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Distributed Computing

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Computational Thinking
Exercise 1

1 Towers of Hanoi

The Towers of Hanoi is a classic mathematical puzzle. Its rules are the following (taken from Wikipedia):

The game consists of three rods and a number of disks of different sizes, which can slide onto any rod. The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top. The objective of the puzzle is to move the entire stack to another rod, obeying the following simple constraints:

- Only one disk can be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
- No larger disk may be placed on top of a smaller disk.

How can you solve this puzzle using recursive functions?

2 Nim Game

a) You are playing a variant of the Nim game with a friend. There are initially 21 sticks on the table. The two players take turns. In a turn, a player can either remove 1 or 3 sticks from the table. Additionally, if there is an even number of remaining sticks, they can remove half of them. If a player cannot make a valid move, i.e. cannot remove any stick, they lose.

You can assume that both players play optimally. How can you model this as a recursion problem? Do you want to start, or make your friend start? Can you also think of a way to speed up the recursive approach using dynamic programming?

b) We now play the same game with a new set of rules.

There are now 3 stacks of sticks, with 13, 21 and 42 sticks, respectively. In each move, the player chooses a single stack, and removes sticks from it according to the rules of the previous game. If at any time all the stacks have the same number of sticks, the last player having made a move loses.

Again, model and solve this problem as a recursion problem.