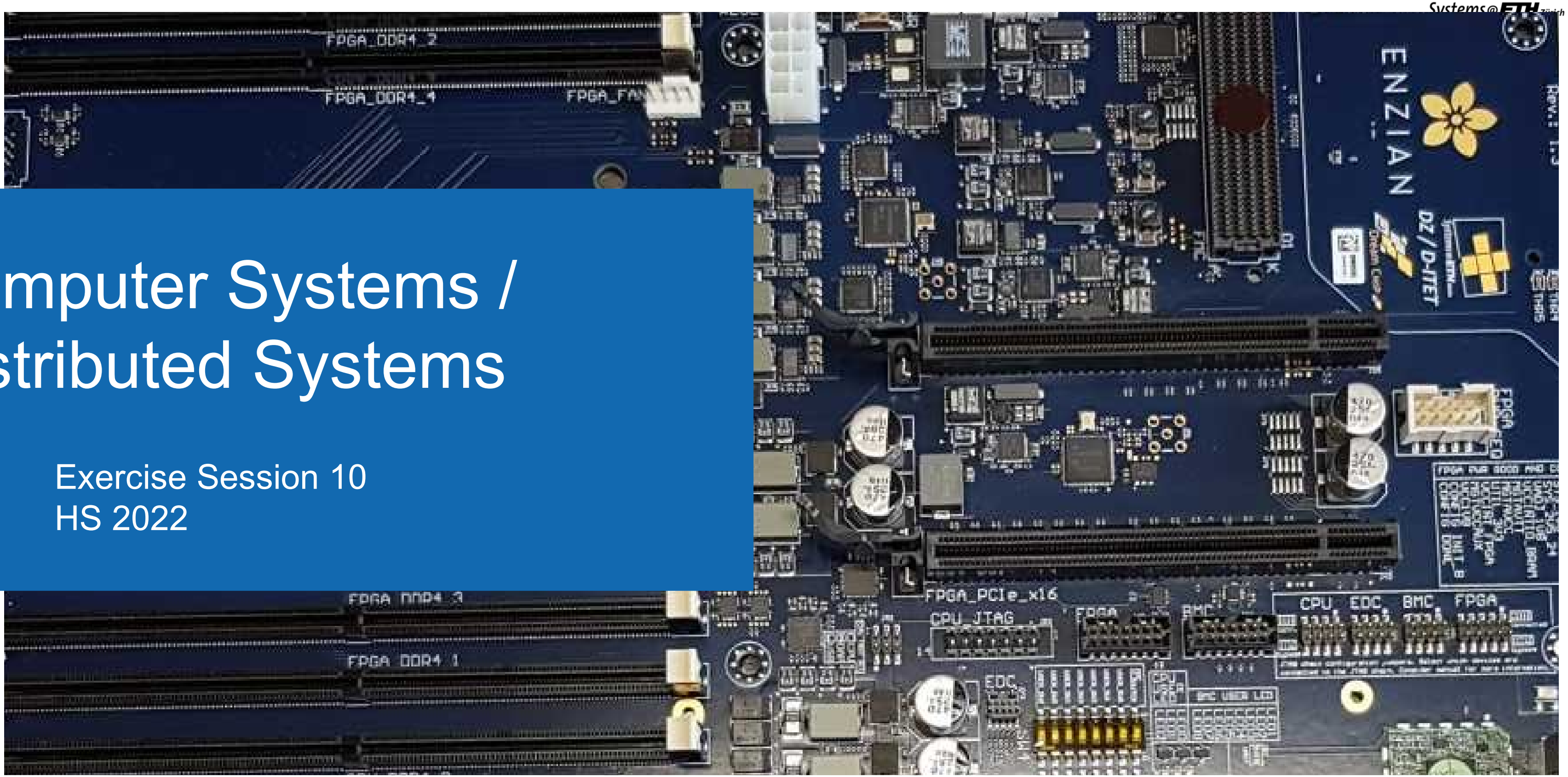
Distributed Computing

Slides last updated: 28.11.2023



Computer Systems / Distributed Systems

Exercise Session 10
HS 2022

Game Theory

Systems@ETH zürich

Distributed Computing

Prisoner's Dilemma - matrix representation of games

u v		Player u	
		Cooperate	Defect
Player v	Cooperate	1 1	0 3
	Defect	3 0	2 2

Game Theory - Terminology

Strategy	move
Strategy profile	set of strategies for all players specifying all actions in a game
Social optimum (SO)	
Dominant strategy (DS)	
Dominant strategy profile	
Nash equilibrium (NE)	

Example: Prisoners Dilemma

u v		Player u	
		Cooperate	Defect
Player v	Cooperate	1 1	0 3
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Strategy: Player v will play “Cooperate”

Strategy profile: Player v will play “Cooperate” and player u will play “Defect”

Dominant Strategy:

Social optimum:

Nash equilibrium:

Game Theory - Terminology

Strategy	move
Strategy profile	set of strategies for all players specifying all actions in a game
Social optimum (SO)	Strategy profile with the best sum of outcomes over players
Dominant strategy (DS)	The move that’s never worse than another strategy for a player
Dominant strategy profile	Every player plays a dominant strategy
Nash equilibrium (NE)	

Example: Prisoners Dilemma

u / v		Player u: Cooperate	Player u: Defect
Player v	Cooperate	1 1	0 3
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Strategy: Player v will play “Cooperate”

Strategy profile: Player v will play “Cooperate” and player u will play “Defect”

Dominant Strategy: Defect (if other player cooperates: 0<1; if other player defects 2<3)

Social optimum: Cooperate-Cooperate (cost: 2)

Nash equilibrium:

Game Theory - Terminology

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Strategy profile	set of strategies for all players specifying all actions in a game
Social optimum (SO)	Strategy profile with the best sum of outcomes over players
Dominant strategy (DS)	The move that's never worse than another strategy for a player
Dominant strategy profile	Every player plays a dominant strategy
Nash equilibrium (NE)	Strategy profile such that nobody can improve by unilaterally changing their move

Example: Prisoners Dilemma

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Strategy profile: Player v will play “Cooperate” and player u will play “Defect”

Dominant Strategy: Defect (if other player cooperates: 0<1; if other player defects 2<3)

Social optimum: Cooperate-Cooperate (cost: 2)

Nash equilibrium: Defect-Defect (cost: 4)

Selfish Caching

Consider a network. Nodes can either cache a file or fetch it through the network from another node. At least one node should store the file.

As a game:

- **Strategy:** cache or not cache
- **Cost:** 1 if cache, otherwise (shortest path to cache) * demand
  (Note: path lengths are symmetric (if undirected) but demands might vary)

Selfish Caching - Algorithm

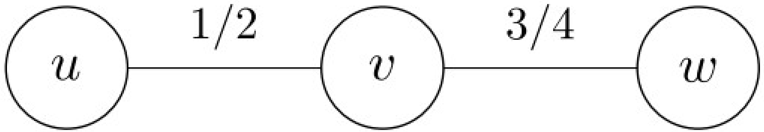
Algorithm 25.7 Nash Equilibrium for Selfish Caching

```
S = {}                                              //set of nodes that cache the file
repeat
    Let v be a node with maximum demand d_v in set V
    S = S ∪ {v}, V = V \ {v}
    Remove every node u from V with c_{u←v} ≤ 1
until V = {}
```

remove all candidates that are better off by fetching

$c_{u \leftarrow v}$ = cost for u of fetching from v, i.e. u-v-path length * demand of u

Selfish Caching - Example



With demands all 1

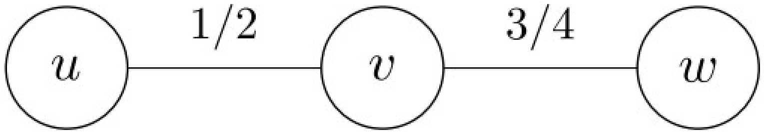
There are 2 NE, both can be found with algorithm depending on the start node:

Optimistic **NE** (start algo at v): **?**

Pessimistic **NE** (start algo at u or w): **?**

Social Optimum: **?**

Selfish Caching - Example



With demands all 1

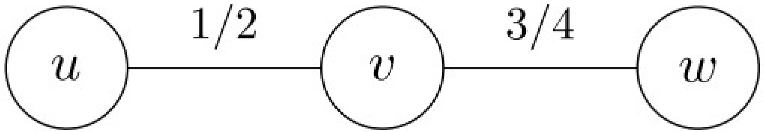
There are 2 NE, both can be found with algorithm depending on the start node:

Optimistic **NE** (start algo at v): **v caches** ⇒ Cost = 1/2 + 1 + 3/4 = **9/4**

Pessimistic **NE** (start algo at u or w): **?**

Social Optimum: **?**

Selfish Caching - Example



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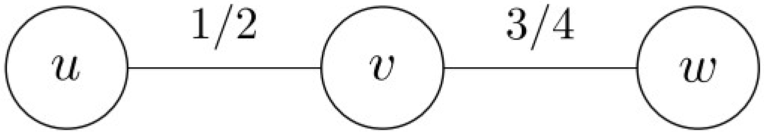
Social Optimum: **?**

Selfish Caching - Example



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Pessimistic **NE** (start algo at u or w): **u & w cache** ⇒ Cost = 1 + 1/2 + 1 = **10/4**

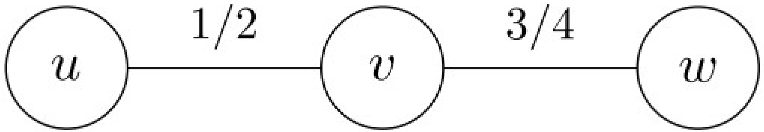
Social Optimum: **v caches** (same as Optimistic NE) ⇒ Cost = **9/4**

Price of Anarchy

Idea: With some rules, we could always enforce the social optimum. But what is the cost of having no rules (anarchy)?

- **Optimistic approach:** players will converge to “best” nash equilibrium.
  - Then, price of anarchy: $OPoA = \frac{\text{cost}(NE_+)}{\text{cost}(SO)}$
- **Pessimistic approach:** players will converge to “worst” nash equilibrium
  - Then, price of anarchy: $PoA = \frac{\text{cost}(NE_-)}{\text{cost}(SO)}$

Selfish Caching - Example



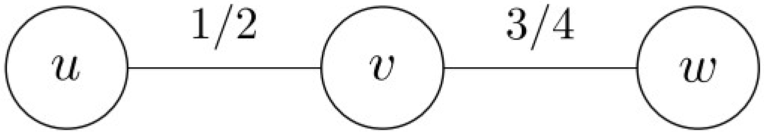
With demands all 1

Optimistic **NE**: **9/4** Pessimistic **NE**: **10/4** **Social Optimum**: **9/4**

PoA: ?

OPoA: ?

Selfish Caching - Example



With demands all 1

Optimistic **NE**: **9/4** Pessimistic **NE**: **10/4** **Social Optimum**: **9/4**

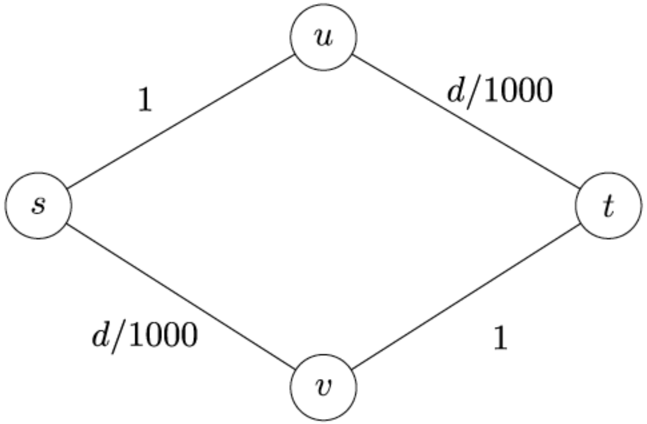
PoA: (10/4) / (9/4) = **10/9** > 1

OPoA: (9/4) / (9/4) = **1**

Braess Paradox

d = #drivers on link

NE for 1000 drivers:
split evenly across
s→u→t and s→v→t
⇒ cost = 1.5

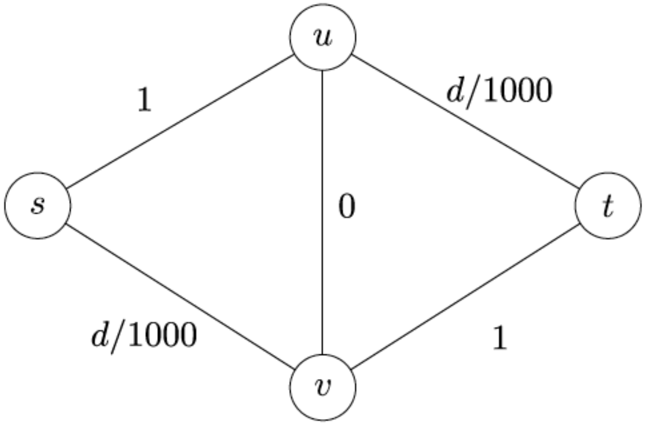


(a) The road network without the shortcut

Braess Paradox

adding link {u,v} makes the NE worse

consider even split, but then s→v→u→t costs just 1, so drivers will start switching until all choose that path ⇒ cost = 2



(b) The road network with the shortcut

Mixed Nash Equilibrium

Definition 25.16 (Mixed Nash Equilibrium). *A Mixed Nash Equilibrium (MNE) is a strategy profile in which at least one player is playing a randomized strategy (choose strategy profiles according to probabilities), and no player can improve their expected payoff by unilaterally changing their (randomized) strategy.*

Theorem 25.17. *Every game has a mixed Nash Equilibrium.*

v \ u		Player u: Rock	Player u: Paper	Player u: Scissors
Player v	Rock	0 / 0	1 / -1	-1 / 1
	Paper	-1 / 1	0 / 0	1 / -1
	Scissors	1 / -1	-1 / 1	0 / 0

Table 23.15: Rock-Paper-Scissors as a matrix.

MNE for rock paper scissors: Both players choose a strategy with ⅓ probability (due to symmetry)

Quiz (Assignment 11)

1.1 Selling a Franc

Form groups of two to three people. Every member of the group is a bidder in an auction for one (imaginary) franc. The franc is allocated to the highest bidder (for his/her last bid). Bids must be a multiple of CHF 0.05. This auction has a crux. Every bidder has to pay the amount of money he/she bid (last bid) – it does not matter if he/she gets the franc. Play the game!

a) Where did it all go wrong?

b) What could the bidders have done differently?

Quorum Systems

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Quorum Systems

High-level functionality:

1. Client selects a free quorum
2. Locks all nodes of the quorum
3. Client releases all locks

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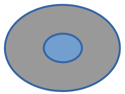
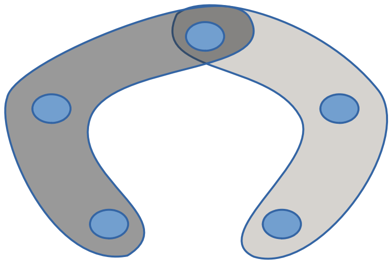
Singleton and Majority Quorum Systems



Singleton quorum system



Majority quorum system
(all sets of n / 2 + 1 nodes)

Load and Work

An access strategy Z defines the probability $P_Z(Q)$ of accessing a quorum Q ∈ S such that:

$$\sum_{Q \in S} P_Z(Q) = 1$$

Load and Work

- **Load** of access strategy Z on a node v_i

$$L_Z(v_i) = \sum_{Q \in S; v_i \in Q} P_Z(Q)$$

- **Load** induced by Z on quorum system S

$$L_Z(S) = \max_{v_i \in S} L_Z(v_i)$$

- **Load** of quorum system S

$$L(S) = \min_Z L_Z(S)$$

- **Work** of quorum Q

$$W(Q) = |Q|$$

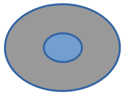
- **Work** induced by Z on quorum system S

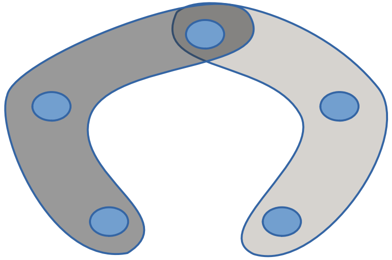
$$W_Z(S) = \sum_{Q \in S} P_Z(Q) \cdot W(Q)$$

- **Work** of quorum system S

$$W(S) = \min_Z W_Z(S)$$

Load and Work





Singleton quorum system

Majority quorum system
(all sets of n / 2 + 1 nodes)

	Singleton	Majority
How many servers need to be contacted? **(Work)**	1	> n/2
What's the load of the busiest server? **(Load)**	100%	≈ 50%
How many server failures can be tolerated? **(Resilience)**	0	< n/2

Basic Grid Quorum System

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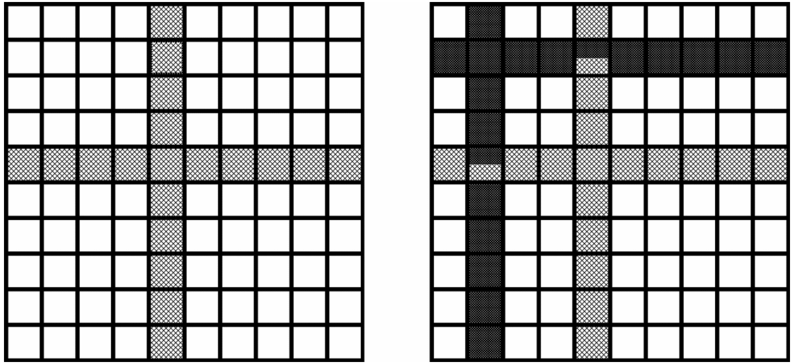
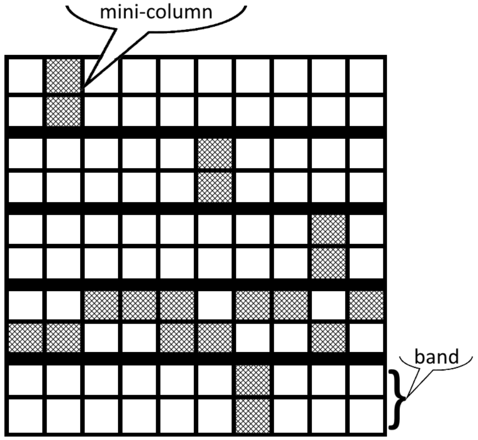
- Nodes arranged in a square matrix
- Each quorum i contains the union of row i and column i



B-Grid Quorum System

- Nodes arranged in rectangular grid with h • r rows
- Group of r rows is a band
- Group of r elements in the same column and band is a mini-column
- **Quorums** consists of one mini-column in every band and one element from each mini-column of one band



Quiz

1. Does a quorum system exist which can tolerate that all nodes of a specific quorum fail?

2. Consider the **nearly all** quorum system, which is made up of n different quorums, each containing n - 1 servers. What is the resilience?

3. Can you think of a quorum system that contains as many quorums as possible? Note: does not have to be minimal.

Quiz Solution

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Quiz Solution

1. Does a quorum system exist which can tolerate that all nodes of a specific quorum fail?

   A: no, as any two quorums intersect!

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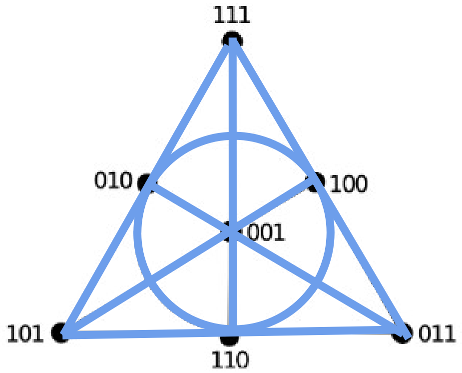
3. Can you think of a quorum system that contains as many quorums as possible? Note: does not have to be minimal.

   A: pick a node and take all quorums containing it. Maximality: between any quorum and its complement at most one can be in the system.

A Quorum System

Consider a quorum system with 7 nodes numbered from 001 to 111, in which each three nodes fulfilling $x \oplus y = z$ constitute a quorum. In the following picture this quorum system is represented: All nodes on a line (such as 111, 010, 101) and the nodes on the circle (010, 100, 110) form a quorum.



- Quorums: 7
- Work: 3
- Load: 3/7

a) Of how many different quorums does this system consist and what are its work and its load?

A Quorum System

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Distributed Computing

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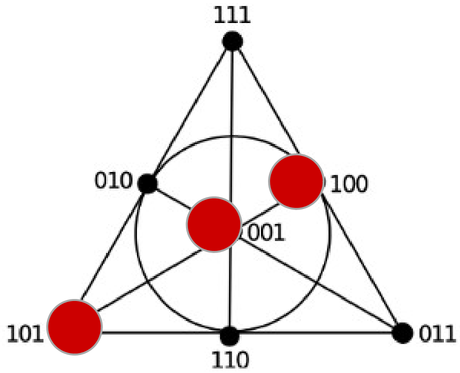

Resilience: 2

Every node is in 3 quorums
=> any two nodes can be contained in at most 2*3 quorums

b) Calculate its resilience f. Give an example where this quorum system does not work anymore with $f + 1$ faulty nodes.