Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Networked Systems Group (NSG)

HS 2018

Prof. L. Vanbever & A. El-Hassany, & M. Apostolaki.
based on Prof. R. Wattenhofer's material

Discrete Event Systems

Exercise Sheet 4

1 Pumping Lemma Revisited

a) Determine whether the language $L = \{1^{n^2} \mid n \in \mathbb{N}\}$ is regular. Prove your claim!

b) Consider a regular language L and a pumping number p such that every word $u \in L$ can be written as $u = xyz$ with $|xy| \leq p$ and $|y| \geq 1$ such that $xy^iz \in L$ for all $i \geq 0$.

Can you use the pumping number p to determine the number of states of a minimal DFA accepting L? What about the number of states of the corresponding NFA?

2 Context Free Grammars

Determine the context free grammar for the following three languages.

a) $L_1 = \{w\#x\#y\#z \mid w, x, y, z \in \{a, b\}^* \text{ and } |w| = |z|, |x| = |y|\}$

b) $L_2 = \{w \mid \text{the length of } w \text{ is odd}\}$

c) $L_3 = \{w \mid \text{contains more 1s than 0s}\}$

Remark: Languages L_2 and L_3 are the same as in Exercice Sheet 3.

3 Pushdown Automata: Reloaded

Consider the following context-free grammar G with non-terminals S and A, start symbol S, and terminals "(", ")", and "0":

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

a) What are the eight shortest words produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.

Remark: **a)** and **b)** are taken from Exercice Sheet 3.

4 Push Down Automata: *The Never Ending Story*

For each of the following context free languages, draw a PDA that accepts L.

a) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{"}u \text{ is a palindrome"}\}$

b) $L = \{u \mid u \in \{0, 1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{"}u \text{ is no palindrome"}\}$