On the Complexity of Computing Minimum Energy Consumption Broadcast Subgraphs

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Abstract. We consider the problem of computing an optimal range assignment in a wireless network which allows a specified source station to perform a broadcast operation. In particular, we consider this problem as a special case of the following more general combinatorial optimization problem, called Minimum Energy Consumption Broadcast Subgraph (in short, MECBS): Given a weighted directed graph and a specified source node, find a minimum cost range assignment to the nodes, whose corresponding transmission graph contains a spanning tree rooted at the source node. We first prove that MECBS is not approximable within a sub-logarithmic factor (unless P=NP). We then consider the restriction of MECBS to wireless networks and we prove several positive and negative results, depending on the geometric space dimension and on the distance-power gradient. The main result is a polynomial-time approximation algorithm for the NP-hard case in which both the dimension and the gradient are equal to 2: This algorithm can be generalized to the case in which the gradient is greater than or equal to the dimension.

1 Introduction

Wireless networking technology will play a key role in future communications and the choice of the network architecture model will strongly impact the effectiveness of the applications proposed for the mobile networks of the future. Broadly speaking, there are two major models for wireless networking: *single-hop* and *multi-hop*. The single-hop model [22], based on the cellular network model, provides one-hop wireless connectivity between mobile hosts and static nodes known as *base stations*. This type of networks relies on a fixed backbone infrastructure that interconnects all base stations by high-speed wired links. On the other hand, the multi-hop model [15] requires neither fixed, wired infrastructure nor predetermined interconnectivity. *Ad hoc* networking [12] is the most popular type of multi-hop wireless networks because of its simplicity:

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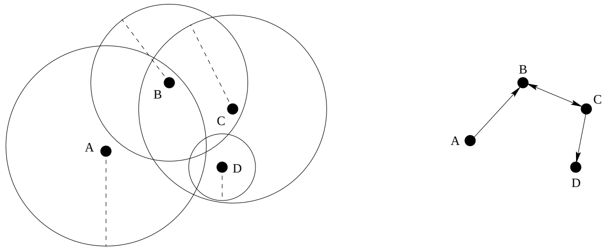

Fig. 1. A Range Assignment and Its Corresponding Directed Transmission Graph.

Indeed, an *ad hoc* wireless network is constituted by a homogeneous system of *mobile* stations connected by wireless links. In ad hoc networks, to every station is assigned a transmission range: The overall range assignment determines a transmission (directed) graph since one station s can transmit to another station t if and only if t is within the transmission range of s (see Fig. 1).

The range transmission of a station depends, in turn, on the energy power supplied to the station: In particular, the power P_s required by a station s to correctly transmit data to another station t must satisfy the inequality

$$\frac{P_s}{d(s,t)^\alpha} > \gamma \tag{1}$$

where $d(s,t)$ is the Euclidean distance between s and t, $\alpha \geq 1$ is the *distance-power gradient*, and $\gamma \geq 1$ is the *transmission-quality* parameter. In an ideal environment (i.e. in the empty space) it holds that $\alpha = 2$ but it may vary from 1 to more than 6 depending on the environment conditions of the place the network is located (see [19]). The fundamental problem underlying any phase of a dynamic resource allocation algorithm in ad-hoc wireless networks is the following: Find a transmission range assignment such that (1) the corresponding transmission graph satisfies a given property π, and (2) the overall energy power required to deploy the assignment (according to Eq. 1) is minimized.

A well-studied case of the above problem consists in choosing π as follows: The transmission graph has to be strongly connected. In this case, it is known that: (a) the problem is not solvable in polynomial time (unless P=NP) [6,14], (b) it is possible to compute a range assignment which is at most twice the optimal one (that is, the problem is 2-approximable), for multi-dimensional wireless networks [14], (c) there exists a constant $r > 1$ such that the problem is not r-approximable (unless P=NP), for d-dimensional networks with $d \geq 3$ [6], and (d) the problem can be solved in polynomial time for one-dimensional networks [14]. Another analyzed case consists in choosing π as follows: The diameter of the transmission graph has to be at most a fixed value h. In this case, while non-trivial negative results are not known, some tight bounds (depending on h) on the minimum energy power have been proved in [7], and an approximation algorithm for the one-dimensional case has been given in [5]. Other trade-offs between connectivity and energy consumption have been obtained in [16,21,24].

In this paper we address the case in which π is defined as follows: *Given a source station s, the transmission graph has to contain a directed spanning tree rooted at s.* This case has been posed as an open question by Ephremides in [10]: Its relevance is due to the fact that any transmission graph satisfying the above property allows the source station to perform a *broadcast* operation. Broadcast is a task initiated by the source station which transmits a message to all stations in the wireless network: This task constitutes a major part of real life multi-hop radio network [2,3].

The Optimization Problem. The broadcast range assignment problem described above is a special case of the following combinatorial optimization problem, called MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH (in short, MECBS). Given a weighted directed graph $G = (V, E)$ with edge weight function $w : E \to \mathcal{R}^+$, a *range assignment* for G is a function $r : V \to \mathcal{R}^+$: The *transmission graph* induced by G and r is defined as $G_r = (V, E')$ where

$$E' = \bigcup_{v \in V} \{(v, u) : (v, u) \in E \wedge w(v, u) \leq r(v)\}.$$

The MECBS problem is then defined as follows: Given a *source node* $s \in V$, find a range assignment r for G such that G_r contains a spanning tree of G rooted at s and $\mathsf{cost}(r) = \sum_{v \in V} r(v)$ is minimized.

Let us consider, for any $d \geq 1$ and for any $\alpha \geq 1$, the family of graphs N_d^α, called *(d-dimensional) wireless networks*, defined as follows: A complete (undirected) graph G belongs to N_d^α if it can be embedded on a d-dimensional Euclidean space such that the weight of an edge is equal to the αth power of the Euclidian distance between the two endpoints of the edge itself. The restriction of MECBS to graphs in N_d^α is denoted by MECBS[N_d^α]: It is then clear that the previously described broadcast range assignment problem in the ideal 2-dimensional environment is MECBS[N_2^2].

Our Results. In this paper, we analyze the complexity of the MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH problem both in the general case and in the more realistic case in which the instances are wireless networks. In particular, we first prove that MECBS *is not approximable within a sub-logarithmic factor, unless* P=NP (see Sect. 2). Subsequently, we consider MECBS[N_d^α], for any $d \geq 1$ and for any $\alpha \geq 1$, and we prove the following results (see Sect. 3):

– *For any $d \geq 1$,* MECBS[N_d^1] *is solvable in polynomial time*: This result is based on a simple observation.
– MECBS[N_d^α] *is not solvable in polynomial time (unless* P=NP*), for any $d \geq 2$ and for any $\alpha > 1$*: This negative result uses the same arguments of [6].
– *For any $\alpha \geq 2$,* MECBS[N_2^α] *is approximable within a constant factor*: This is the main result of the paper. A major positive aspect of the approximation algorithm lies on the fact that it is just based on the computation of a standard minimum spanning tree (shortly, MST). In a network with dynamic power control, the range assigned to the stations can be modified at any time: Our algorithm can thus take advantage of all known techniques to dynamically maintain MSTs (see, for example,

[9,11,18]). MSTs have already been used in order to develop approximation algorithms for range assignment problems in wireless networks: However, we believe that the analysis of the performance of our algorithm (which is based on computational geometry techniques) is rather interesting by itself.

Finally, in Sect. 4 we first observe that our approximation algorithm can be generalized in order to deal with MECBS[N_d^α], for any $d \geq 2$ and for any $\alpha \geq d$: However, we also prove that the approximation ratio grows at least exponentially with respect to d. We then briefly consider the behavior of our approximation algorithm when applied to MECBS[N_d^α] with $\alpha < d$ and we summarize some questions left open by this paper.

Prerequisites. We assume the reader to be familiar with the basic concepts of computational complexity theory (see, for example, [4,20]) and with the basic concepts of the theory of approximation algorithms (see, for example, [1]).

2 The Complexity of MECBS

In this section, we prove that the MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH problem is not approximable within a sub-logarithmic factor (unless P= NP). To this aim, we provide a reduction from MIN SET COVER to MECBS. Recall that MIN SET COVER is defined as follows: given a collection C of subsets of a finite set S, find a minimum cardinality subset $C' \subseteq C$ such that every element in S belongs to at least one member of C'. It is known that, unless P=NP, MIN SET COVER is not approximable within $c \log n$, for some $c > 0$, where n denotes the cardinality of S [23] (see, also, the list of optimization problems contained in [1]).

Theorem 1. *If* $\mathsf{P} \neq \mathsf{NP}$*, then* MECBS *is not approximable within a sub-logarithmic factor.*

Proof (Sketch). Let x be an instance of the MIN SET COVER problem. In the full version of the paper, we show how to construct an instance y of MECBS such that there exists a feasible solution for x whose cardinality is equal to k if and only if there exists a feasible solution for y whose cost is equal to $k + 1$. This clearly implies that if MECBS is approximable within a sub-logarithmic factor, then MIN SET COVER is approximable within a sub-logarithmic factor: The theorem hence follows from the non-approximability of MIN SET COVER. □

One interesting feature of the reduction used in the previous proof is that it also allows us to show that MECBS is not approximable within a constant factor (unless P=NP), when the problem is restricted to undirected graphs.

3 The Restriction to Wireless Networks

In this section we analyze the complexity of the MINIMUM ENERGY CONSUMPTION BROADCAST SUBGRAPH problem restricted to wireless networks, that is, MECBS[N_d^α] with $d, \alpha \geq 1$. First of all, observe that if $\alpha = 1$ (that is, the edge weights coincide with the Euclidian distances), then the optimal range assignment is simply obtained by assigning to s the distance from its farthest node and by assigning 0 to all other nodes. We then have that the following result holds.

Theorem 2. *For any $d \geq 1$, then* $\mathrm{MECBS}[\mathsf{N}_d^1]$ *is solvable in polynomial time.*

It is, instead, possible to prove the following result, whose proof is an adaptation of the one given in [6] to prove the NP-hardness of computing a minimum range assignment that guarantees the strong connectivity of the corresponding transmission graph (the proof will be given in the full version of the paper).

Theorem 3. *For any $d \geq 2$ and for any $\alpha > 1$,* $\mathrm{MECBS}[\mathsf{N}_d^\alpha]$ *is not solvable in polynomial time (unless* P= NP*).*

Because of the above negative result, it is reasonable to look for polynomial-time algorithms that compute approximate solutions for MECBS restricted to wireless networks. We now present and analyze an efficient approximation algorithm for $\mathrm{MECBS}[\mathsf{N}_2^\alpha]$, for any $\alpha \geq 2$. In what follows, given a graph $G \in \mathsf{N}_2^\alpha$, we denote by $G^{1/\alpha}$ the graph obtained from G by setting the weight of each edge to the αth root of the weight of the corresponding edge in G: Hence, $G^{1/\alpha} \in \mathsf{N}_2^1$, that is, there exists an embedding of $G^{1/\alpha}$ on the plane such that the Euclidean distance $d(u, v)$ between two nodes u and v coincides with the weight of the edge (u, v) in $G^{1/\alpha}$.

The Approximation Algorithm MST-ALG**.** Given a graph $G \in \mathsf{N}_2^\alpha$ and a specified source node s, the algorithm first computes a MST T of G (observe that this computation does not depend on the value of α). Subsequently, it makes T downward oriented by rooting it at s. Finally, the algorithm assigns to each vertex v the maximum among the weights of all edges of T outgoing from v. Clearly, the algorithm runs in polynomial time and computes a feasible solution.

3.1 The Performance Analysis of the Approximation Algorithm

The goal of this section is to prove that, for any instance $x = \langle G = (V, E), w, s\rangle$ of $\mathrm{MECBS}[\mathsf{N}_2^\alpha]$ with $\alpha \geq 2$, the range assignment r computed by MST-ALG satisfies the following inequality:

$$\mathsf{cost}(r) \leq 10^{\alpha/2} \cdot 2^\alpha \mathsf{opt}(x), \tag{2}$$

where $\mathsf{opt}(x)$ denotes the cost of an optimal range assignment. First notice that

$$\mathsf{cost}(r) \leq w(T),$$

where, for any subgraph G' of G, $w(G')$ denotes the sum of the weights of the edges in G'. As a consequence of the above inequality, it now suffices to show that there exists a spanning subgraph G' of G such that $w(G') \leq 10^{\alpha/2} \cdot 2^\alpha \mathsf{opt}(x)$. Indeed, since the weight of T is bounded by the weight of G', we have that Eq. 2 holds.

In order to prove the existence of G', we make use of the following theorem whose proof is given in Sect. 3.2.

Theorem 4. *Let $G \in \mathsf{N}_2^\alpha$ with $\alpha \geq 2$ and let R be the diameter of $G^{1/\alpha}$, that is, the maximum distance between two nodes in $G^{1/\alpha}$. Then, for any* MST *T of G,*

$$w(T) \leq 10^{\alpha/2} R^\alpha.$$

Let r_{opt} be an optimal assignment for x. For any $v \in V$, let

$$S(v) = \{u \in V : w(v,u) \leq r_{\mathsf{opt}}(v)\}$$

and let $T(v)$ be a MST of the subgraph of G induced by $S(v)$. From Theorem 4, it follows that $w(T(v)) \leq 10^{\alpha/2} \cdot 2^\alpha r_{\mathsf{opt}}(v)$. Consider the spanning subgraph $G' = (V, E')$ of G such that

$$E' = \bigcup_{v \in V} \{e \in E : e \in T(v)\}.$$

It then follows that

$$w(G') \leq \sum_{v \in V} w(T(v)) \leq 10^{\alpha/2} \cdot 2^\alpha \sum_{v \in V} r_{\mathsf{opt}}(v) = 10^{\alpha/2} \cdot 2^\alpha \mathsf{opt}(x).$$

We have thus proved the following result.

Theorem 5. *For any $\alpha \geq 2$,* MECBS[N_2^α] *is approximable within $10^{\alpha/2} \cdot 2^\alpha$.*

3.2 Proof of Theorem 4

Given a graph $G \in \mathsf{N}_2^\alpha$ with $\alpha \geq 2$, we identify the nodes of G with the points corresponding to an embedding of $G^{1/\alpha}$ on the plane: Recall that the Euclidean distance $d(u,v)$ between two points u, v coincides with the weight of the edge (u,v) in $G^{1/\alpha}$.

Let us first consider the case $\alpha = 2$ and let $e_i = (u_i, v_i)$ be the ith edge in T, for $i = 1, \ldots, |V| - 1$ (any fixed ordering of the edges is fine). We denote by D_i the *diametral open circle* of e_i, that is, the open disk whose center c_i is on the midpoint of e_i and whose diameter is $d(u_i, v_i)$. From Lemma 6.2 of [17], it follows that D_i contains no point from the set $V - \{u_i, v_i\}$. The following lemma, instead, states that, for any two diametral circles, the center of one circle is not contained in the other circle.

Lemma 1. *For any $i, j \in \{1, \ldots, |V| - 1\}$ with $i \neq j$, c_i is not contained in D_j.*

Proof. Suppose by contradiction that there exist two diametral circles D_i and D_j such that c_i is contained in D_j. We will show that the longest edge between e_i and e_j can be replaced by a strictly shorter one, still maintaining the connectivity of T: Since T is a MST the lemma will follow. Let us assume, without loss of generality, that $d(u_j, v_j) \geq d(u_i, v_i)$. We first prove that

$$\max\{d(u_i, u_j), d(v_i, v_j)\} < d(u_j, v_j) \tag{3}$$

Let Y^+ and Y^- be the half-planes determined by the line identified by c_i and c_j: Without loss of generality, we may assume that v_i and v_j (respectively, u_i and u_j) are both contained in Y^+ (respectively, Y^-), as shown in Fig. 2. Assume also that

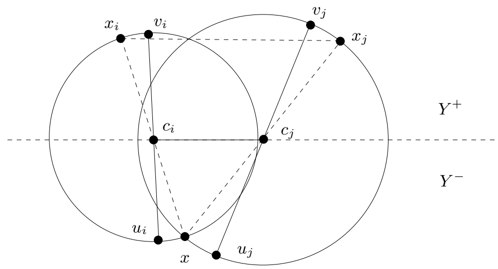

Fig. 2. The Proof of Lemma 1.

$d(v_i, v_j) \geq d(u_i, u_j)$ (the other case can be proved in a similar way). Let x be the intersection point in Y^- between the two circumferences determined by D_i and D_j (notice that, since D_i and D_j are open disks, neither D_i nor D_j contains x) and let x_i and x_j be the points diametrically opposite to x with respect to c_i and c_j, respectively. Clearly, $d(v_i, v_j) \leq d(x_i, x_j)$. Eq. 3 easily follows from the following

Fact 1. $d(x_i, x_j) < d(u_j, v_j)$.

Proof (of Fact 1). By definition, c_i (respectively, c_j) is the median of the segment $\overline{xx_i}$ (respectively, $\overline{xx_j}$). Thus, the triangles $\triangle(xx_ix_j)$ and $\triangle(xc_ic_j)$ are similar. From the hypothesis that $c_i \in D_j$, it follows that $d(c_i, c_j) < d(x, c_j)$. Thus, by similarity, it must hold that

$$d(x_i, x_j) < d(x, x_j) = d(u_j, v_j)$$

and the fact follows. □

As a consequence of Eq. 3, we can replace in T, $e_j = (u_j, v_j)$ by either (u_i, u_j) or (v_i, v_j) (the choice depends on the topology of T), thus obtaining a better spanning tree. □

We now use the above lemma in order to bound the number of diametral circles any point on the plane belongs to.

Lemma 2. *For any point p on the plane, p is contained in at most five diametral circles.*

Proof. Suppose by contradiction that there exist a point p covered by (at least) six diametral circles. Then, there must exist two circles D_1 and D_2 such that their respective centers c_1 and c_2 form with p an angle $\beta \leq \pi/3$ (see Fig. 3(a)). Let R_1 and R_2 denote the diameters of D_1 and D_2, respectively. Since $\beta \leq \pi/3$, we have that

$$d(c_1, c_2) \leq \max\{d(c_1, p), d(c_2, p)\} < \max\{R_1, R_2\}$$

where the strict inequality is due to the fact that $p \in D_1 \cap D_2$ and that both D_1 and D_2 are open disks. Hence, either $c_1 \in D_2$ or $c_2 \in D_1$, thus contradicting Lemma 1. □

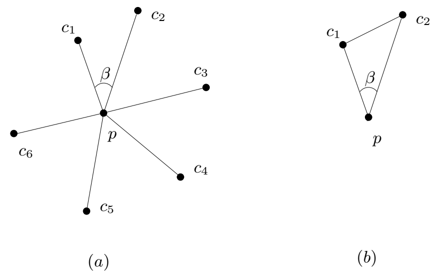

Fig. 3. The Proof of Lemma 2

For any i with $1 \leq i \leq |V| - 1$, let $\overline{D_i}$ denote the smallest closed disk that contains D_i. The last lemma of this section states that the union of all $\overline{D_i}$s is contained in a closed disk whose diameter is comparable to the diameter of $G^{1/\alpha}$.

Lemma 3. *Let $D = \bigcup_{e_i \in T} \overline{D_i}$. Then, D is contained into the closed disk whose diameter is equal to $\sqrt{2}R$ and whose center coincides with the center of D.*

Proof. Consider any two points x and y within D. It is easy to see that the worst case corresponds to the case in which both x and y are on the boundary of D. Consider the closed disk whose diameter is equal to $d(x, y)$ and whose center c' is on the midpoint of the segment $\overline{xy}$, and let z be any point on its boundary (see Fig. 4). It holds that $d(c, z) \leq \sqrt{2}R/2$, where c is the center of D. Indeed, from the triangular inequality we have that

$$d(c, z) \leq d(c, c') + d(c', z) = d(c, c') + d(x, y)/2.$$

Moreover, since the angle $cc'y$ is equal to $\pi/2$,

$$d(c, c')^2 + d(c', y)^2 = d(c, y)^2 = R^2/4.$$

Thus,

$$d(c, z) \leq \sqrt{\frac{R^2 - d(x, y)^2}{4}} + d(x, y)/2.$$

The right end of this equation reaches its maximum when $d(x, y) = \sqrt{2}R/2$, which implies $d(c, z) \leq \sqrt{2}R/2$. Hence the lemma follows. □

We are now able to prove Theorem 4. In particular, we have to prove that

$$\sum_{i=1}^{|V|-1} d(u_i, v_i)^2 \leq 10R^2, \tag{4}$$

where (u_i, v_i) is the ith edge in T, for $i = 1, \ldots, |V| - 1$. Indeed, let $\mathsf{Area}(D_i)$ denote the area of $\overline{D_i}$. It then holds that

$$\sum_{i=1}^{|V|-1} d(u_i, v_i)^2 = \frac{4}{\pi} \sum_{i=1}^{|V|-1} \mathsf{Area}(D_i). \tag{5}$$

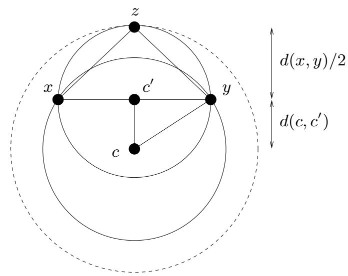

Fig. 4. The Proof of Lemma 3.

By combining Lemma 3 and 2, we have that

$$\sum_{i=1}^{|V|-1} \mathsf{Area}(D_i) \leq 5 \cdot \left[\pi \left(\frac{\sqrt{2}R}{2}\right)^2\right] = \frac{5}{2}\pi R^2. \tag{6}$$

By combining Eq. 5 and 6 we obtain Eq. 4, which proves the lemma for $\alpha = 2$.

Finally, we consider the case $\alpha > 2$. By using simple computations, we get

$$\begin{aligned}\mathsf{cost}(r) &= \sum_{i=1}^{|V|-1} d(u_i,v_i)^\alpha = \sum_{i=1}^{|V|-1} \left(d(u_i,v_i)^2\right)^{\alpha/2} \\ &\leq \left(\sum_{i=1}^{|V|-1} d(u_i,v_i)^2\right)^{\alpha/2} \leq 10^{\alpha/2}R^\alpha,\end{aligned}$$

where the last inequality follows from Eq. 4. This completes the proof of Theorem 4.

4 Further Results and Open Questions

Algorithm MST-ALG can be generalized to higher dimensions. In particular, it is possible to prove the following result.

Theorem 6. *There exists a function $f : \mathcal{N} \times \mathcal{R} \to \mathcal{R}$ such that, for any $d \geq 2$ and for any $\alpha \geq d$,* MECBS[N_d^α] *is approximable within factor $f(d,\alpha)$.*

The proof of the above theorem is again based on the computation of a MST of the input graph: Indeed, the algorithm is exactly the same. Unfortunately, the following result (whose proof is based on results in [8,13,25] and will be given in the full version of the paper) shows that the function f in the statement of the theorem grows exponentially with respect to d.

Theorem 7. *There exists a positive constant γ such that, for any d and for any k, an instance $x_{k,d}$ of* MECBS[N_d^d] *exists such that $\mathsf{opt}(x_{k,d}) = k^d$ while the cost of the range assignment computed by* MST-ALG *is at least $k^d \cdot 2^{\gamma d}$.*

One could also ask whether our algorithm approximates MECBS[N_d^α] in the case in which $d \geq 2$ and $\alpha < d$. Unfortunately, it is not difficult to produce an instance x such that $\mathsf{opt}(x) = O(n^{\alpha/d})$ while the cost of the range assignment computed by MST-ALG is $\Omega(n)$, where n denotes the number of vertices: For example, in the case $d = 2$, we can just consider the two dimensional grid of side $\sqrt{n}$ and the source node positioned on its center.

Open Problems. Three main problems are left open by this paper. The first one is to improve the analysis of MST-ALG (or to develop a different algorithm with a better performance ratio). Actually, we have performed several experiments and it turns out that the practical value of the performance ratio of MST-ALG (in the case in which $d = 2$ and $\alpha = 2$) is between 2 and 3. The second open problem is to analyze the approximability properties of MECBS[N_d^α] when $\alpha < d$: In particular, it would be very interesting to study the three-dimensional case. As previously observed, the MST-based algorithm does not guarantee any approximation, and it seems thus necessary to develop approximation algorithms based on different techniques. The last open problem is to consider MECBS[N_1^α], for any $\alpha \geq 1$: In particular, we conjecture that this problem is solvable in polynomial time.

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