Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich
Networked Systems Group (NSG)

HS 2024
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based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 4

1 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

a) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

b) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

2 Regular and Context-Free Languages

a) Consider the context-free grammar G with the production $S \to SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

3 Push Down Automata

For each of the following context free languages, draw a PDA that accepts L.

a) $L = \{u \mid u \in \{0,1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{“}u \text{ is a palindrome”}\}$

b) $L = \{u \mid u \in \{0,1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{“}u \text{ is no palindrome”}\}$

4 Ambiguity

Consider the following context-free grammar G with non-terminals S and A, start symbol S, and terminals "(", ")", and "0":

$$
\begin{aligned}
S &\to SA \mid \varepsilon \\
A &\to AA \mid (S) \mid 0
\end{aligned}
$$

a) What are the eight shortest words produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

5 Counter Automaton

A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional *counter* C, i.e., a register that can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a *Counter Automaton* M. M can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let $\mathcal{L}_{count}$ be the set of languages recognized by counter automata.

a) Let $\mathcal{L}_{reg}$ be the set of regular languages. Prove that $\mathcal{L}_{reg} \subseteq \mathcal{L}_{count}$.

b) Prove that the opposite is not true, that is, $\mathcal{L}_{count} \nsubseteq \mathcal{L}_{reg}$. Do so by giving a language which is in $\mathcal{L}_{count}$, but not in $\mathcal{L}_{reg}$. Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.

c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.

6 Inequality-checking with PDAs [Exam HS20]

Draw a PDA that recognizes $L = \{x\#y \mid x, y \in \{0,1\}^*,\ x \neq y\}$. Use at most 12 states.

Hint: Can two strings x and y with $|x| < |y|$ ever be equal?