1 Selling a Franc

Form groups of three to five people. One person is the auctioneer who has to provide one (imaginary) franc. Every other member of the group is a bidder. The franc is allocated to the highest bidder (for his/her last bid). Bids must be a multiple of CHF 0.05. This auction has a crux. Every bidder has to pay the amount of money he/she bid (last bid) – it does not matter if he/she gets the good. Play the game!

   a) Where did it all go wrong?
   b) What could the bidders have done differently?

Basic

2 Selfish Caching

   a) For each of the following caching networks, compute the social optimum, the pure Nash equilibria, the price of anarchy (PoA) as well as the optimistic price of anarchy (OPoA):

      i. \( d_u = d_v = d_w = d_x = 1 \)

      ![Diagram 1]

      ii. The demand is written next to a node.

      ![Diagram 2]
3 Selfish Caching with variable caching cost

The selfish caching model introduced in the lecture assumed that every peer incurs the same caching cost. However, this is a simplification of the reality. A peer with little storage space could experience a much higher caching cost than a peer who has terabytes of free disc space available. In this exercise, we omit the simplifying assumption and allow variable caching costs \( \alpha_i \) for node \( i \).

What are the Nash Equilibria in the following caching networks given that

i. \( \alpha_u = 1, \alpha_v = 2, \alpha_w = 2 \),

ii. \( \alpha_u = 3, \alpha_v = 3/2, \alpha_w = 3 \) ?

\[
\begin{array}{ccc}
  u & 3 & v \\
  d_u = 1/2 & d_v = 1/3 & d_w = 1/3 \\
  & 5 & w
\end{array}
\]

Does any of the above instances have a dominant strategy profile? What is the PoA of each instance?

Advanced

4 Matching Pennies

Tobias and Stephan like to gamble, and came up with the following game: Each of them secretly turns a penny to heads or tails. Then they reveal their choices simultaneously. If the pennies match Tobias gets both pennies, otherwise Stephan gets them.

Write down this 2-player game as a bi-matrix, and compute its (mixed) Nash equilibria!

Mastery

5 PoA Classes

The PoA of a class \( \mathcal{C} \) is defined as the maximum PoA over all instances in \( \mathcal{C} \). Let

- \( A_n^{[a,b]} \) be the class of caching networks with \( n \) peers, \( a \leq \alpha_i \leq b \), \( d_i = 1 \), and each edge has weight 1,

- \( W_n^{[a,b]} \) be the class of networks with \( n \) peers, \( a \leq d_i \leq b \), \( \alpha_i = 1 \), and each edge has weight 1.

Show that \( PoA(A_n^{[a,b]}) \leq \frac{b}{a} \cdot PoA(W_n^{[\frac{1}{2}, \frac{1}{2}]}) \) for all \( n > 0 \).