1 The Resilience of a Quorum System

a) Does a quorum system exist, which can tolerate that all nodes of a specific quorum fail? Give an example or prove its nonexistence.

b) Consider the nearly all quorum system, which is made up of \( n \) different quorums, each containing \( n - 1 \) servers. What is the resilience of this quorum system?

c) Can you think of a quorum system that contains as many quorums as possible? Note: the quorum system does not have to be minimal.

Basic

2 A Quorum System

Consider a quorum system with 7 nodes numbered from 001 to 111, in which each three nodes fulfilling \( x \oplus y = z \) constitute a quorum. In the following picture this quorum system is represented: All nodes on a line (such as 111, 010, 101) and the nodes on the circle (010, 100, 110) form a quorum.

a) Of how many different quorums does this system consist and what are its work and its load?

b) Calculate its resilience \( f \). Give an example where this quorum system does not work anymore with \( f + 1 \) faulty nodes.
3 Uniform Quorum Systems

Definitions:

\textbf{s-Uniform:} A quorum system $\mathcal{S}$ is \textit{s-uniform} if every quorum in $\mathcal{S}$ has exactly $s$ elements.

\textbf{Balanced access strategy:} An access strategy $Z$ for a quorum system $\mathcal{S}$ is \textit{balanced} if it satisfies $L_Z(v_i) = L$ for all $v_i \in V$ for some value $L$.

\textbf{Claim:} An $s$-uniform quorum system $\mathcal{S}$ reaches an optimal load with a balanced access strategy, if such a strategy exists.

\textbf{a)} Describe in your own words why this claim is true.

\textbf{b)} Prove the optimality of a balanced access strategy on an $s$-uniform quorum system.