Counter-example to the solution proposed for the Kleene* construction during Lecture 2

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Let $\Sigma = \{0, 1\}$. We consider the NFA $N_1$ depicted below which recognizes the language $1^*0$.

![Figure 1: NFA $N_1$](image1)

We want to build $N_2$, a NFA recognizing $(1^*0)^*$. We distinguish between two cases:

**Case 1: Following the lecture’s recipe** We obtain NFA $N_2$ (Figure 2).

![Figure 2: Case 1: NFA $N_2$](image2)

**Case 2: Following the suggestion in the lecture** As a reminder, the suggestion was to not add the extra starting state but turning the existing starting state into an accepting one instead (the rest of the recipe is kept as in case 1). Doing so, we obtain the NFA $N'_2$ (Figure 3).

![Figure 3: Case 2: NFA $N'_2$](image3)

One can see that $L(N'_2) \neq (1^*0)^*$. For instance, it accepts the string 11111 while it should not.