Computational Thinking
Exercise 14

1 PCP warm-up

Do the following PCPs have a solution?

- a) Domino set $\left[ \frac{aa}{aaa} \right]$, $\left[ \frac{abba}{bbb} \right]$, $\left[ \frac{aba}{aab} \right]$, $\left[ \frac{bbab}{bab} \right]$.
- b) Domino set $\left[ \frac{ab}{ba} \right]$, $\left[ \frac{abab}{baab} \right]$, $\left[ \frac{ba}{ab} \right]$.
- c) Domino set $\left[ \frac{abbb}{b} \right]$, $\left[ \frac{bb}{bba} \right]$, $\left[ \frac{ac}{ca} \right]$, $\left[ \frac{ac}{ca} \right]$, $\left[ \frac{bb}{bba} \right]$.
- d) Domino set $\left[ \frac{ad}{bad} \right]$, $\left[ \frac{bc}{cba} \right]$, $\left[ \frac{ca}{ac} \right]$, $\left[ \frac{dc}{bc} \right]$.

2 PCP variants

Are the following variants of the PCP problem decidable or undecidable?

- a) $ab^k$ PCP: each word $\alpha$ and each word $\beta$ has the following form: it starts with a single letter $a$, and then an arbitrary number of letters $b$. Some examples for valid words are $a$, $abb$ or $abbbbbb$.
- b) Limited-use PCP: given an integer parameter $k$ in the input, we only accept domino sequences that contain each domino at most $k$ times.
- c) Unique-triplet PCP: we only accept domino sequences where no consecutive triplet of dominoes appears two times, i.e. there are no distinct indices $i,j$ such that each of the following three pairs of dominoes are the same: those at positions $i$ and $j$, those at positions $(i+1)$ and $(j+1)$, and those at positions $(i+2)$ and $(j+2)$.
- d) Two-color PCP: besides the two words $(\alpha, \beta)$, dominoes also have a color: each domino is painted red or blue. We only accept domino sequences that are alternating, i.e. a red domino is always followed by a blue domino, and vice versa.
- e) Half-used PCP: given the input set of dominoes $S$, we only accept domino sequences that use at most half of the domino types (possibly with repetitions), i.e. there are at least $\frac{1}{2} \cdot |S|$ input dominoes that never occur in the sequence.
- f) Silly PCP: for each domino $(\alpha, \beta)$ of the input set, the two words have the same length, i.e. we have $|\alpha| = |\beta|$.
- g) Almost-silly PCP: for some constant integer $c > 1$, the length of each word $\alpha$ and each word $\beta$ has to be a multiple of $c$.
- h) Binary PCP: the size of the alphabet is restricted to two characters, i.e. $\Sigma = \{0, 1\}$.