1. Pumping Lemma

Is following language regular?

\[ L = \{0^a1^b0^c1^d \mid a, b, c, d \geq 0 \text{ and } a = 1, b = 2 \text{ and } c = d\} \]
1. Pumping Lemma

Assume for contradiction that $L$ is regular, $p$ is the pumping length. Let $w = 0110^p1^p$, $w \in L$ and $|w| > p$. We therefore consider the various cases.

* If $y$ starts anywhere within the first three symbols (i.e. 011) of $w$, deleting $y$ creates a word with an illegal prefix (e.g. 1 0 1 for $y = 01$).

* If $y$ consists of only 0s from the second block, the word $w' = xy_2z$ has more 0s than 1s in the last $|w'| - 3$ symbols and hence $c \neq d$. Note that $y$ cannot contain 1s from the second block because of the requirement $|xy| \leq p$.

Therefore, $L_1$ cannot be regular and we have a contradiction.
1. Pumping Lemma

Assume for contradiction that \( L \) is regular, \( p \) is the pumping length. Let \( w = 0110^p1^p \), \( w \in L \) and \( |w| > p \).

From \( pL \), \( w \) can be split into 3 parts: \( w = xyz \), where \( |xy| \leq p \) and for any \( i \geq 0 \), we have \( xy^iz \in L \).

* If \( y \) starts anywhere within the first three symbols (i.e. 011) of \( w \), deleting \( y \) creates a word with an illegal prefix (e.g. 1 \( 0 \) \( 1 \) for \( y = 01 \)).

* If \( y \) consists of only 0s from the second block, the word \( w' = xy^2z \) has more 0s than 1s in the last \( |w'| - 3 \) symbols and hence \( c \neq d \).

Note that \( y \) cannot contain 1s from the second block because of the requirement \( |xy| \leq p \).

Therefore, \( L \) cannot be regular and we have a contradiction.
1. Pumping Lemma

Assume for contradiction that $L$ is regular, $p$ is the pumping length.
Let $w = 0110^p1^p$, $w \in L$ and $|w| > p$.

From $pL$, $w$ can be split into 3 parts: $w = xyz$, where $|xy| \leq p$ and for any $i \geq 0$, we have $xy^iz \in L$.

We therefore consider the various cases.
* If $y$ starts anywhere within the first three symbols (i.e. $011$) of $w$, deleting $y$ creates a word with an illegal prefix (e.g. $101$ for $y = 01$).
* If $y$ consists of only $0$s from the second block,

Note that $y$ cannot contain $1$s from the second block because of the requirement $|xy| \leq p$. \
1. Pumping Lemma

Assume for contradiction that $L$ is regular, $p$ is the pumping length. Let $w = 0110^p1^p$, $w \in L$ and $|w| > p$.

From $pL$, $w$ can be split into 3 parts: $w = xyz$, where $|xy| \leq p$ and for any $i \geq 0$, we have $xy^iz \in L$.

We therefore consider the various cases.

* If $y$ starts anywhere within the first three symbols (i.e. 011) of $w$, deleting $y$ creates a word with an illegal prefix (e.g. $10^p1^p$ for $y = 01$).
* If $y$ consists of only 0s from the second block, the word $w' = xy^2z$ has more 0s than 1s in the last $|w'| - 3$ symbols and hence $c \neq d$.

Note that $y$ cannot contain 1s from the second block because of the requirement $|xy| \leq p$.

Therefore, $L$ cannot be regular and we have a contradiction.
2. Deterministic Finite Automata [Exam]

Transform the NFA into an equivalent DFA, while assuming $\Sigma = \{0, 1\}$. (Hint: Only construct states which are necessary!)

---

Diagram:

- States: $q_1, q_2, q_3, q_4, q_5, q_6$
- Transitions:
  - $q_1 \xrightarrow{0} q_2$
  - $q_1 \xrightarrow{1} q_3$
  - $q_2 \xrightarrow{1} q_3$
  - $q_2 \xrightarrow{\varepsilon} q_4$
  - $q_3 \xrightarrow{1} q_5$
  - $q_4 \xrightarrow{1} q_5$
  - $q_4 \xrightarrow{\varepsilon} q_6$
  - $q_5 \xrightarrow{1} q_6$
  - $q_6 \xrightarrow{0} q_1$
2. Deterministic Finite Automata [Exam]
2 Deterministic Finite Automata [Exam]
2. Deterministic Finite Automata [Exam]
2. Deterministic Finite Automata [Exam]
2. Deterministic Finite Automata [Exam]
2. Deterministic Finite Automata [Exam]
2. Deterministic Finite Automata [Exam]
3. Transforming Automata [Exam]

Consider the DFA over the alphabet \( \Sigma = \{0, 1\} \). Give a regular expression for the language \( L \) accepted by the automaton below. If you like, you can do this by ripping out states as presented in the lecture.

Hint: remove \( q_2, q_1, q_3 \)
3. Transforming Automata [Exam]

Add start and accept

Ripe out q2
3. Transforming Automata [Exam]

Add start and accept

Ripe out q2
3. Transforming Automata [Exam]

Add start and accept

Ripe out q2
3. Transforming Automata [Exam]

Ripe out q1
3. Transforming Automata [Exam]

Ripe out q3

\((01^*0)^*1(0 \cup 11^*0(01^*0)^*1)^*\)
4. Pumping Lemma

a) \( L = 1^n 02^n \geq 0 \)

Is \( L \) regular?

Assume \( L \) is regular.
We take \( w = 1^p 0 2^p \in L \),
\( w = xyz \) with \( |xy| \leq p \) and \( |y| \geq 1 \), because of \( |xy| \leq p \), \( xy \) can only consist of 1s.
According to the pumping lemma, we should have \( xy^iz \in L \).
However, by choosing \( i = 0 \) we delete at least one 1 and get a word \( w' = 1^{p-|y|} 0 2^p \) with \( |y| \geq 1 \).
\( w' \) is not in \( L \) since it has fewer 1s than 2s.
This means that \( w \) is not pumpable and hence, \( L \) is not regular.