Discrete Event Systems

Exercise Session 4

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1  Context-Free Grammars

Give context-free grammars for the following languages over the alphabet \( \Sigma = \{0, 1\} \):

a) \( L_1 = \{ w \mid \text{the length of } w \text{ is odd} \} \)

b) \( L_2 = \{ w \mid \text{contains more 1s than 0s} \} \)
2 Regular and Context-Free Languages

a) Consider the context-free grammar $G$ with the production $S \rightarrow SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language $L$ that is regular.
3 Push Down Automata

For each of the following context free languages, draw a PDA that accepts $L$.

a) $L = \{ u \mid u \in \{0, 1\}^* \text{ and } u^{\text{reverse}} = u \} = \{ u \mid \text{“}u\text{ is a palindrome”}\}$

b) $L = \{ u \mid u \in \{0, 1\}^* \text{ and } u^{\text{reverse}} \neq u \} = \{ u \mid \text{“}u\text{ is no palindrome”}\}$
4 Ambiguity

Consider the following context-free grammar $G$ with non-terminals $S$ and $A$, start symbol $S$, and terminals “(”, “)”, and “0”:

$$
S \rightarrow SA | \varepsilon \\
A \rightarrow AA | (S) | 0
$$

a) What are the eight shortest words produced by $G$?

b) Context-free grammars can be ambiguous. Prove or disprove that $G$ is unambiguous.

c) Design a push-down automaton $M$ that accepts the language $L(G)$. If possible, make $M$ deterministic.