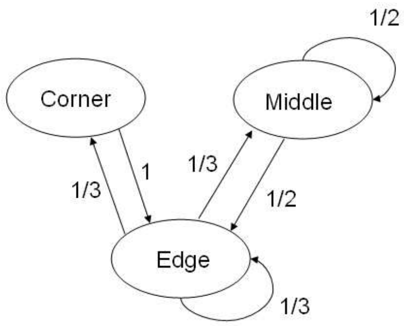
Discrete Event Systems
Exercise 8: Sample Solution

1 Night Watch

a) Observe that in the steady state, in does not matter in which corner Stefan and Thomas are: the four states are equivalent. Moreover, the same symmetry argument applies for rooms at the border and for rooms in the middle. Thus, we can analyze the following simplified Markov chain:



Hence, in the steady state, it holds that

$$P_c = 1/3 \cdot P_b; \quad P_b = 1/3 \cdot P_b + 1/2 \cdot P_m + P_c; \quad 1 = P_c + P_b + P_m$$

Solving this equation system gives: $P_c = 1/6$. The probability of being in a specific corner is therefore $1/6 \cdot 1/4 = 1/24$.

b) Since the two walks are independent, we have

$$1/24 + 1/24 - (1/24)^2 = 0.082.$$

2 Probability of Arrival

The proof is similar to the one about the transition time h_{ij} (see script). We express f_{ij} as a condition probability that depends on the result of the first step in the Markov chain. Recall that the random variable T_{ij} is the *hitting time*, that is, the number of steps from i to j. We get $Pr[T_{ij} < \infty | X_1 = k] = Pr[T_{ij} < \infty]$ for $k \neq j$ and $Pr[T_{ij} < \infty | X_1 = j] = 1$. We can therefore write f_{ij} as

$$\begin{aligned} f_{ij} &= Pr[T_{ij} < \infty] = \sum_{k \in S} Pr[T_{ij} < \infty | X_1 = k] \cdot p_{ik} \\ &= p_{ij} \cdot Pr[T_{ij} < \infty | X_1 = j] + \sum_{k \neq j} Pr[T_{ij} < \infty | X_1 = k] \cdot p_{ik} \\ &= p_{ij} + \sum_{k \neq j} p_{ik} f_{kj}. \end{aligned}$$