Discrete Event Systems
Exercise Sheet 5

1 Minimum Pumping length
Consider the regular language $L = 1^*0^+1^*0^+ \cup 111^+0^+$. Give the minimum pumping length and briefly explain the intuition behind your answer.

2 The art of being regular
Assume that the alphabet $\Sigma$ is $\{0, 1\}$ and consider the language $L = \{x#y \mid x + y = 3y\}$ in which $x$ and $y$ are binary numbers. For instance, the string 1000#100 belongs to $L$. Is $L$ regular? If so, exhibit a finite automaton (deterministic or not) or a regular expression recognizing it. If not, prove it formally using the pumping lemma or the closure properties of regular languages.

3 Counter Automaton
A push-down automaton is basically a finite automaton augmented by a stack. Consider a finite automaton that (instead of a stack) has an additional counter $C$, i.e., a register that can hold a single integer of arbitrary size. Initially, $C = 0$. We call such an automaton a Counter Automaton $M$. $M$ can only increment or decrement the counter, and test it for 0. Since theoretically, all possible data can be coded into one single integer, a counter automaton has unbounded memory. Further, let $L_{\text{count}}$ be the set of languages recognized by counter automata.

   a) Let $L_{\text{reg}}$ be the set of regular languages. Prove that $L_{\text{reg}} \subseteq L_{\text{count}}$.

   b) Prove that the opposite is not true, that is, $L_{\text{count}} \nsubseteq L_{\text{reg}}$. Do so by giving a language which is in $L_{\text{count}}$, but not in $L_{\text{reg}}$. Characterize (with words) the kind of languages a counter automaton can recognize, but a finite automaton cannot.

   c) Which automaton is stronger? A counter automaton or a push-down automaton? Explain your decision.