Discrete Event Systems

Solution to Exercise Sheet 11

1 Set Representation

1.1 Warm-up

a) $\psi_X = 1$

b) $N \cup E = X \iff \psi_N + \psi_E = 1$

c) $N \cap O = \emptyset \iff \psi_N \cdot \psi_O = 0$

d) $Q_1 = E \setminus O \iff \psi_{Q_1} = \psi_E \cdot \overline{\psi_O}$

e) $Q_2 = (O \cap E) \cup \overline{O} = (O \cup \overline{O}) \cap (E \cup \overline{O}) = X \cap (E \cup \overline{O}) = E \cup \overline{O} \iff \psi_{Q_2} = \psi_E + \psi_O$
1.2 Specification Composition

a) The specification for C1, C2 and C3 are the following:

\[ \psi_{C1} = (x_1 + x_2 + x_3) \rightarrow x_s \]
\[ \psi_{C1} = (x_1 + x_2 + x_3) x_s + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} = x_s + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} \]

\[ \psi_{C2} = x_1 \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_1} \cdot x_2 \cdot \overline{x_3} + \overline{x_1} \cdot \overline{x_2} \cdot x_3 + \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} \]

\[ \psi_{C3} = x_b \rightarrow (x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3}) \]
\[ \psi_{C3} = x_b \cdot x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_b} = x_s \cdot \overline{x_1} \cdot \overline{x_2} \cdot \overline{x_3} + \overline{x_b} \]

b) The specification consists in satisfying all constraints at all times:

\[ \psi_N = \psi_{C1} \cdot \psi_{C2} \cdot \psi_{C3} \]
2 Binary Decision Diagrams

2.1 Verification using BDDs

a) $f_2 : y = x_1 + x_2 + x_3 + x_1 + x_2 + x_3 + x_1 + x_2 + x_3$

b) for $f_1$, we have
   - case $x_1 = 0$:
     - case $x_2 = 0$:
       - $y_{x_1=0,x_2=0} = x_3$
     - case $x_2 = 1$:
       - $y_{x_1=0,x_2=1} = x_3$
   - case $x_1 = 1$:
     - case $x_2 = 0$:
       - $y_{x_1=1,x_2=0} = 1$
     - case $x_2 = 1$:
       - $y_{x_1=1,x_2=1} = x_3$

for $f_2$, we have
   - case $x_1 = 0$:
     - case $x_2 = 0$:
       - $y_{x_1=0,x_2=0} = x_3 + 1 + x_3 = x_3$
     - case $x_2 = 1$:
       - $y_{x_1=0,x_2=1} = 1 + x_3 = x_3$
   - case $x_1 = 1$:
     - case $x_2 = 0$:
       - $y_{x_1=1,x_2=0} = 1$
     - case $x_2 = 1$:
       - $y_{x_1=1,x_2=1} = x_3$

The two ROBDDs have identical falls, therefore they are equivalent.
2.2 BDDs with Respect to Different Orderings

a) \[ g = x_1 \left\{ x_2 [y_1 (y_2) + \overline{y_1} (0)] + \overline{x_2} [y_1 (\overline{y_2}) + \overline{y_1} (0)] \right\} + \overline{x_1} \left\{ x_2 [y_1 (0) + \overline{y_1} (y_2)] + \overline{x_2} [y_1 (0) + \overline{y_1} (\overline{y_2})] \right\} \]

b) The ROBDD for \( g \) is the following:

![ROBDD Diagram]

c) Using the new ordering \( \pi' \), the Boole-Shannon decomposition becomes

\[ g = x_1 \left\{ y_1 [x_2 (y_2) + \overline{x_2} (y_2)] + \overline{y_1} (0) \right\} + \overline{x_1} \left\{ y_1 [0] + \overline{y_1} [x_2 (y_2) + \overline{x_2} (y_2)] \right\} . \]

This is a better ordering as it leads to a ROBDD with fewer nodes with respect to \( \pi' \) (6 instead of 9).