Principles of Distributed Computing

Exercise 14

1 Pancake Networks

In the lecture, you have encountered several different graphs as underlying network structures for peer-to-peer (P2P) networks. Here, we will look at another prominent example, the Pancake graph\(^1\) \(P_n\).

The pancake graph \(P_n\) is defined as follows: The vertex set is
\[
V(P_n) = \{(v_1, v_2, \ldots, v_n) \mid v_i \in \{1, \ldots, n\} \text{ and } v_i \neq v_j \forall i \neq j\}.
\]

In other words, \(V(P_n) = S_n\), the group of all permutations on \(n\) elements. There exists an edge of dimension \(i\) for \(2 \leq i \leq n\) when
\[
eq \{(v_1, v_2, v_{i+1}, \ldots, v_n) \leftrightarrow (v_1, v_i, v_{i+1}, \ldots, v_n)\).
\]

For the following questions, where appropriate, give your answers in terms of \(N := |V(P_n)|\) (approximately), the number of vertices, as well as \(n\).

a) Draw (nicely!) \(P_n\) for \(n = 2, 3, 4\). Try to describe a pattern for drawing \(P_n\) for any \(n\).

b) What is the degree of each vertex in \(P_n\)?

c) Can you give bounds on the diameter \(D(P_n)\) of the pancake network?

d) Show that \(P_n\) is Hamiltonian for \(n \geq 3\).

e) How can the pancake graph be used to implement a distributed hash table (DHT)? In other words, where are files, indexed by bitstrings of a certain length \(b\), stored in the pancake graph, and how can these files be looked up (given the corresponding bitstring)?\(^2\)

The pancake graph has been proposed for P2P networks partly because of the properties analyzed in this exercise.


\(^2\)You can ignore churn in this exercise.