Principles of Distributed Computing
Exercise 6

1 Deterministic Consensus

In this exercise, we assume that communication takes place in synchronous communication rounds. Consider the following simple deterministic algorithm:

Algorithm 1 Simple Deterministic Consensus
1: Broadcast own value to all other processors
2: Receive values from all other processors
3: Decide on the minimum value

We assume that nodes can crash at any point in time, in particular while sending messages to other processors.

a) Give an example that shows that Algorithm 1 does not guarantee that all processors reach consensus!

b) Assuming that at most \( f \) nodes crash, give an algorithm, similar to Algorithm 1, that guarantees that nodes reach consensus!\(^1\) How many rounds does your algorithm need?

2 Randomized Consensus

In the lecture, we studied a randomized consensus algorithm (Algorithm 27). In the remarks it says that the algorithm can be modified to tolerate up to \( f < n/4 \) crashing nodes.

a) Show that the algorithm tolerates \( f < n/8 \) crash failures, but is incorrect if \( f \geq n/8 \).

   **Hint:** The algorithm breaks down even if no processes crash at all!

b) Modify the algorithm such that it deals with \( f < n/4 \) crash failures.

c\(*\) Give an algorithm that solves the (asynchronous, crash-failure) consensus problem for arbitrary initial inputs from an alphabet \( \alpha_1, \ldots, \alpha_m \)! Prove that the number \( f(n, m) \) of tolerated failures is optimal.

\(^1\)You can use the number \( f \) in your algorithm.