Principles of Distributed Computing
Exercise 3

1 Deterministic distributed algorithms in the port-numbering model

Deterministic algorithms in the port-numbering model are very limited. Prove that[1] the following problems cannot be solved in this model at all (**hint:** consider cycles):

a) Find a maximal independent set.

b) Find a maximal matching.

c) Find a factor-$100^{100^{100}}$ approximation of maximum-size independent set.

d) Find any node coloring.

e) Find any edge coloring.

f) Find a factor-1.999 approximation of minimum-size vertex cover.

How would your answers to the above questions change if you considered randomized algorithms in the port-numering model, i.e., each node has access to an unlimited source of random bits?

Hint: When condisering randomized algorithms, try to solve leader election with non-zero probability first.

2 Calculations with the $\log^*$ function

In the lecture, we have seen how to obtain an approximation to a minimum-size weighted vertex cover in $O(\Delta + \log^* k)$ rounds, where $k := (W(\Delta!)^{\Delta})^{\Delta}$ and weights are from $1, \ldots, W$. We now want to check that indeed $O(\Delta + \log^* k) \subseteq O(\Delta + \log^* W)$ as stated in the lecture.[2]

a) Define $M = \max\{W, \Delta, 4\}$. Show that $k \leq M^{2M^3}$.

Hint: You can use very crude and simple estimates.

b) Show that $2 \log M \leq M$ and hence $\log k \leq 2M^3 \log M \leq M^4$.

c) Conclude that $\log \log k \leq 4 \log M$, and show that $\log^* k \in O(\log^* M)$.

d) Infer that $\log^* k \in O(\log^*(\Delta) + \log^*(W))$.

e) To complete the analysis, show that $\Delta + \log^* k \in O(\Delta + \log^* W)$.

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[1] in the worst case – omitting this phrase is a common abuse of language whenever it is clear from the context that one is interested in worst-case analysis

[2] When using Big-O notation, frequently $\leq$, $<$, etc. are used in the notation. However, one has to keep in mind that e.g. $O(n)$ describes a set of functions—both $f(n) = 0$ and $g(n) = 100n + 20000$ are in $O(n)$, but certainly are very different!