Principles of Distributed Computing
Exercise 9

1 Sorting Networks

For each of the following questions, prove or disprove the given claim.

a) The network of 6 wires and 12 comparators in Figure 1 above is a sorting network, that is, it sorts each input sequence of numbers correctly.

b) Given any correct sorting network, adding another comparator at the end destroys the sorting property.

c) Given any correct sorting network, adding another comparator at the front does not destroy the sorting property.

d) Every correct sorting network needs to have at least one comparator between each two consecutive wires.

e) A network which contains all \( \binom{n}{2} \) comparators between any two of the \( n \) wires, in whatever order they are placed, is a correct sorting network.

f) Given any correct sorting network, adding another comparator anywhere does not destroy the sorting property. (Hint: Study examples with a small number of wires.)

g) Given any correct sorting network, entering a sequence at the “wrong” end (i.e., using the network backwards by calling the input wires output wires and vice versa) sorts the sequence as well.