 and 1 with probability .5, independently of the past. Markus on the other hand writes 0 and 1 with probabilities .4 and .6 respectively. Who will win more games on average?
- Assume that Anna changes her strategy as follows: She writes the number with which she would have won the last game, i.e., if Markus has written 0 in the last round, Anna writes 0, and if Markus has written 1, Anna writes 1. Assuming Markus’ strategy is unchanged, who will win more games now (on average)?
- Finally, Markus changes his strategy as well: While Anna writes the number with which she would have won the last game, Markus writes the number with which he would have won *two* games ago. Analyze these strategies as well!