Discrete Event Systems
Exercise 10

1 "Hopp FCB!"

Besides its moodiness, the *FC Basel* (FCB) soccer club is confronted with yet another problem: sporadically, its players get sick. Assume that the whole team consists of n players. Assume that the time period a fit player remains fit is exponentially distributed with parameter μ (independently of the state of the other players). On the other hand, the time a sick player remains sick is exponentially distributed with parameter λ.

a) Model the situation as a birth-and-death Markov process where the states denote the number of players which are fit.

b) Derive a formula for the probability that exactly i players are fit.

c) Assume that the FCB has 20 players, and that $\lambda^{-1} = 4$ months and $\mu^{-1} = 10$ months. Calculate the probability that the FCB can participate at a given match.

2 A Binary Game

Anna and Markus play the following game: Well hidden from the other player, they write either 0 or 1 onto a note. Then, they disclose their decision. If the sum modulo 2 is 0, Anna wins, and vice versa if the sum is 1.

a) Anna's strategy is to write both 0 and 1 with probability .5, independently of the past. Markus on the other hand writes 0 and 1 with probabilities .4 and .6 respectively. Who will win more games on average?

b) Assume that Anna changes her strategy as follows: She writes the number with which she would have won the last game, i.e., if Markus has written 0 in the last round, Anna writes 0, and if Markus has written 1, Anna writes 1. Assuming Markus' strategy is unchanged, who will win more games now (on average)?

c) Finally, Markus changes his strategy as well: While Anna writes the number with which she would have won the last game, Markus writes the number with which he would have won *two* games ago. Analyze these strategies as well!