Principles of Distributed Computing
Exercise 5: Sample Solution

1 Shared Sum

In the following, let $X$ (initialized to 0) always denote the shared register used to hold the sum $x = \sum_{i=1}^{n} x_i$, and assume that all $x_i$ (and thus also $x$) are initially 0. Denote by $\Delta x_i$ the amount by which $x_i$ is changed by process $p_i$ at some time, i.e., if $x_i := x_i'$ is assigned by $p_i$, then $\Delta x_i = x_i' - x_i$.

a) To update $x$, $p_i$ calls fetch-and-add($X, \Delta x_i$). Therefore, $X$ changes exactly the same as $x_i$ and holds the correct value. Since no process has to wait or retry, we have neither lockouts nor deadlocks. A simple read on $X$ (or fetch-and-add($X, 0$)) gets the current value of $x$.

b) An update is done by the following code:

1: $x := X$
2: while not compare-and-swap($X, x, x + \Delta x_i$) do
3:   $x := X$
4: end while

The loop is left after $X$ changed by $\Delta x_i$ exactly once, thus the code is correct. Again, $x$ can be obtained by a simple read. Since the compare-and-swap may only fail if another process $p_j$ changed the value of $X$ between $p_i$ reading it and calling compare-and-swap, there is no deadlock. However, other updates may delay a change by some $p_i$ indefinitely, hence lockouts are possible.

c) A write is implemented by

1: $x := \text{load-link}(X)$
2: while not store-conditional($X, x + \Delta x_i$) do
3:   $x := \text{load-link}(X)$
4: end while

and is correct for the same reasons as in b). Reads are again simple. However, the solution differs from b) in that we may have deadlocks, since e.g., two processes can fight endlessly for getting the register linked to them for sufficiently long to write a value (At least for the case where a load-link can destroy the link to the register of another processor; i.e. weak LL/SC).

d) It can be done. We use a special encoding on $X$. Either it stores a regular value and $\perp$ (i.e., $(x, \perp)$) or the value and an additional identifier $\text{id}(i)$ of a process $p_i$. A node will effectively acquire a lock on $X$ by writing its ID to $X$ and only afterwards write its update to $X$. 
When \( x_i \) is changed, \( p_i \) executes

\begin{verbatim}
1: while true do
2:   \( (x, id) := X \) // simple read
3:   \( (x, id) := \text{compare-and-swap}(X, (x, \bot), (x, id(i))) \) // try to lock \( X \) with own ID
4:   if \( id = id(i) \) then
5:     \( X := (x + \Delta x_i, \bot) \) // regular write, but compare-and-swap would also do
6:       break while loop
7:   end if
8: end while
\end{verbatim}

Because writing by compare-and-swap works only if the second argument equals the value of the register, once a process “locks” \( X \) with its identifier, no other process may do so until the same process performs the write enclosed in the if-condition. Thus, this write happens exactly if the compare-and-swap was successful. The only reason to check the identifier by an if-statement rather than using compare-and-swap again is that we need to ensure that the process leaves the loop after changing \( X \) by \( \Delta x_i \). On the other hand, the while loop can only be left after a successful write, thus \( X \) is updated correctly. Reads are again plain reads.

As before, the solution is free of deadlocks: At least one process can write, because after each write the ID part of \( X \) contains \( \bot \), i.e., one process will succeed in “locking” \( X \). As in b) and c), the solution is prone to lockouts.