

# Discrete Event Systems

## Exam Questions 1 - 4

Saturday, 3<sup>rd</sup> February 2018, 09:00–11:00.

**Do not open until told to by the supervisors!**

The exam lasts 120 minutes, and there is a total of 120 points. The maximal number of points for each question is indicated in parentheses. Your answers must be in English. Be sure to always justify your answers.

Give your solutions on the page corresponding to the exam question and/or the empty one(s) following it. In case you run out of space, we also added extra pages – in case you use them, please indicate which question you are solving! Should even this not be enough, please contact a supervisor.

Please write down your name and Legi number (student ID) in the boxes below. Once the exam starts, also write your name on every page in the top right corner.

Name	Legi-Nr.

## Points

Question	Topic	Achieved Points	Maximal Points
1	Regular Languages		25
2	Context-free Languages		15
3	King of the Hill		15
4	GPU Rental		25

# 1 Regular Languages

(25 points)

## 1.1 NFA, DFA, REX [15 points]

Consider the language  $L = \{w \mid w \in \{0,1\}^* \text{ such that } w \text{ contains an even number of zeros or exactly two ones}\}$ . For example, the strings 00011 and 111 belong to  $L$ , while the strings 100000 and 1110 do not. The empty string belongs to  $L$ .

- a) [5] Draw a NFA that recognizes  $L$  with 6 or less states.
- b) [6] Draw a DFA that recognizes  $L$  with 8 or less states.
- c) [4] Give a Regular Expression for  $L$ .

Additional space for 1.1

## 1.2 Regular or not? [10 points]

Are the following languages regular? If so, exhibit a finite automaton (deterministic or not) or a regular expression for it. If not, prove it formally using the pumping lemma or the closure properties of regular languages.

- a) [5]  $L = \{0^n \mid n \text{ is a power of } 2\}$ .
- b) [5]  $L = \{w \mid w \in \{0, 1\}^* \wedge \text{there is no } x \text{ such that } w = xx\}$ .

Additional space for 1.2

## 2 Context-free languages

(15 points)

### 2.1 Write me a grammar [5 points]

Give a context-free grammar (the production rules) for the following language:

$$L = \{a^n b^m \mid 0 \leq n \leq m \leq 3n\}$$

## 2.2 Ambiguous or Not? [5 points]

Let  $G$  be the context-free grammar given by the following production rules:

$$S \rightarrow 0A \mid 1B$$

$$A \rightarrow 0AA \mid 1S \mid 1$$

$$B \rightarrow 1BB \mid 0S \mid 0$$

Is  $G$  ambiguous? Explain your answer.

**2.3 Draw me a PDA** [5 points]

Consider the following Language:

$$L = \{a^{2n}b^{3n} \mid n \geq 0\}$$

Design a PDA that accepts  $L$ . The designed PDA must have *9 or less* states.

Additional space for question 2

### 3 King of the Hill

(15 points)

King of the Hill is a game played between two players P1 and P2. The playground is a terrain with seven hills. There is a cage on top of some hill. After some random time (Poisson process with expected time 1 minute) the cage will disappear on the hill, and immediately appear again on another hill. The objective of the players is to be in the cage alone. Whenever a player is alone in the cage for time  $t$ , the player will earn  $t$  points. The time taken by P1 and P2 to get to the cage is a Poisson process with expected value  $1/2$  minute and 1 minute, respectively. When both players are inside the cage, no player will earn points, and there is a shootout between them. The shootout duration is a Poisson process and takes  $1/2$  minute on average. P1 wins the shootout with probability  $p$  and P2 with probability  $1 - p$ . The player who wins the shootout remains in the cage, whereas the other player again spawns somewhere in the playground, heading again for the cage.

- a) [5] Model the game as a Continuous Time Markov Chain.
- b) [5] If the game is played long enough, what is the probability that the cage is empty?
- c) [5] Assume P2 wins each and every shootout, i.e.,  $p = 0$ . If the game is played long enough, which player gains more points in expectation?

Additional space for question 3

## 4 GPU

(25 points)

Alice needs a graphics processing unit (GPU) to do deep learning research. She can

- buy a GPU at time  $t$  for the price  $b(t)$ ,
- or rent a GPU with rental cost rate  $r(t)$ : If Alice rents a GPU from time  $t_1$  to  $t_2$ , then the total rental cost is  $\int_{t_1}^{t_2} r(t)dt$ .

Alice does not know the time (denoted by  $T$ ) when she will not be interested in deep learning anymore. That is, after unknown time  $T$ , she does not need a GPU. Let's assume that

- $b: R_+ \mapsto R_+$  is a non-increasing function, i.e.,  $b(t_1) \geq b(t_2)$  if  $t_1 < t_2$ ,
- $r$  is positive and  $\int_0^\infty r(t)dt > b(0)$ ,
- and in any time interval  $(t_1, t_2)$  the price reduction is always smaller than the rental cost, i.e.,  $b(t_1) - b(t_2) < \int_{t_1}^{t_2} r(t)dt$ .

We focus on the cost in the interval  $[0, T]$ .

- a) [5] What is the optimal offline strategy, i.e., when  $T$  is known?
- b) [6] Design a deterministic online algorithm with a competitive ratio of 2 for any  $b(t)$  and  $r(t)$ .
- c) [6] Assume  $b$  is a constant function, that is, the machine always costs the same. Now show a lower bound of the competitive ratio of deterministic algorithms.
- d) [8] Suppose  $b(t) = \max\{1000 - t, 200\}$  and  $r(t) = 3$ . Design a deterministic online algorithm with competitive ratio strictly smaller than 2.

Additional space for question 4

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