Discrete Event Systems

Exercise Session 4

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1 Context-Free Grammars

Give context-free grammars for the following languages over the alphabet $\Sigma = \{0, 1\}$:

a) $L_1 = \{w \mid \text{the length of } w \text{ is odd}\}$

b) $L_2 = \{w \mid \text{contains more 1s than 0s}\}$

2 Regular and Context-Free Languages

a) Consider the context-free grammar G with the production $S \to SS \mid 1S2 \mid 0$. Describe the language $L(G)$ in words, and prove that $L(G)$ is not regular.

b) The regular languages are a subset of the context-free languages. Give the context-free grammar for an arbitrary language L that is regular.

3 Push Down Automata

For each of the following context free languages, draw a PDA that accepts L.

a) $L = \{u \mid u \in \{0,1\}^* \text{ and } u^{reverse} = u\} = \{u \mid \text{“}u \text{ is a palindrome”}\}$

b) $L = \{u \mid u \in \{0,1\}^* \text{ and } u^{reverse} \neq u\} = \{u \mid \text{“}u \text{ is no palindrome”}\}$

4 Ambiguity

Consider the following context-free grammar G with non-terminals S and A, start symbol S, and terminals "(", ")", and "0":

$$\begin{aligned} S &\rightarrow SA \mid \varepsilon \\ A &\rightarrow AA \mid (S) \mid 0 \end{aligned}$$

a) What are the eight shortest words produced by G?

b) Context-free grammars can be ambiguous. Prove or disprove that G is unambiguous.

c) Design a push-down automaton M that accepts the language $L(G)$. If possible, make M deterministic.