Automata & languages
A primer on the Theory of Computation

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Part 4 out of 4
Last week, we showed the equivalence of DFA, NFA and REX is equivalent to

\[
\text{DFA} \simeq \text{NFA} \quad \text{and} \quad \text{REX}
\]
We also started to look at non-regular languages

Pumping lemma

If $A$ is a regular language, then there exist a number $p$ s.t. any string in $A$ whose length is at least $p$ can be divided into three pieces $xyz$ s.t.

- $xyz \in A$, for each $i \geq 0$ and
- $|y| > 0$ and
- $|xy| \leq p$
To prove that a language $A$ is not regular:

1. Assume that $A$ is regular

2. Since $A$ is regular, it must have a pumping length $p$

3. Find one string $s$ in $A$ whose length is at least $p$

4. For any split $s=xyz$,
   Show that you cannot satisfy all three conditions

5. Conclude that $s$ cannot be pumped
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5. Conclude that $s$ cannot be pumped $\rightarrow$ $A$ is not regular
Pumping lemma

If $A$ is a regular language, then there exist a number $p$ s.t.

Any string in $A$ whose length is at least $p$ can be divided into three pieces $xyz$ s.t.

- $xyz \in A$, for each $i \geq 0$ and
- $|y| > 0$ and
- $|xy| \leq p$

Wait…

What happens if $A$ is a finite language?!
Pumping lemma

If *A* is a regular language, then there exist a number $p$ s.t.

As we saw two weeks ago, all finite languages are regular…

So what's $p$?

$p := \text{len}(\text{longest\_string}) + 1$

makes the lemma hold vacuously
Non-regular languages are not closed under most operations

If $L_1$ and $L_2$ are regular,
then so are

$\begin{align*}
L_1 \cup L_2 \\
L_1 \cdot L_2 \\
L_1^*
\end{align*}$

If $L_1$ and $L_2$ are not regular,
then

$\begin{align*}
L_1 \cup L_2 \\
L_1 \cdot L_2 \\
L_1^*
\end{align*}$

may or may not be regular!

$(L_1)^C$ is not regular

non RL are closed under complement
This week is all about

Context-Free Languages

a superset of Regular Languages