Discrete Event Systems
Exercise 8

1 An Unsolvable Problem

It's the first day of your internship at the software firm Bug Inc., and your boss calls you to his office in order to explain your task for the next three months. He says that many clients complain that the programs of Bug Inc. often contain faulty loops that never terminate. In order to prevent such errors in future, you are asked to implement a program that may check whether a given program will halt on all possible inputs or not.

a) Try to find a proof that convinces your boss that this is not possible for general programs. Hint: The proof works by contradiction. Assume a procedure $halt(P : Program) : boolean$ that takes a program P and decides whether P halts on all possible inputs or not. Now construct a program X that terminates if $halt(X)$ is false and loops endlessly if $halt(X)$ is true, which yields the desired contradiction.

b) Your boss still disagrees and proposes the following method: $halt(X)$ simply simulates the execution of program X. If the program terminates it returns true, and if it loops it returns false. Where is the problem of this approach?

c) Your boss is finally convinced but argues that your proof is a very special case that hardly reflects reality. Are there assumptions under which it is always possible to check whether a program halts or not?

2 Soccer Betting

The *FC Basel* soccer club is a particularly moody team. Upon winning a game, they tend to win subsequent games. After losing a game, however, they often end up losing the next game as well. A group of international scientists, consisting of soccer experts, mathematicians, and psychologists, has recently conducted a thorough analysis of this behavior. In particular, they have discovered that upon winning a game, the FCB wins the next game with a probability of 60% as well. With probabilities 0.2 each, the next game will be a tie or a loss. After a loss, the FCB will win/tie/lose its next game with probability 0.1/0.2/0.7, respectively. Finally, after a tie, the next game being a win or a loss is equally probable. The probability that the next game also ends up being a tie is 0.4.

a) Model the FCB's moodiness using a Markov chain.

b) In *three games* from now, the FCB will play against the Grasshoppers from Zurich. The Swiss TOTO offers your the following odds:

Win: 3.5 Tie: 3.5 Loss: 2.0

Given that the FCB won two games ago, but lost its last game, what would be your bet? Why?

c) More recent studies have shown that the FCB is even moodier than expected. In fact, after losing *two games in a row*, the probability of winning its next game reduces to 0.05, that of getting at least a tie to 0.1. Change your Markov chain model to incorporate the new circumstances. How does the change influence your bet?

3 The Winter Coat Problem

While exploring the sea with his boat, Mr. Robinson lost orientation and ended up on a strange island. After living there for several years, he observed that the weather on the island follows a strict probabilistic pattern: The weather of a given day only depends on the weather the day before. When it was sunny (S), the following day is sunny again with 50%, but it gets cloudy (C) with 30% and it starts to rain (R) with 20%. Once its cloudy, it says cloudy with 10%, gets sunny with 70% and starts to rain with 20%. Finally, after a rainy day, it gets sunny with 10%, cloudy with 40% and remains rainy with 50%.

a) Model this special weather condition using a Markov chain.

b) After spending a sunny day, how many days does Mr. Robinson have to wait until its sunny again (in expectation)?

Due to the global warming of earth, the weather conditions are actually slightly different: After a sunny day, it remains sunny only with 49%, but gets hot (H) with 1%. Once it's hot, it remains hot forever. Similarly, after a rainy day, it remains rainy with 49%, but starts to snow (W) with 1%. Again, once it snows, it continues to snow forever.

c) Adapt your Markov chain to model the new situation.

d) What is the probability that Mr. Robinson ever needs a winter coat, given that he arrived on a sunny day on the island?