Principles of Distributed Computing
Exercise 2: Sample Solution

1 Leader Election in an “Almost Anonymous” Ring

   a) Yes, it is possible:

   \textbf{Algorithm 1} Leader Election (all but one nodes have the same ID)
   \begin{align*}
   1: & \text{ send ID to each neighbor} \\
   2: & \text{ if if both received ID’s differ from the own ID then} \\
   3: & \quad \text{I am the leader} \\
   4: & \text{ end if}
   \end{align*}

   b) No, not always: If both differing processors have the same ID, the number of nodes is even, and these nodes are exactly opposite to each other, their local views will always remain completely identical. Thus they both must take the same decision or never terminate.\footnote{In an odd, directed ring, however, symmetry can be broken by means of the distance between the two nodes.}

2 Distributed Computation of the AND

a) Because the size of the ring is not known to the nodes, the case where all nodes have a one as input and the case where all but one nodes have a one as input cannot be distinguished.

b) All input values have to be sent all around the ring. In order to detect the returning of the own message, we add a hop counter to each message. If the message has made $n$ hops, it has arrived where it started.

c) The following algorithm calculates the AND in a synchronous, non-uniform ring:

Algorithm 2 AND in the Ring: asynchronous, non-uniform ($n$ is the number of nodes)

1: if input bit = 0 then
2:    send 0 to the neighbor in the ring
3: end if;
4: for $i := 2$ to $n$ do
5:    if received a 0 and have not already sent a 0 then
6:        send 0 to the neighbor in the ring
7:    end if
8: end for;
9: if received at least one 0 then
10:   result := 0
11: else
12:   result := 1
13: end if;

If the result is 1, no message is sent, otherwise there is exactly one message over each link. Thus, time and message complexity are both $n$. 