

# Bitcoin Privacy

João Pedro Monteiro

# Overview

1. What is Bitcoin?
2. Privacy Issues
3. ZeroCoin
4. eZC: ZeroCoin Reloaded

# What is Bitcoin?

“We're in the 21<sup>st</sup> Century and I can call someone in Indonesia,  
see him on screen and talk to him for free...

“We're in the 21<sup>st</sup> Century and I can call someone in Indonesia,  
see him on screen and talk to him for free...  
... and yet I can't send him **1 cent.**”

(Wences Casares, Xapo CEO)



WikiLeaks

@wikileaks

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WikiLeaks now accepts anonymous  
Bitcoin donations on  
**1HB5XMLmzFVj8ALj6mfBsbifRoD4miY36v**



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# WikiLeaks

Addresses are identifiers which you use to send bitcoins to another person.

## Summary

Address [1HB5XMLmzFVj8ALj6mfBsbifRoD4miY36v](#)

Hash 160 [b169f2b0b866db05900b93a5d76345f18d3afb24](#)

Tools [Taint Analysis](#) - [Related Tags](#) - [Unspent Outputs](#)

## Transactions

No. Transactions 2931 

Total Received **\$ 955,004.61** 

Final Balance **\$ 143.31** 

[Request Payment](#)

[Donation Button](#)

# PayPal stops payment delivery to Mega, citing 'business reasons'

By [Russell Brandom](#) on February 27, 2015 04:25 pm [Email](#) [@russellbrandom](#)

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Let's give Bitcoin a boost :-) #Mega

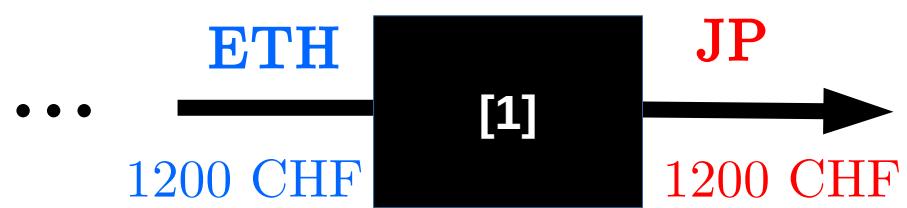
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7,636 RETWEETS 687 FAVORITES

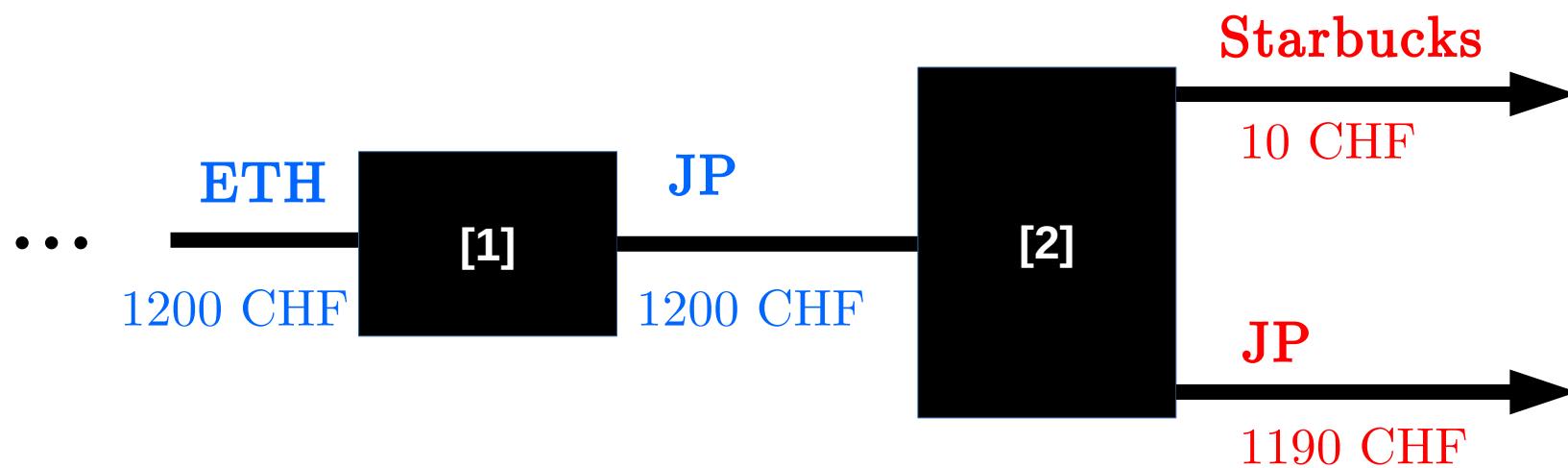
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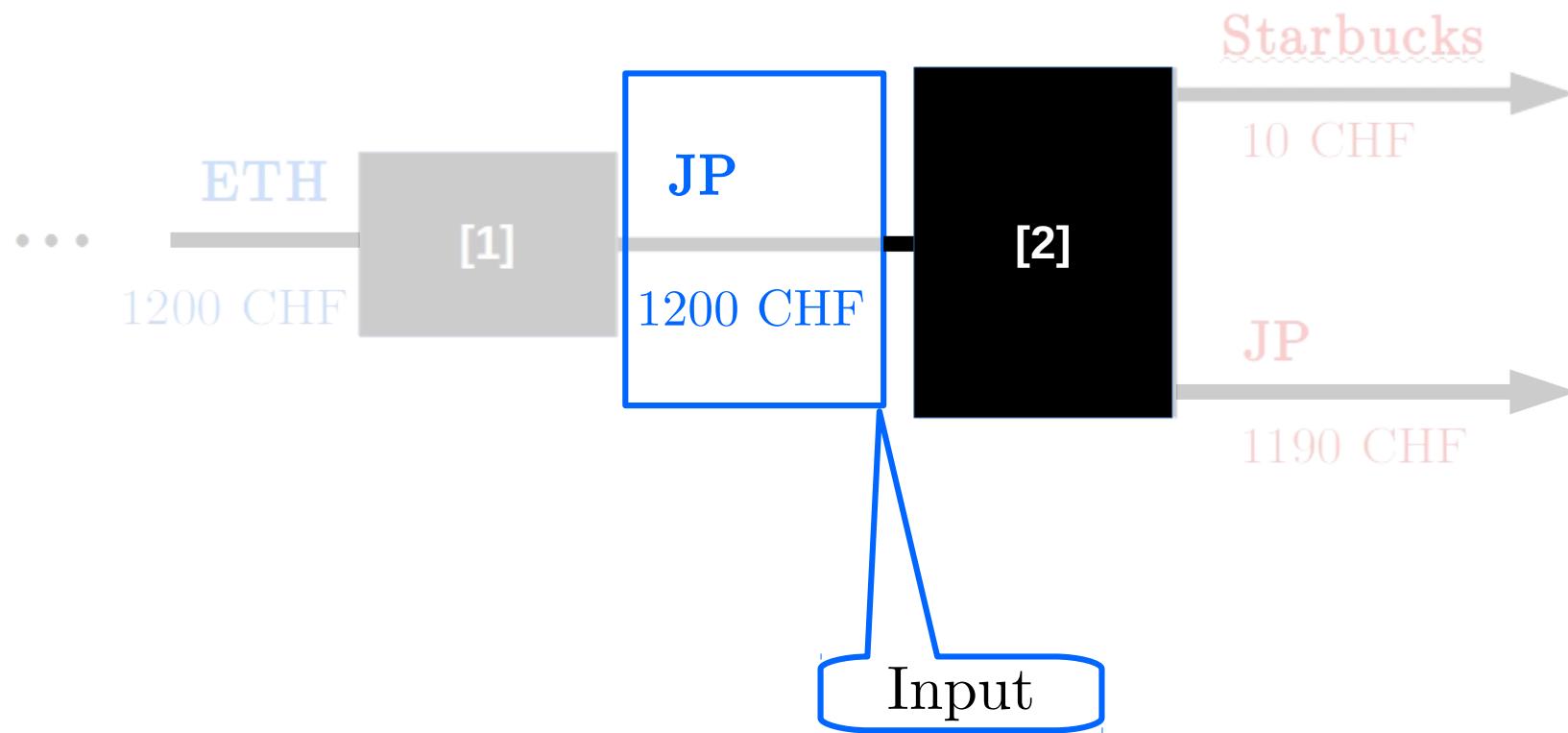
# Transactions



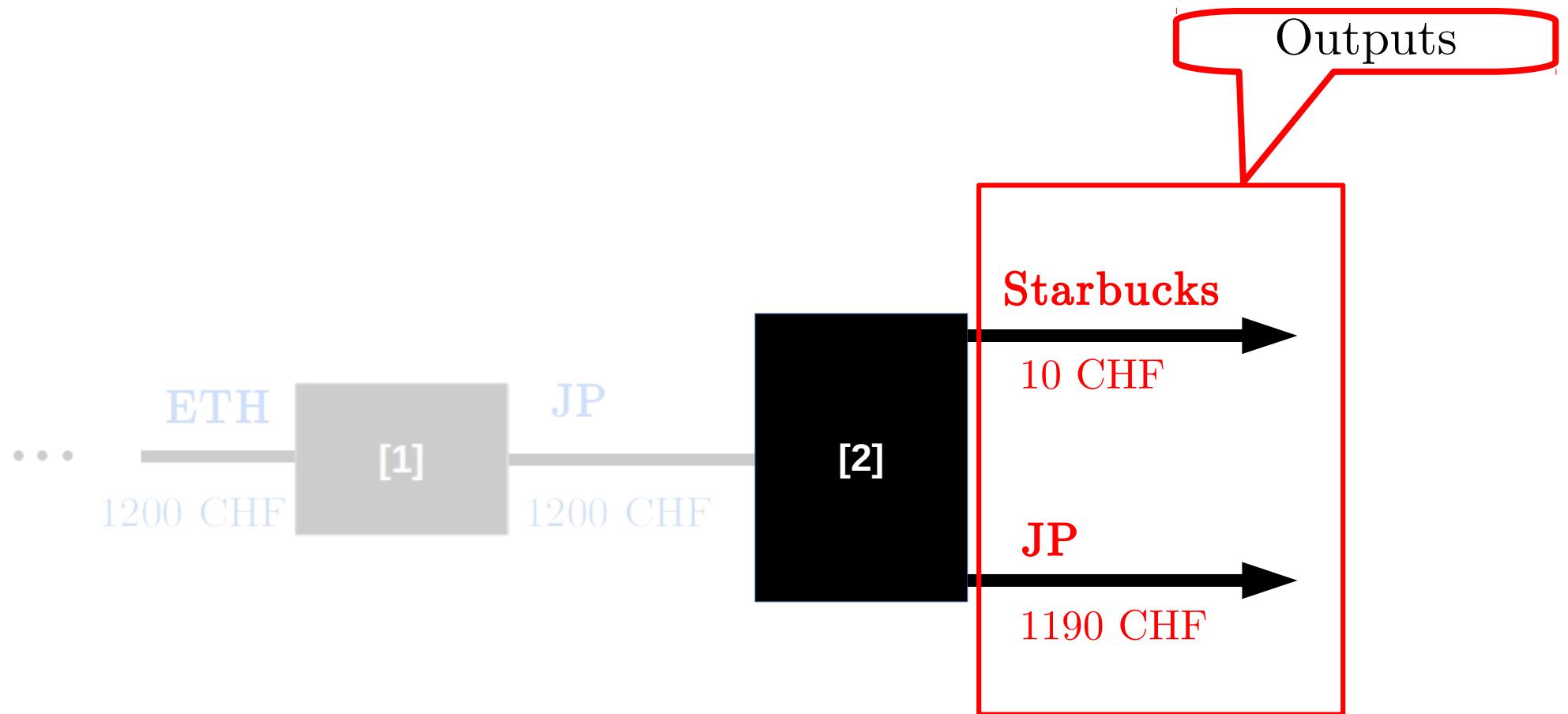
# Transactions



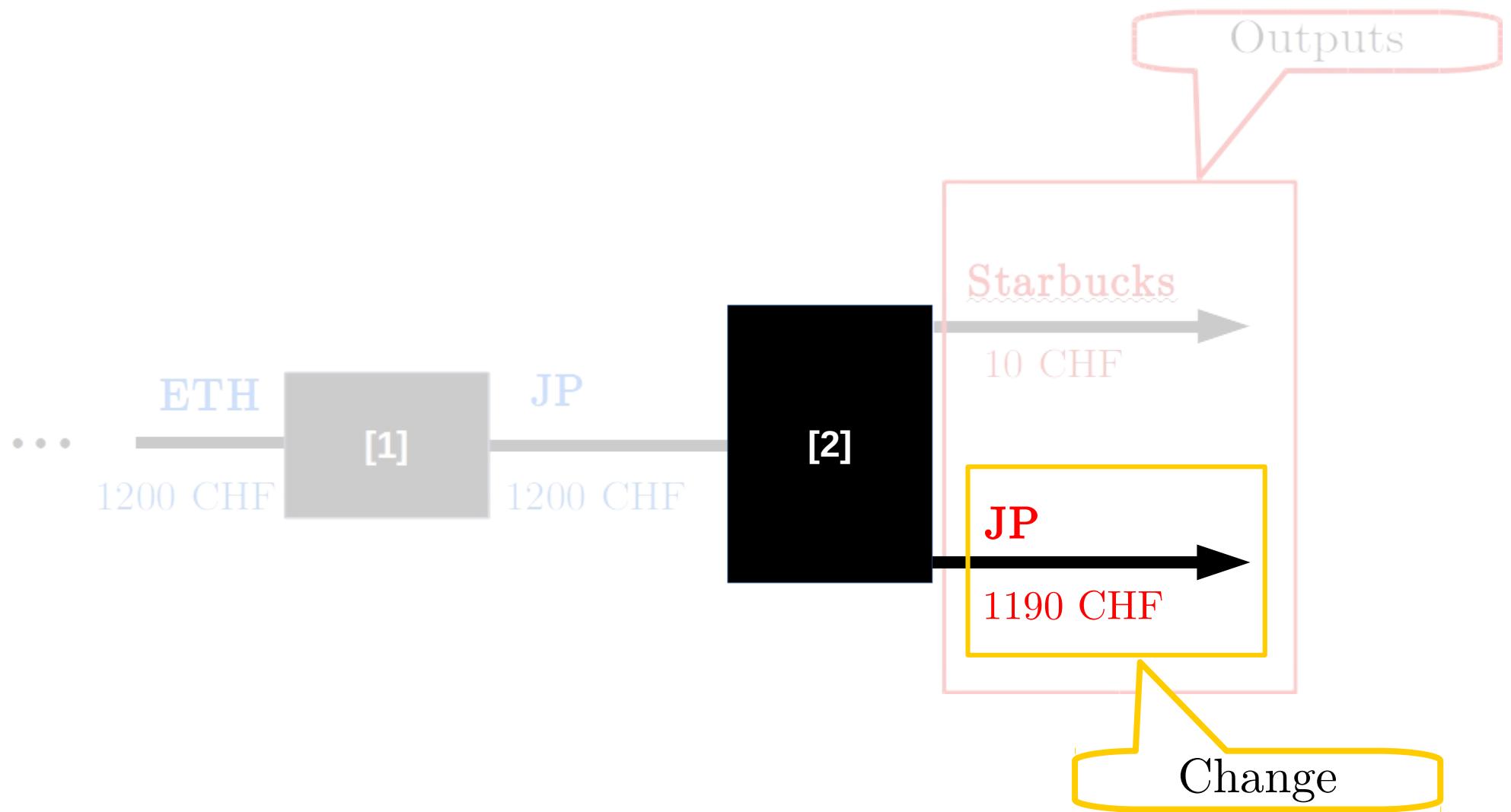
# Transactions



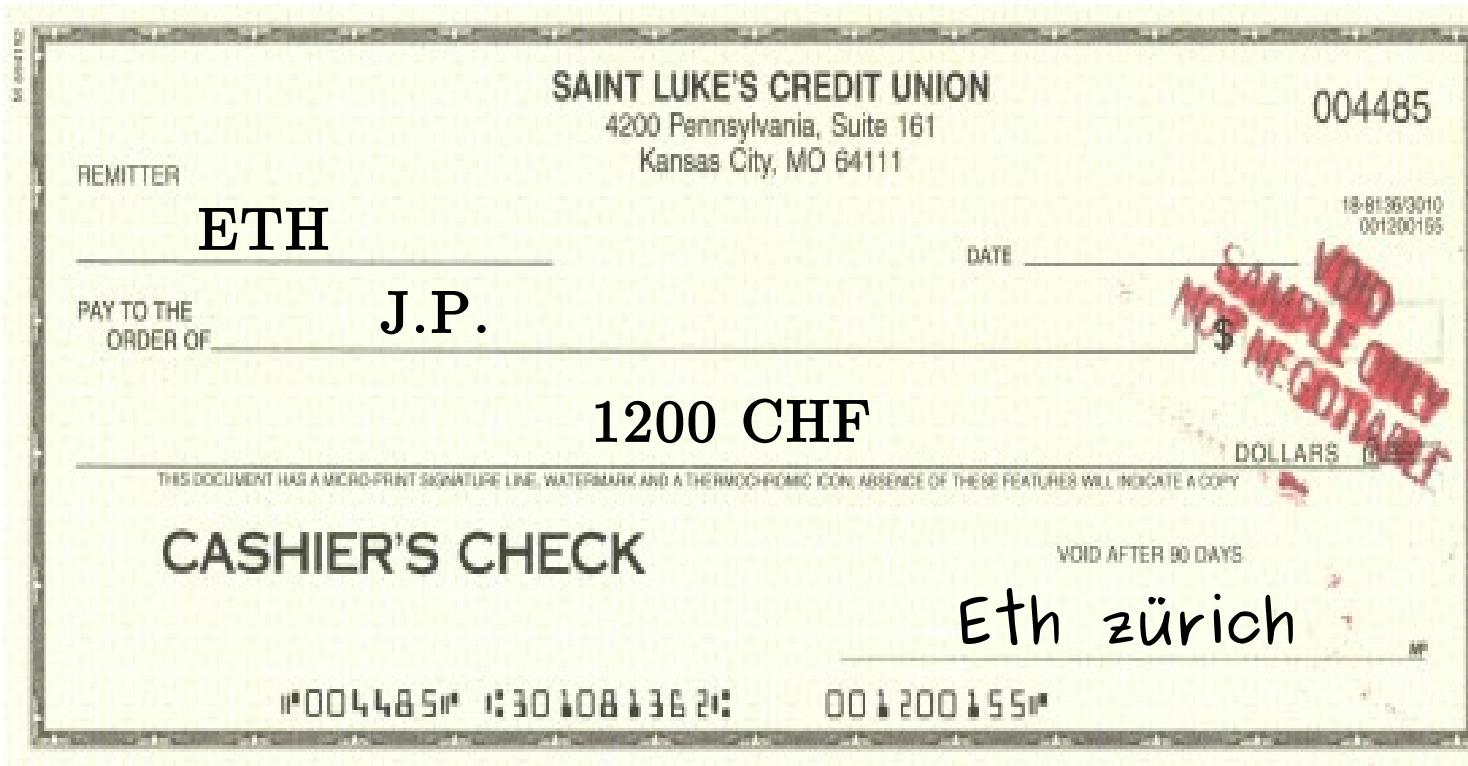
# Transactions



# Transactions



# Transactions



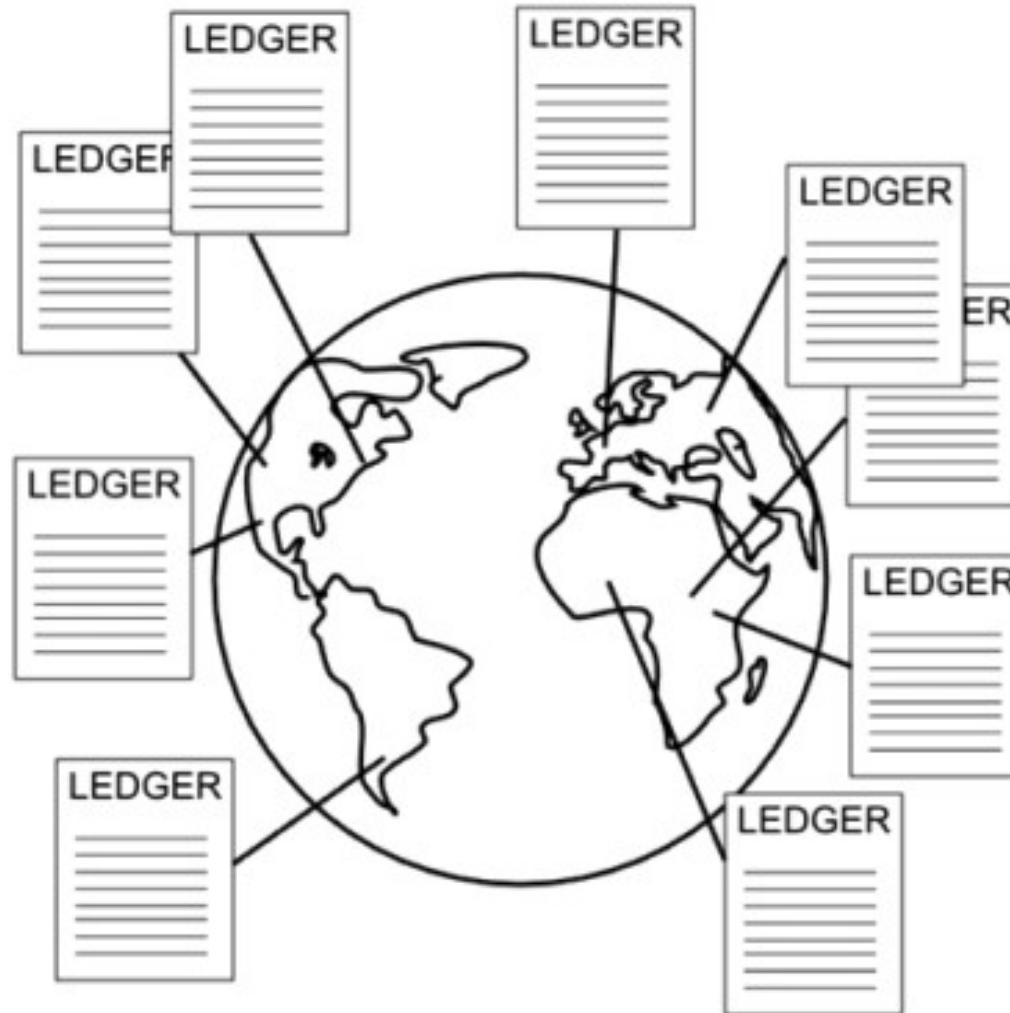
# In a centralized system...



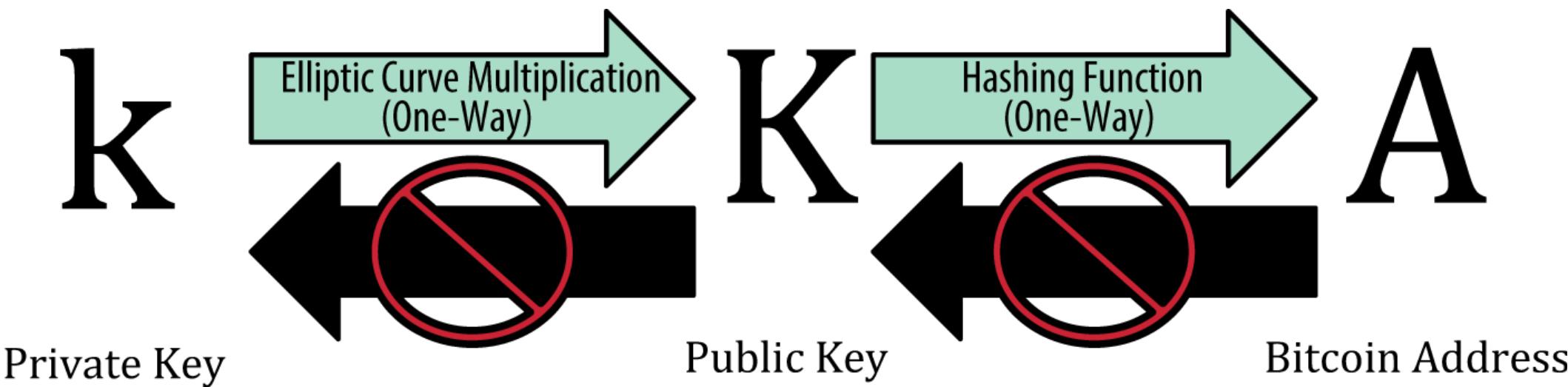
# In a decentralized system...



# In a decentralized system...



# Key Triplets



Who can spend the created outputs?

# Who can spend the created outputs?

## **Standard Transaction to Bitcoin address (pay-to-pubkey-hash)**

```
scriptPubKey: OP_DUP OP_HASH160 <pubKeyHash> OP_EQUALVERIFY OP_CHECKSIG  
scriptSig: <sig> <pubKey>
```

```
1
2
3 {
4     "txid" : "9ca8f969bd3ef5ec2a8685660fdbf7a8bd365524c2e1fc66c309acbae2c14ae3",
5     "version" : 1,
6     "locktime" : 0,
7     "vin" : [
8         {
9             "txid" : "d3c7e022ea80c4808e64dd0a1dba009f3eaee2318a4ece562f8ef815952717d7",
10            "vout" : 0,
11            "scriptSig" : {
12                "asm" :
13                    "3045022100a4ebbeec83225dedead659bbde7da3d026c8b8e12e61a2df0dd0758e227383b302203301768ef878007e9ef7c304f70ffaf1f2c975b192d34c5b9b2ac1bd193dfba20104793ac8a58ea751f9710e39aad2e2
14                "hex" :
15                    "483045022100a4ebbeec83225dedead659bbde7da3d026c8b8e12e61a2df0dd0758e227383b302203301768ef878007e9ef7c304f70ffaf1f2c975b192d34c5b9b2ac1bd193dfba2014104793ac8a58ea751f9710e39aa
16            },
17            "sequence" : 4294967295
18        }
19    ],
20    "vout" : [
21        {
22            "value" : 0.05000000,
23            "n" : 0,
24            "scriptPubKey" : {
25                "asm" : "OP_DUP OP_HASH160 07bdb518fa2e6089fd810235cf1100c9c13d1fd2 OP_EQUALVERIFY OP_CHECKSIG",
26                "hex" : "76a91407bdb518fa2e6089fd810235cf1100c9c13d1fd288ac",
27                "reqSigs" : 1,
28                "type" : "pubkeyhash",
29                "addresses" : [
30                    "1hvzSofQwT8cjB8JU7nBsCSfEVQX5u9CL"
31                ]
32            }
33        },
34        {
35            "value" : 1.03362847,
36            "n" : 1,
37            "scriptPubKey" : {
38                "asm" : "OP_DUP OP_HASH160 107b7086b31518935c8d28703d66d09b36231343 OP_EQUALVERIFY OP_CHECKSIG",
39                "hex" : "76a914107b7086b31518935c8d28703d66d09b3623134388ac",
40                "reqSigs" : 1,
41                "type" : "pubkeyhash",
42                "addresses" : [
43                    "12W9goQ3P7Waw5JH8fRVs1e2rVAKoGnvoy"
44                ]
45            }
46        }
47    ]
48 }
```

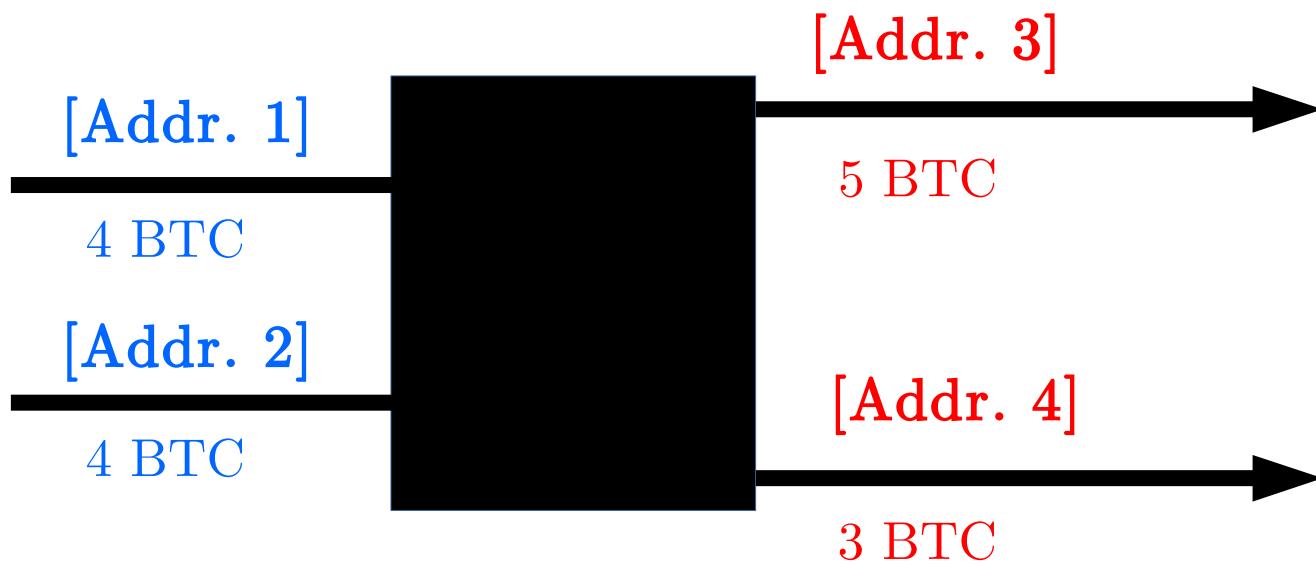
# Privacy Issues with Bitcoin



**NOT SURE IF BITCOIN WAS CREATED  
AGAINST THE NSA**

**OR BY THE NSA**

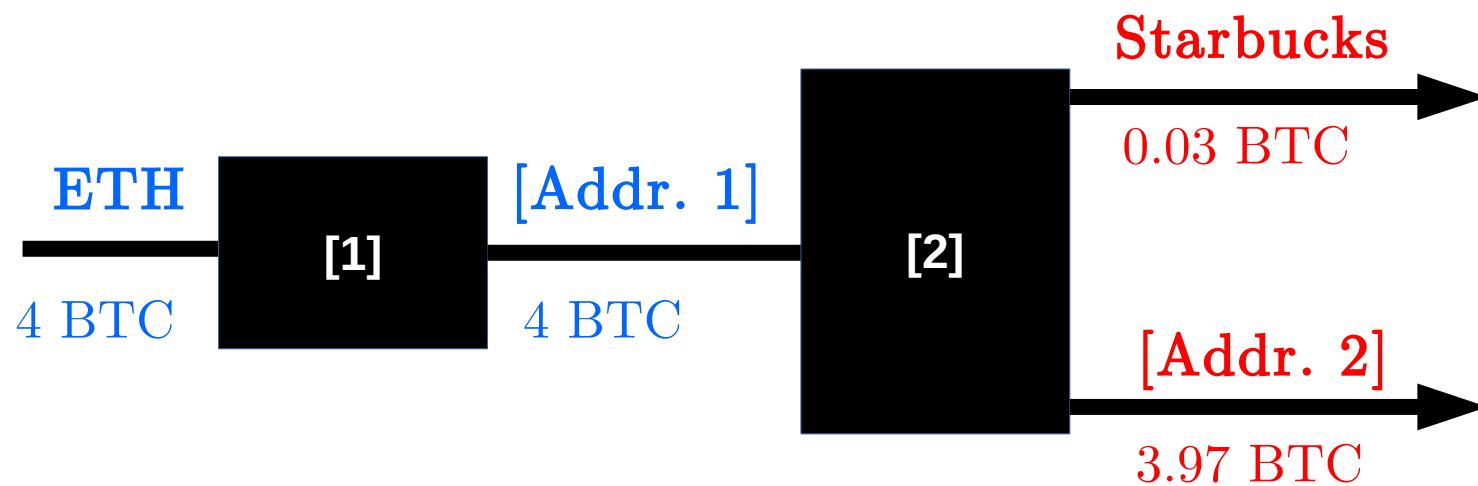
# Isn't Bitcoin anonymous?!



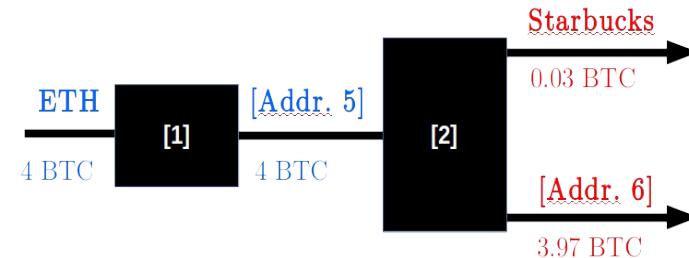
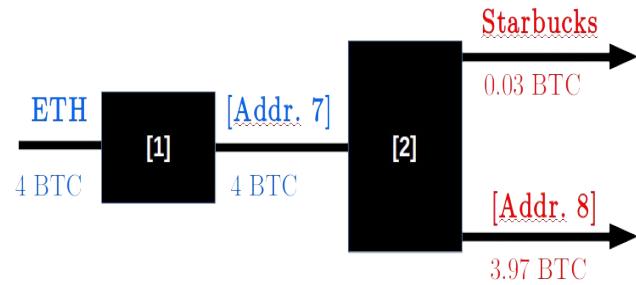
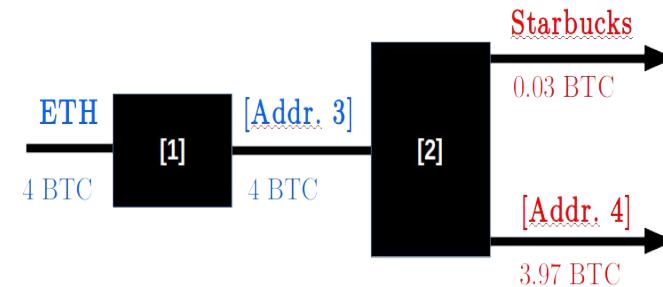
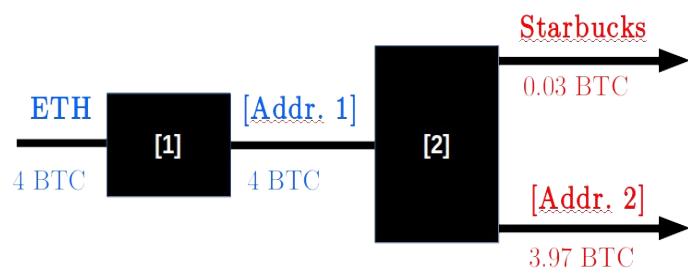
# Isn't Bitcoin anonymous?!

7e91e648710a99017d500c3953bb4e5dd684c97b235e50c8c52b3ec037e6bdcf	(Fee: 0.00 BTC - Size: 306 bytes) 2011-06-28 21:06:01
13vFf3MZKxSA3Q9e14c8xUXbMpHQn1wCgq (50 BTC - Output)	90 BTC
17zeTMh8xXeXXjZnbULXV3g3t3f7pftnEh (50 BTC - Output)	10 BTC
	WikiLeaks  - (Spent) 1Ne2mvY6Du6kQjjwi6tnHy2nLZup2n8Kgv - (Spent) <span style="background-color: green; color: white; padding: 2px;">90 BTC</span>

# Isn't Bitcoin anonymous?!



# Isn't Bitcoin anonymous?!



# Why Anonymity?

"What we used to call liberty and freedom

we now call privacy."

(Jacob Appelbaum)



WikiLeaks now accepts anonymous  
Bitcoin donations on  
**1HB5XMLmzFVj8ALj6mfBsbifRoD4miY36v**



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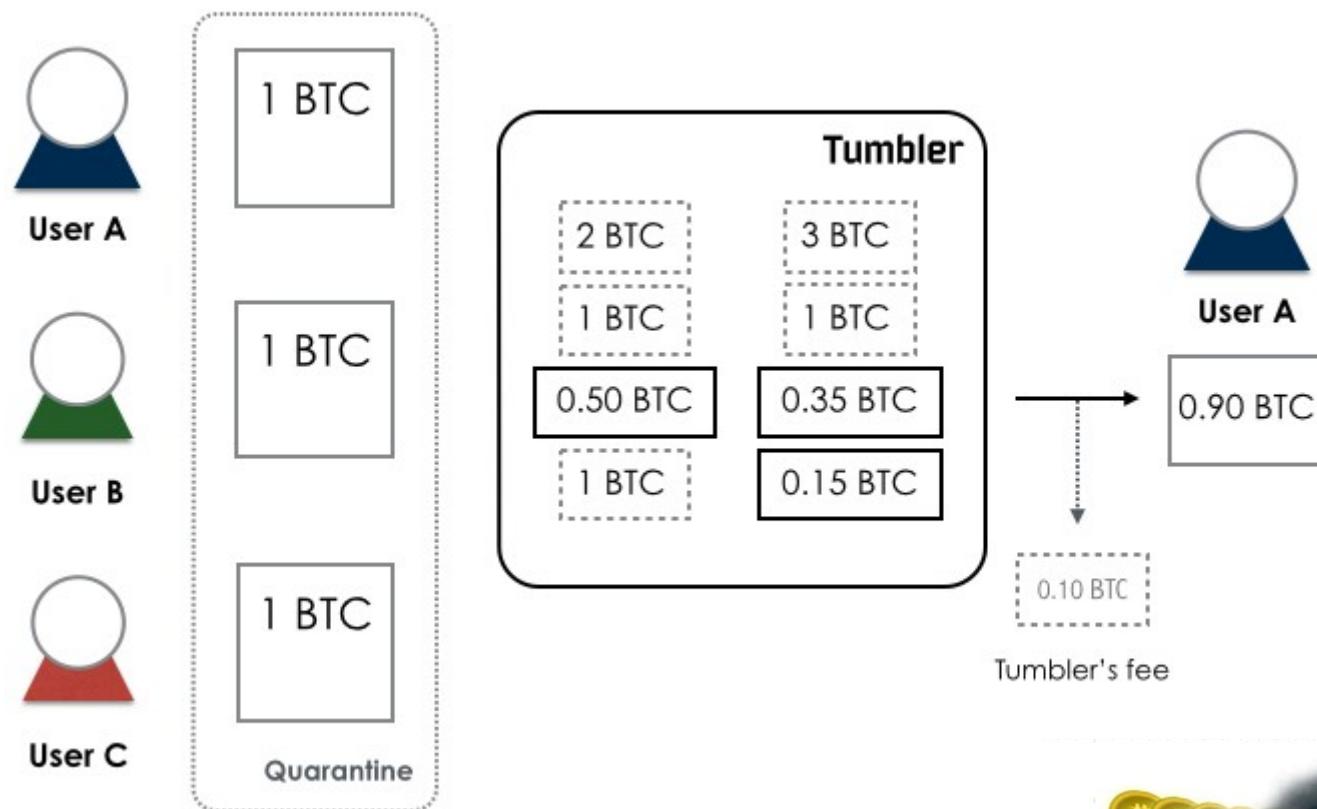
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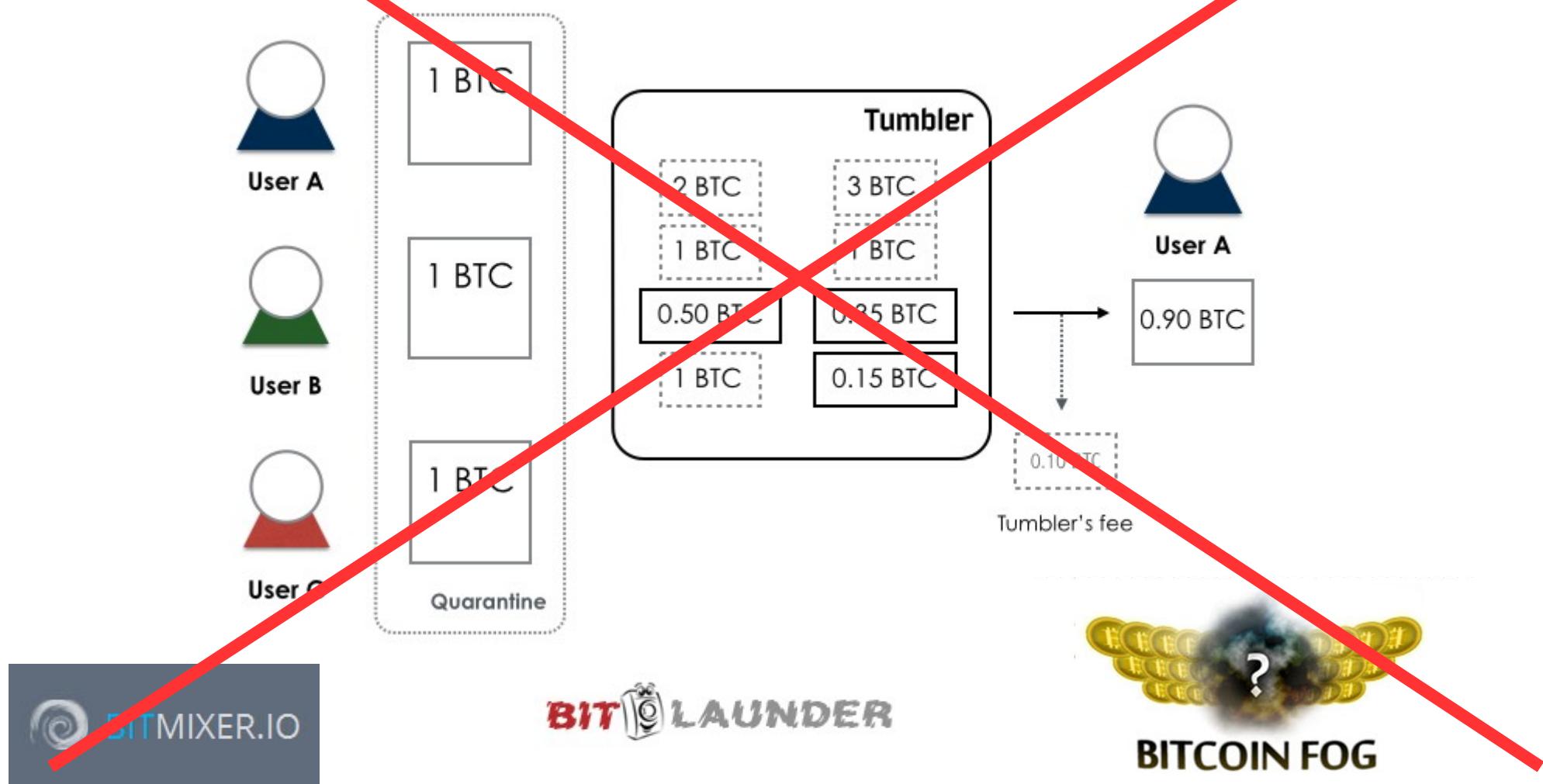
# But... how?

- Mixing Services



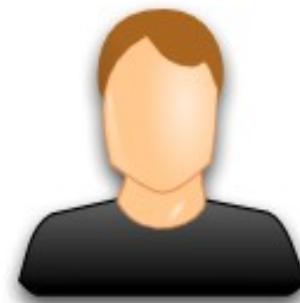
# But... how?

- Mixing Services



# ZeroCoin

or “how to prove you have money without showing it”



Bob

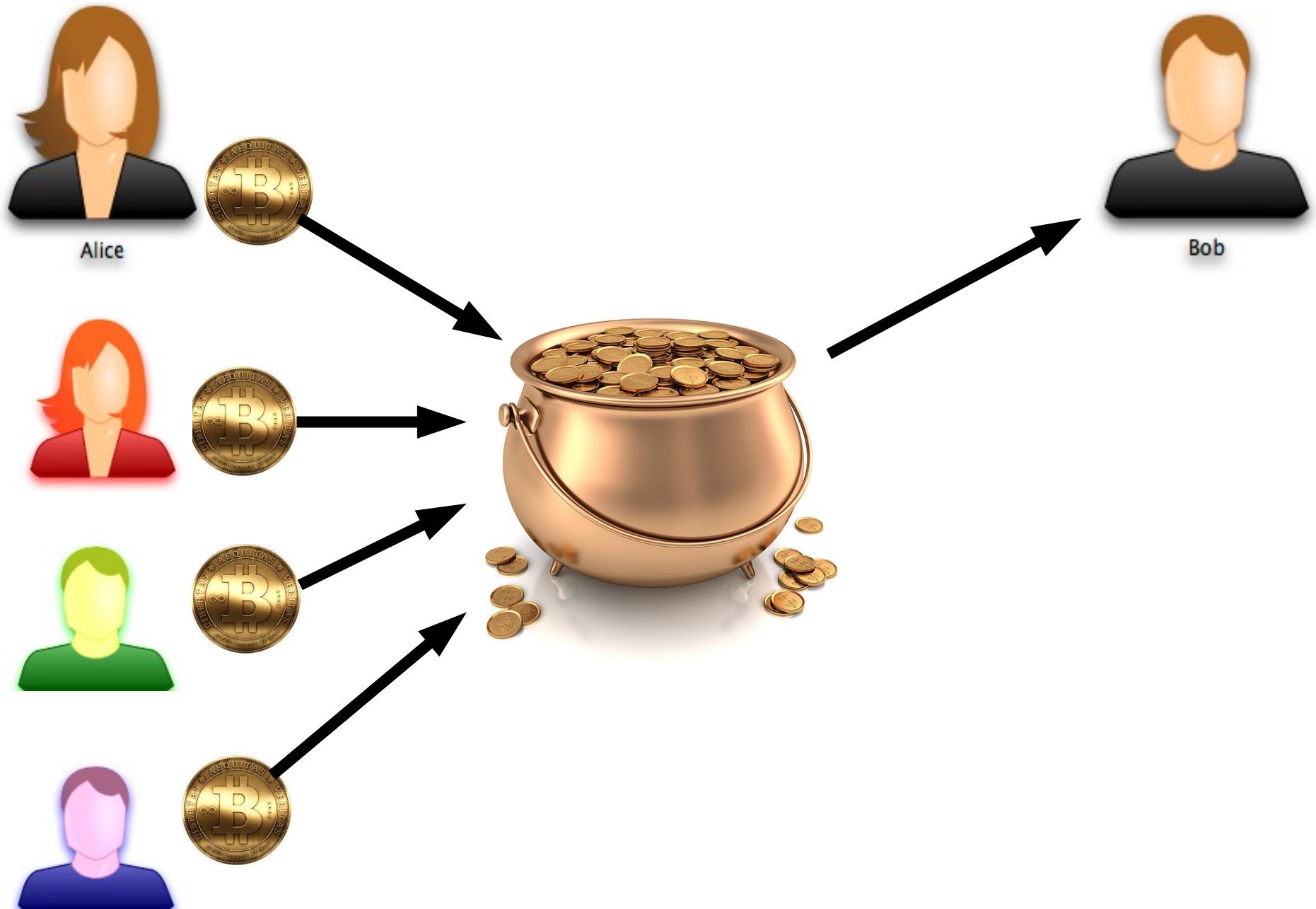


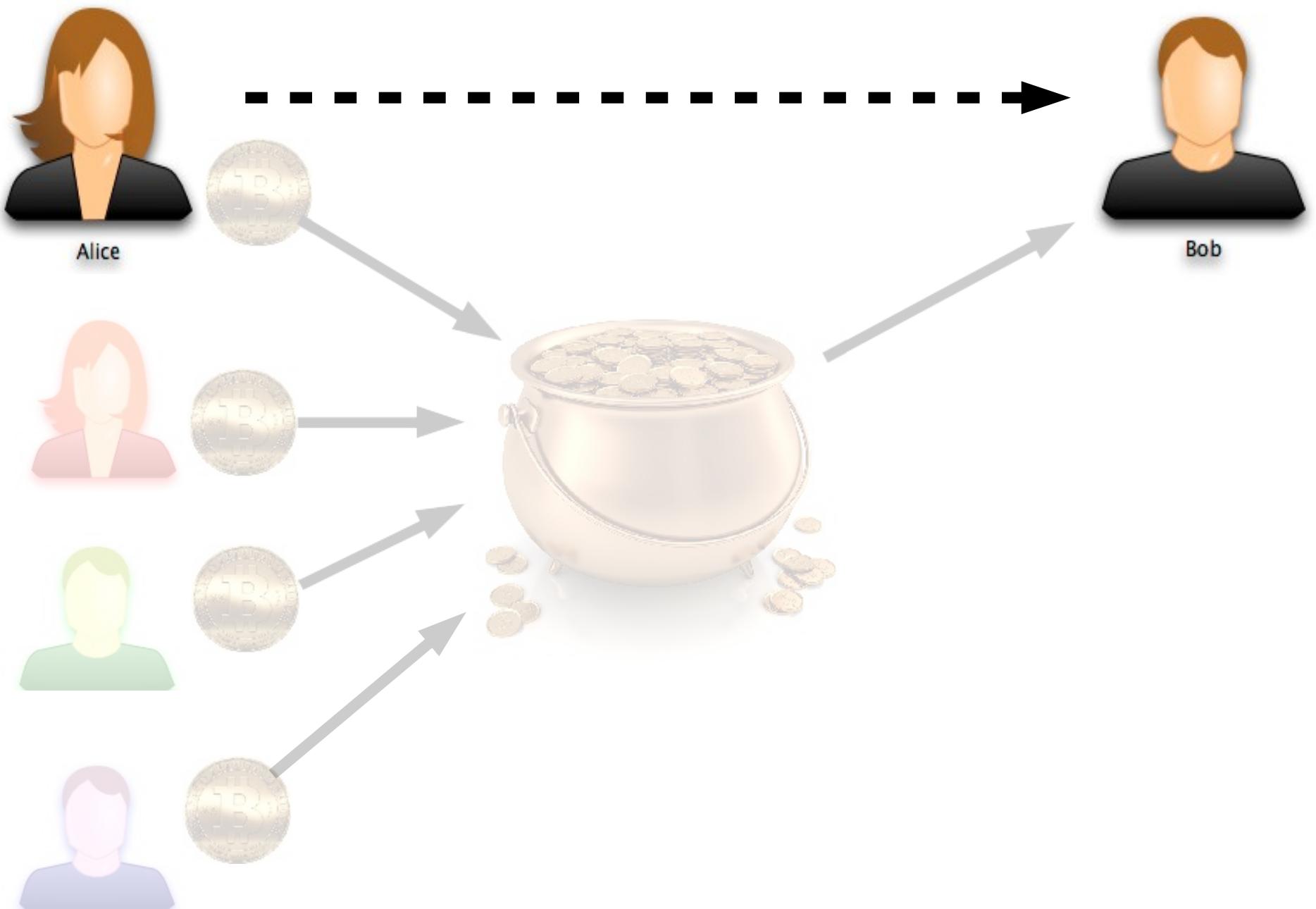




Alice







# Basic Idea

- 4 operations
  - `setup()`
  - `mint()`
  - `spend()`
  - `verify()`

# `mint()`



`mint()`



1 BTC



`mint()`



1 BTC



`mint()`



1 BTC



`mint()`



`mint()`



1 BTC



`mint()`



1 BTC



`mint()`



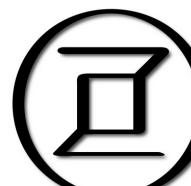
1 BTC



Alice



`mint()`



1 ZC



`mint()`



`mint()`



1 BTC



`mint()`



1 BTC



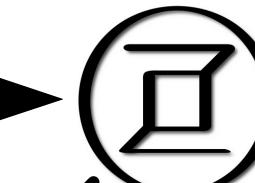
`mint()`



1 BTC



`mint()`



1 ZC

`pubzc`



# spend()



Alice

# spend()

proof



Alice

$$(f+g)'(x) = f'(x) + g'(x)$$

proof from book:

$$\begin{aligned} (f+g)'(x) &= \lim_{h \rightarrow 0} \frac{(f+g)(x+h) - (f+g)(x)}{h} \\ &\rightarrow = \lim_{h \rightarrow 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\ &\rightarrow = \lim_{h \rightarrow 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h} \\ \text{how does this happen?} &\rightarrow = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ &\rightarrow = f'(x) + g'(x) \end{aligned}$$

why isn't  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  zero?  $f(x) = f(x)$ ?

# spend()

proof



Alice

$$(f+g)'(x) = f'(x) + g'(x)$$

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why isn't  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  zero?  $f(x) = f(x)$ ?

- “I've minted one ZC”
- “I haven't spend it”

# spend()

proof

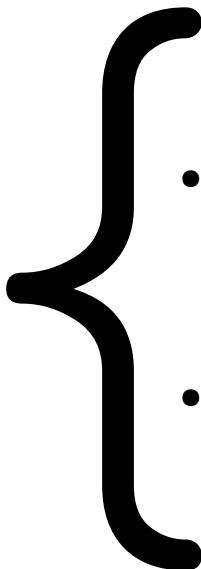


Alice

$$(f+g)'(x) = f'(x) + g'(x)$$

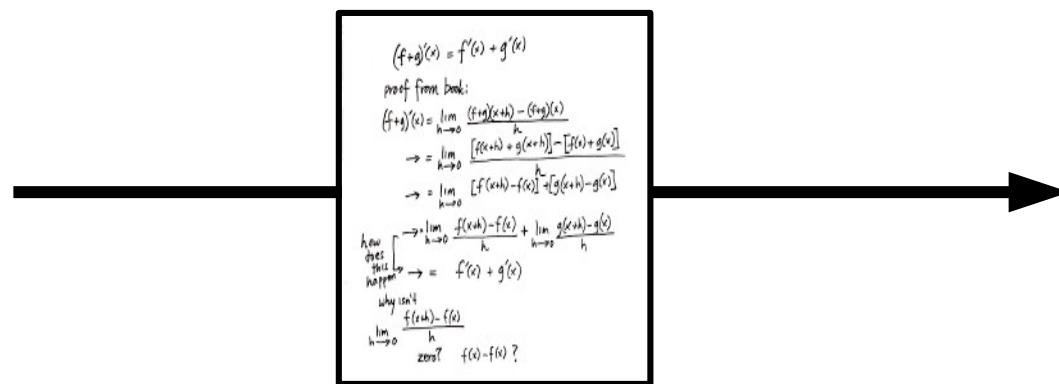
proof from book:

$$\begin{aligned}(f+g)'(x) &= \lim_{h \rightarrow 0} \frac{(f+g)(x+h) - (f+g)(x)}{h} \\ &\rightarrow = \lim_{h \rightarrow 0} \frac{[f(x+h) + g(x+h)] - [f(x) + g(x)]}{h} \\ &\rightarrow = \lim_{h \rightarrow 0} \frac{[f(x+h) - f(x)] + [g(x+h) - g(x)]}{h} \\ &\rightarrow = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ &\text{how does this happen?} \rightarrow = f'(x) + g'(x) \\ &\text{why isn't } \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ zero? } f(x) = f(x) ?\end{aligned}$$

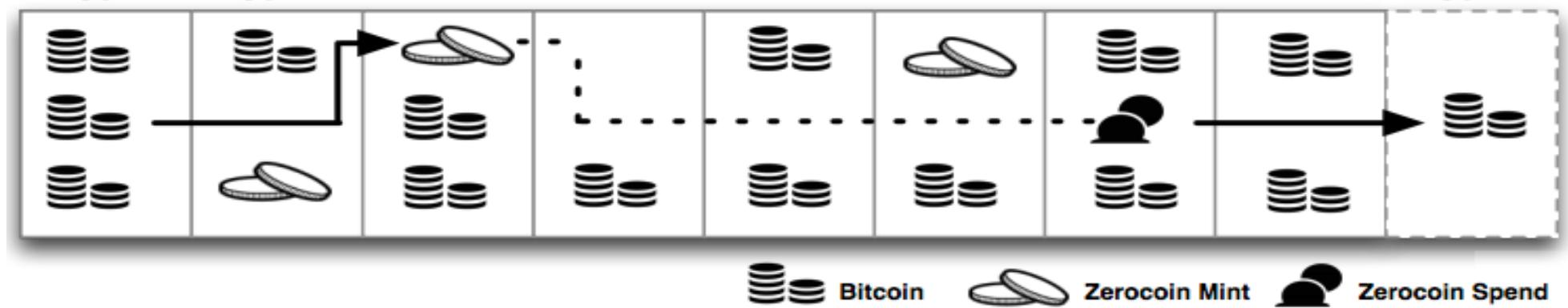


- “I know one (unspent)  $\text{pub}_{zc}$ ”
- “I know the construction of  $\text{pub}_{zc}$ ”

# spend()







# verify()



Challenges?

# Cryptographic Building Blocks

- Commitment Scheme
- Zero-Knowledge Proofs
- Accumulator

# Commitment Scheme

- “How would you flip a coin over the phone?”

1. flips coin = x



Alice



Bob



Alice



Bob

1. flips coin =  $x$
2. random  $r_A$



Alice

1. flips coin =  $x$
  2. random  $r_A$
  3.  $h(x, r_A)$
- 



Bob



Alice



Bob

1. flips coin =  $x$
2. random  $r_A$
3.  $h(x, r_A)$



4. chooses  $y$





Alice



Bob

1. flips coin =  $x$

2. random  $r_A$

3.  $h(x, r_A)$

---

4. chooses  $y$

5.  $x, r_A$

---



Alice



Bob

1. flips coin =  $x$

2. random  $r_A$

3.  $h(x, r_A)$



4. chooses  $y$



5.  $x, r_A$



$$h(x, r_A) == h(y, r_A)$$

# Pedersen Commitment Scheme (\*)

- group  $G = \langle g \rangle = \langle h \rangle$

(\*) simplified

# Pedersen Commitment Scheme (\*)

- group  $G = \langle g \rangle = \langle h \rangle$

- commit to value  $s$ :

choose random  $r$

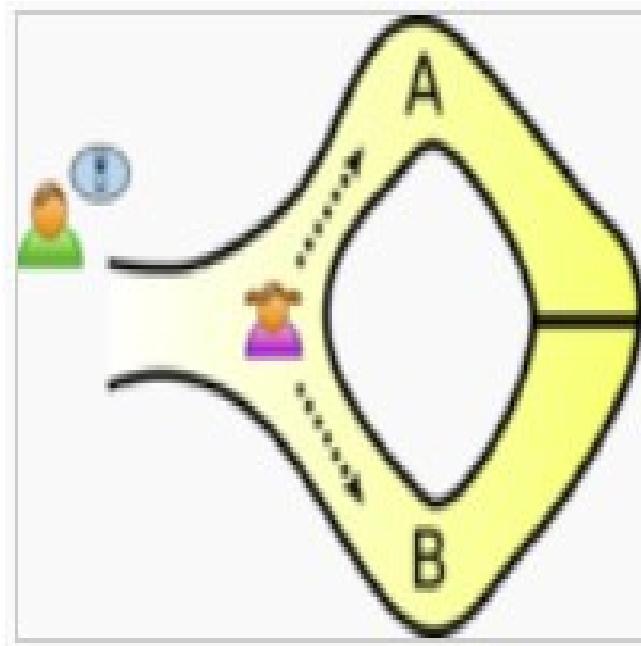
$$\text{pub} = g^s h^r$$

$$\text{sec} = (s, r)$$

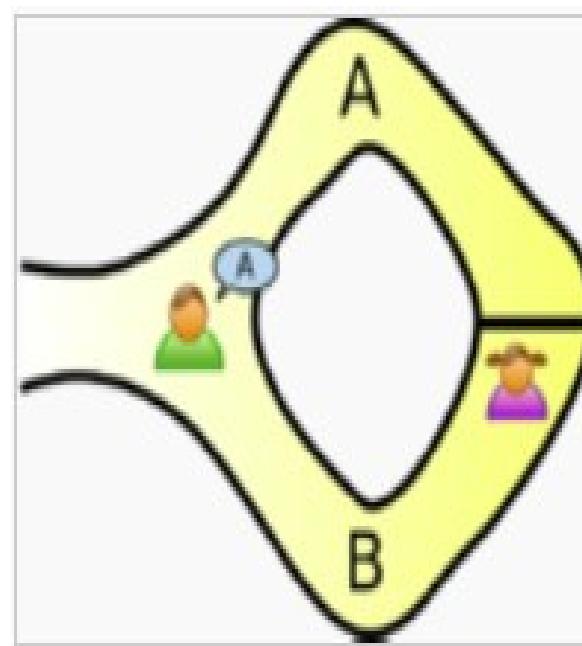
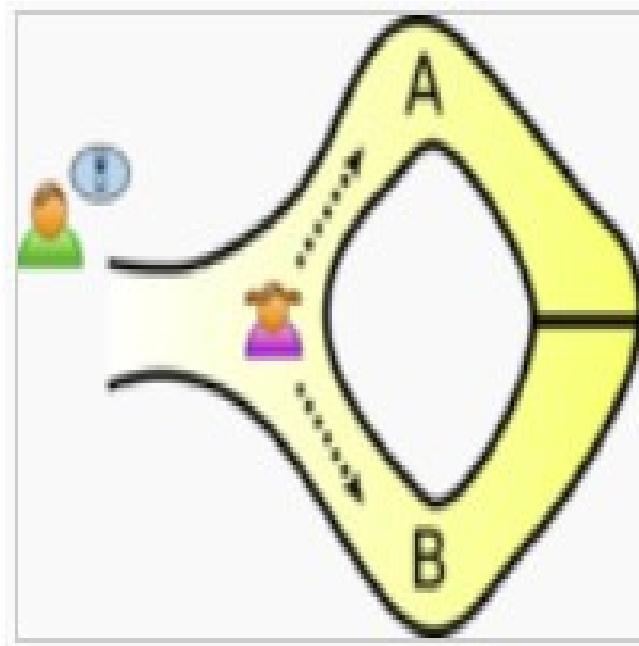
(\*) simplified

# Zero-Knowledge Proofs

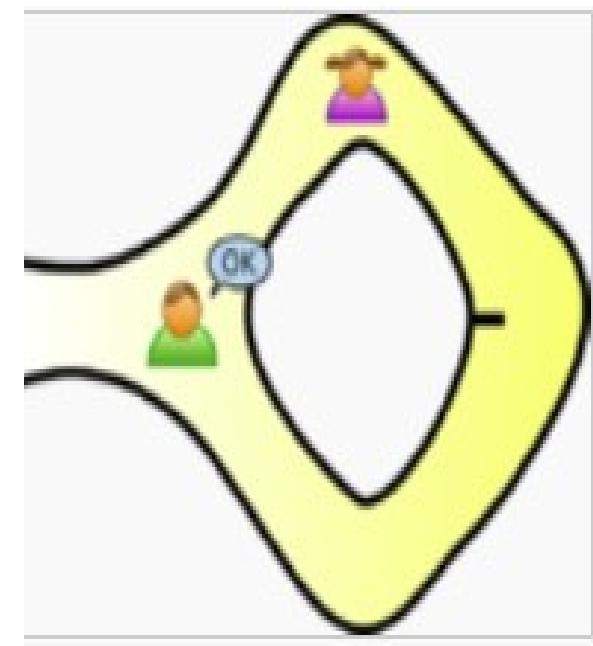
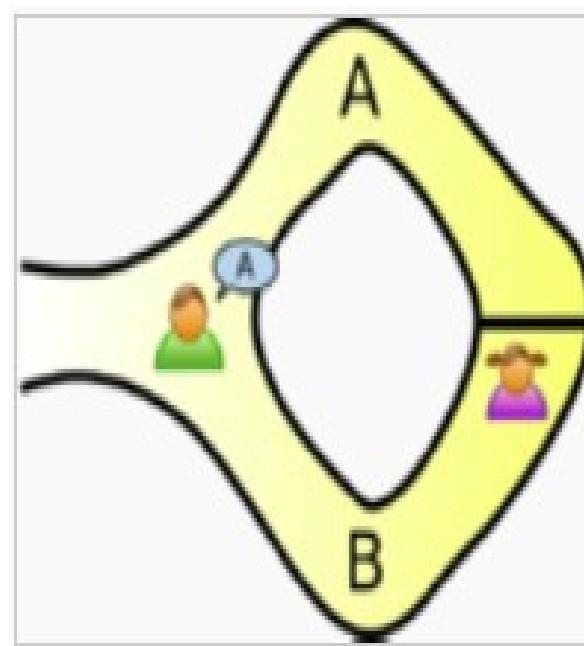
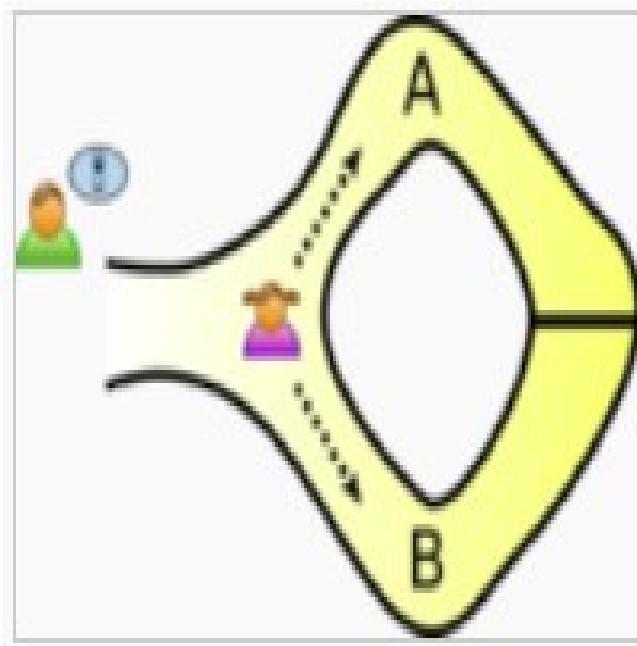
# Interactive Zero-Knowledge Proofs



# Interactive Zero-Knowledge Proofs



# Interactive Zero-Knowledge Proofs



# Non-Interactive Zero-Knowledge Proofs

- Fiat-Shamir Heuristic
- can be used as a Signature of Knowledge (**ZKSoK**)

# Accumulator

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n\}$
- Show: “I know  $\text{pub}_{zc, j} \in C$ ” without revealing it

# Accumulator

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n\}$
- Show: “I know  $\text{pub}_{zc, j} \in C$ ” without revealing it

$$(\text{pub}_{zc, j} = \text{pub}_{zc, 1}) \vee \dots \vee (\text{pub}_{zc, j} = \text{pub}_{zc, n})$$

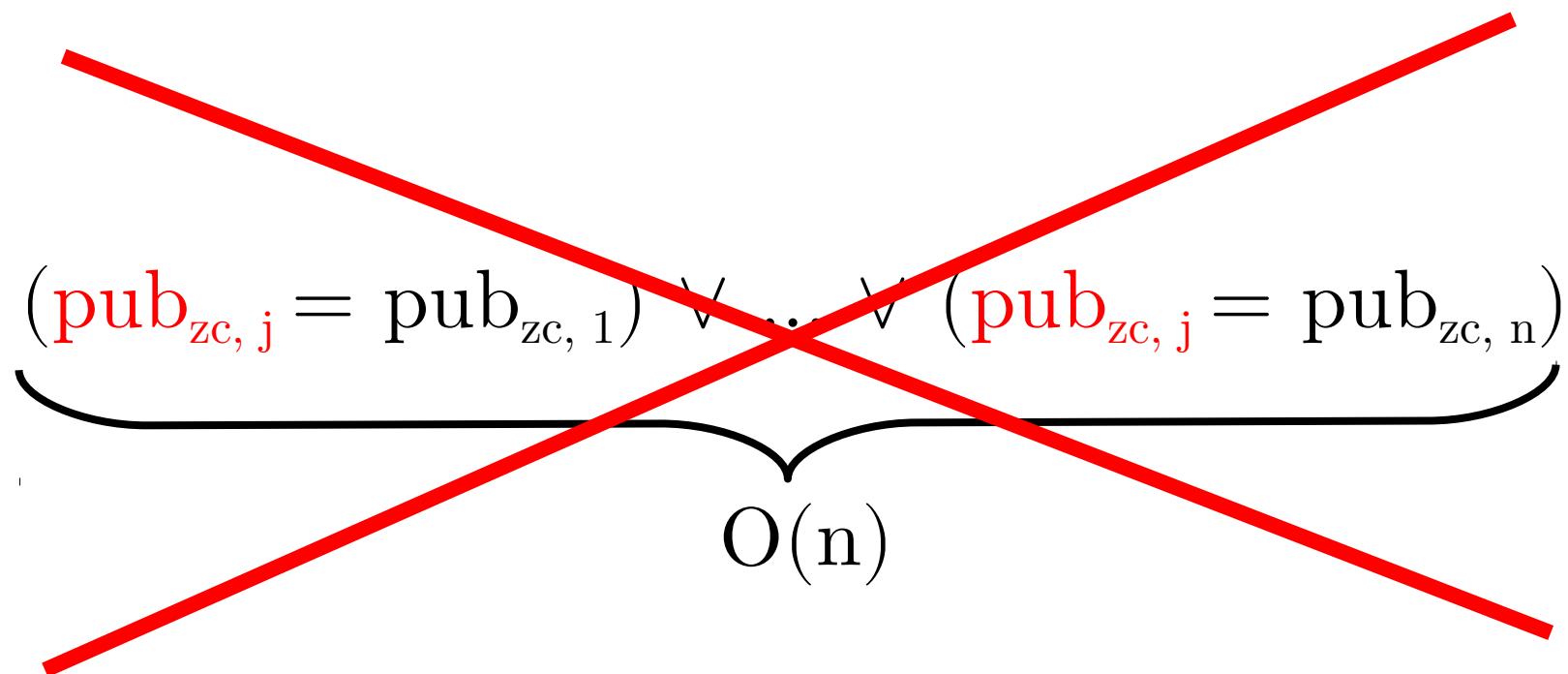
# Accumulator

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n\}$
- Show: “I know  $\text{pub}_{zc, j} \in C$ ” without revealing it

$$\underbrace{(\text{pub}_{zc, j} = \text{pub}_{zc, 1}) \vee \dots \vee (\text{pub}_{zc, j} = \text{pub}_{zc, n})}_{O(n)}$$

# Accumulator

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n\}$
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- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n \}$
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public one-way accumulator:

$$\text{Acc} = \text{accumulate}(C)$$

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$\text{Acc} = \text{accumulate}(C)$

$\text{wit} = \text{generateWitness}(C, \text{value})$

# Accumulator

- **Given:**  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n \}$
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public one-way accumulator:

$\text{Acc} = \text{accumulate}(C)$

$\text{wit} = \text{generateWitness}(C, \text{value})$

$\text{accVerify}(\text{Acc}, \text{value}, \text{wit}) \rightarrow \{0, 1\}$

# Accumulator

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n \}$
- Show: “I know  $\text{pub}_{zc, j} \in C$ ” without revealing it

public one-way accumulator:

$$\text{Acc} = \text{accumulate}(C) = \prod_{i=1}^n \text{pub}_{zc, i}$$

# Accumulator

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n \}$
- Show: “I know  $\text{pub}_{zc, j} \in C$ ” **without revealing it**

public one-way accumulator:

$$\text{Acc} = \text{accumulate}(C) = \prod_{i=1}^n \text{pub}_{zc, i}$$

# Accumulator (\*)

- Given:  $C := \{ \text{pub}_{zc, i} \mid i = 1, \dots, n \}$
- Show: “I know  $\text{pub}_{zc, j} \in C$ ” **without revealing it**

public one-way accumulator:

$$\text{Acc} = \text{accumulate}(C) = u \prod_{i=1}^n \text{pub}_{zc, i}$$

(\*) simplified

$$\text{Acc} = \text{accumulate}(\mathbf{C}) = \prod_{i=1}^n \text{pub}_{\text{zc}, i}$$

$$\text{Acc} = \text{accumulate}(\mathbf{C}) = \prod_{i=1}^n \text{pub}_{zc, i}$$

$$\text{wit} = \text{generateWitness}(\mathbf{C}, \text{value}) = \text{accumulate}(\mathbf{C} \setminus \{\text{value}\})$$

$$\text{Acc} = \text{accumulate}(\mathbf{C}) = \mathbf{U} \prod_{i=1}^n \text{pub}_{zc, i}$$

$$\text{wit} = \text{generateWitness}(\mathbf{C}, \text{value}) = \text{accumulate}(\mathbf{C} \setminus \{\text{value}\})$$

$$\text{accVerify}(\text{Acc}, \text{value}, \text{wit}) = 1 \text{ iff } \text{wit}^{\text{value}} = \text{Acc}$$

# ZeroCoin Protocol

- 4 operations
  - `setup()`
  - `mint()`
  - `spend()`
  - `verify()`

# setup()

- setup accumulator
- setup commitment parameters

$$G = \langle g \rangle = \langle h \rangle$$

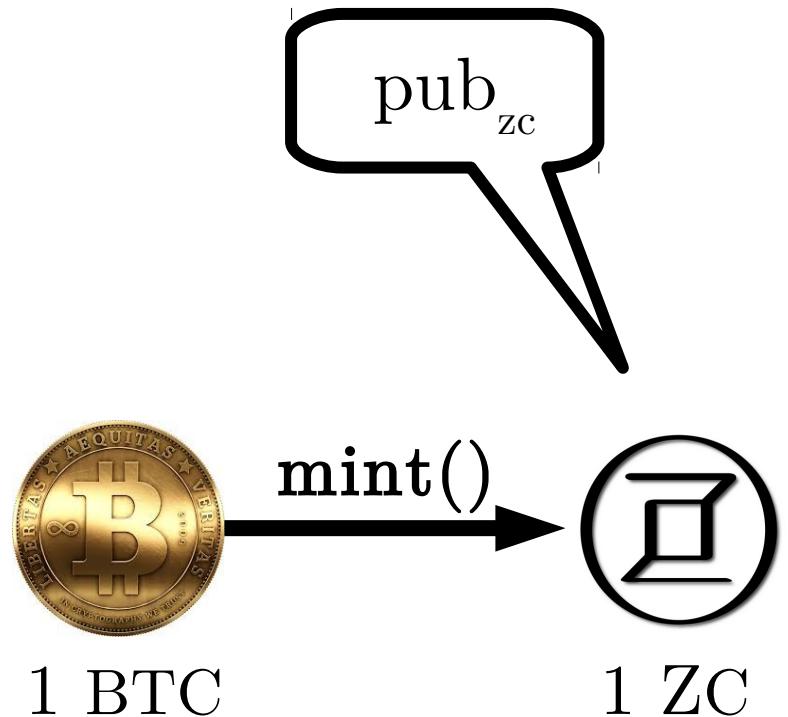
$$\text{mint}(I_{\text{btc}}) \rightarrow (\text{pub}_{\text{zc}}, \text{sec}_{\text{zc}})$$

- commit to value s:

choose random r

$$\text{pub}_{\text{zc}} = g^s h^r$$

$$\text{sec}_{\text{zc}} = (s, r)$$



$$\text{spend}(\text{pub}_{\text{zc}}, \text{sec}_{\text{zc}}, \mathbf{C}, \mathbf{O}_{\text{btc}}) \rightarrow (\pi, s)$$

- $\text{Acc} = \text{accumulate}(\mathbf{C})$

$$\text{spend}(\text{pub}_{\text{zc}}, \text{sec}_{\text{zc}}, \mathbf{C}, \mathbf{O}_{\text{btc}}) \rightarrow (\pi, s)$$

- $\text{Acc} = \text{accumulate}(\mathbf{C})$
- $\text{wit} = \text{generateWitness}(\mathbf{C}, \text{pub}_{\text{zc}})$

$$\text{spend}(\text{pub}_{\text{zc}}, \text{sec}_{\text{zc}}, \mathbf{C}, \mathbf{O}_{\text{btc}}) \rightarrow (\pi, s)$$

- $\text{Acc} = \text{accumulate}(\mathbf{C})$
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- $s \in \text{sec}_{\text{zc}} = (s, r)$

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- $\text{Acc} = \text{accumulate}(\mathbf{C})$
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- $s \in \text{sec}_{\text{zc}} = (s, r)$
- $\pi = \text{ZKSoK } [\mathbf{O}_{\text{btc}}] \{ \dots \}$

$$\text{spend}(\text{pub}_{\text{zc}}, \text{sec}_{\text{zc}}, \mathbf{C}, \mathbf{O}_{\text{btc}}) \rightarrow (\pi, \mathbf{s})$$

- $\text{Acc} = \text{accumulate}(\mathbf{C})$
- $\text{wit} = \text{generateWitness}(\mathbf{C}, \text{pub}_{\text{zc}})$
- $\mathbf{s}$  in  $\text{sec}_{\text{zc}} = (\mathbf{s}, \mathbf{r})$
- $\pi = \text{ZKSoK } [\mathbf{O}_{\text{btc}}] \{ (\text{pub}_{\text{zc}}, \text{wit}, \mathbf{r}) :$

$\underbrace{\text{AccVerify}(\text{Acc}, \text{pub}_{\text{zc}}, \text{wit}) = 1 \wedge \dots}$

“I know one  $\text{pub}_{\text{zc}}$  in  $\mathbf{C}$ ”

$$\text{spend}(\text{pub}_{\text{zc}}, \text{sec}_{\text{zc}}, \mathbf{C}, \mathbf{O}_{\text{btc}}) \rightarrow (\pi, \mathbf{s})$$

- $\text{Acc} = \text{accumulate}(\mathbf{C})$
- $\text{wit} = \text{generateWitness}(\mathbf{C}, \text{pub}_{\text{zc}})$
- $\mathbf{s}$  in  $\text{sec}_{\text{zc}} = (\mathbf{s}, \mathbf{r})$
- $\pi = \text{ZKSoK } [\mathbf{O}_{\text{btc}}] \{ (\text{pub}_{\text{zc}}, \text{wit}, \mathbf{r}) : \underbrace{\text{AccVerify}(\text{Acc}, \text{pub}_{\text{zc}}, \text{wit}) = 1}_{\text{“I know one } \text{pub}_{\text{zc}} \text{ in } \mathbf{C}”} \wedge \underbrace{\text{pub}_{\text{zc}} = g^s h^r}_{\text{“I know its construction”}} \}$

$$\text{verify}(\pi, s, O_{\text{btc}}, C) \rightarrow \{0, 1\}$$

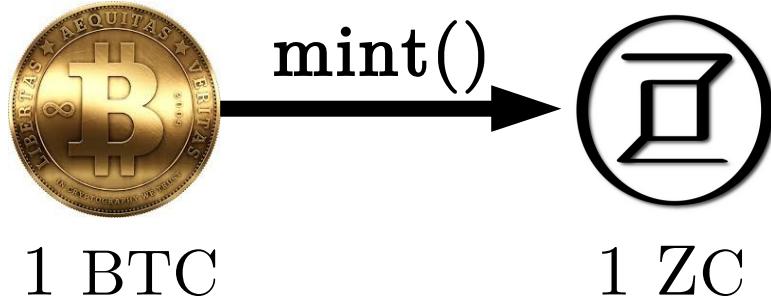
- verify if  $s$  unspent
- verify correctness of  $\pi$  as a signature on  $O_{\text{btc}}$

# Remarks

- accumulator checkpoint
- proof size → memory issues
- proof complexity → longer verification time

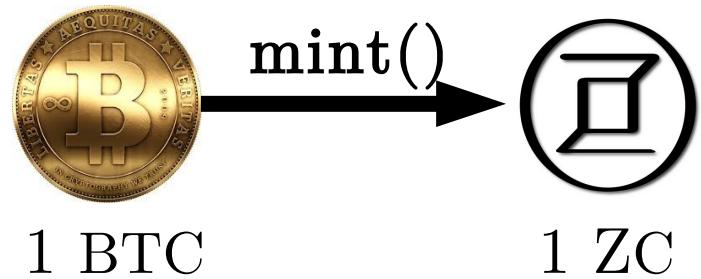
One more thing...

# Is ZeroCoin enough?

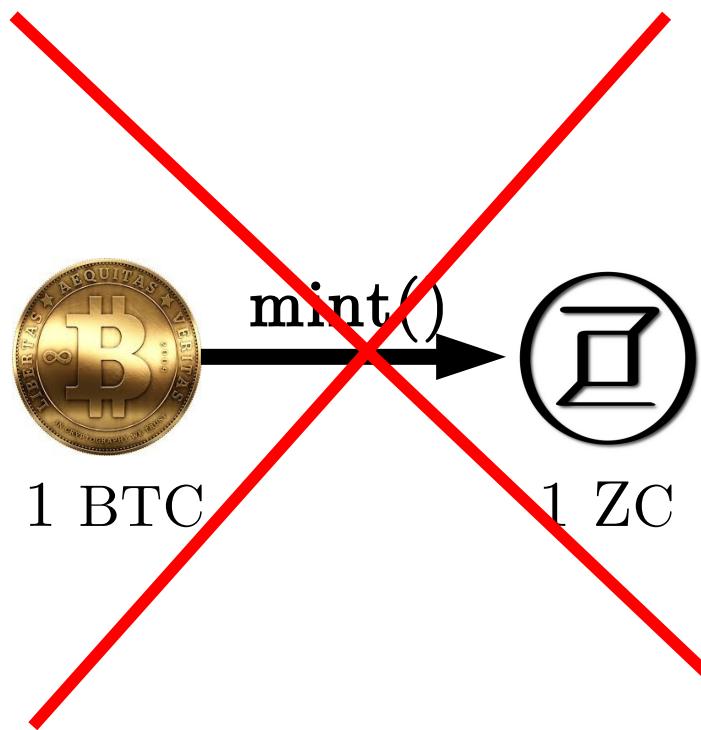


# eZC: ZeroCoin Reloaded

# New Ideas



# New Ideas



# New Ideas



amount = 10 BTC

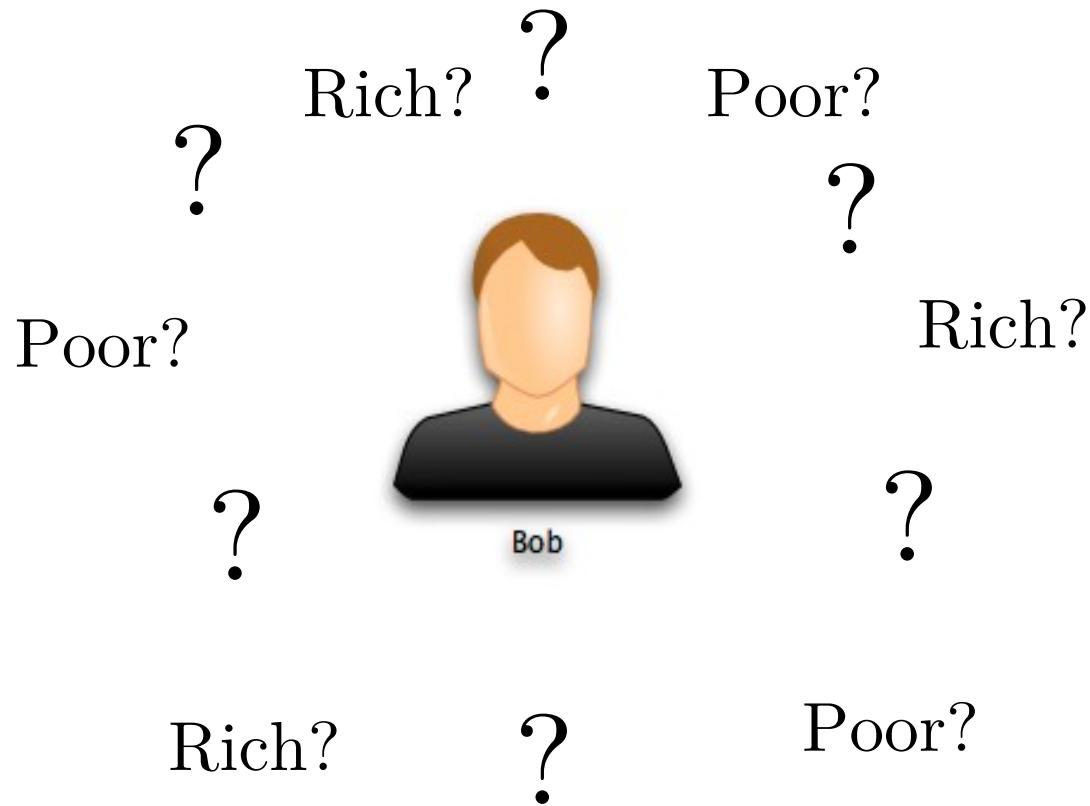


amount = 10 BTC



Bob

# New Ideas



# But... How?

- 5 operations:
  - `setup()`
  - **`mint(amount)`**
  - `spendEZCtoBTC()`
  - `spendEZCtoEZC()`
  - `verify()`

setup()

G = <g> = <h> = <**w**>

$$\text{mint}(I_{\text{btc}}) \rightarrow (\pi_{\text{pub}}, \text{pub}_{\text{ezc}}, \text{sec}_{\text{ezc}})$$

- commit to value s and transaction amount (a):

$$\text{pub}_{\text{ezc}} = g^s h^r \textcolor{red}{w^a}$$

$$\text{sec}_{\text{ezc}} = (s, r)$$

$$\pi_{\text{pub}} = \text{ZKPoK}\{(s, r) : \text{pub}_{\text{ezc}} = g^s h^r w^a\}$$

# spendEZCtoEZC



Alice

1. Proves eZC<sub>send</sub>'s validity



Bob

2. Mints eZC<sub>change</sub>

3. Pre-mints eZC<sub>receive</sub>

4. Mints eZC<sub>receive</sub>

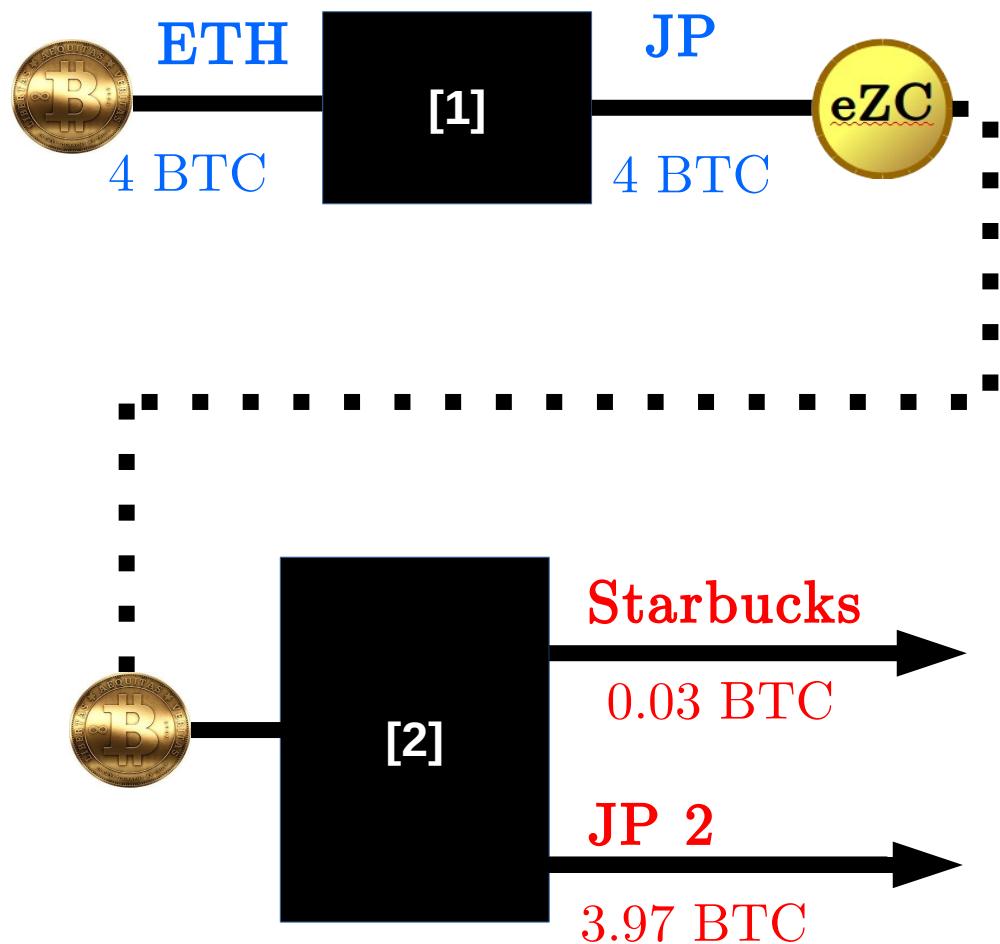
# Transactions Revisited



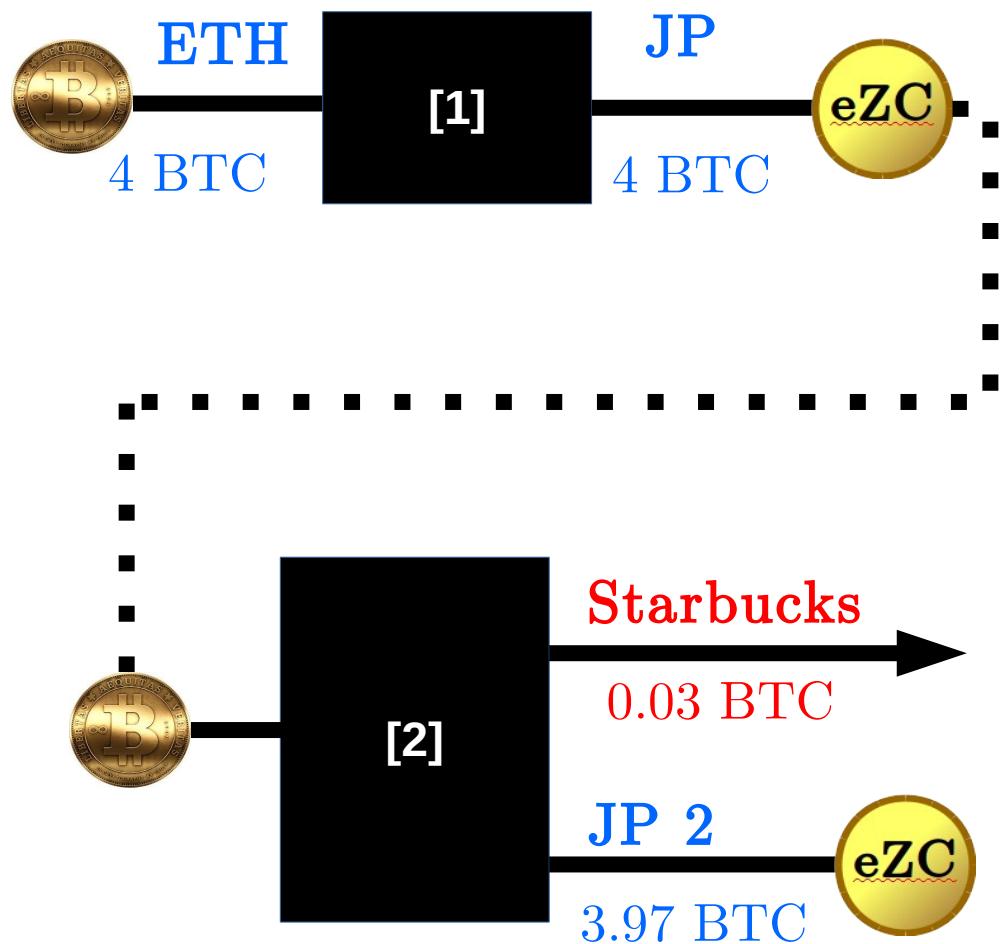
# Transactions Revisited



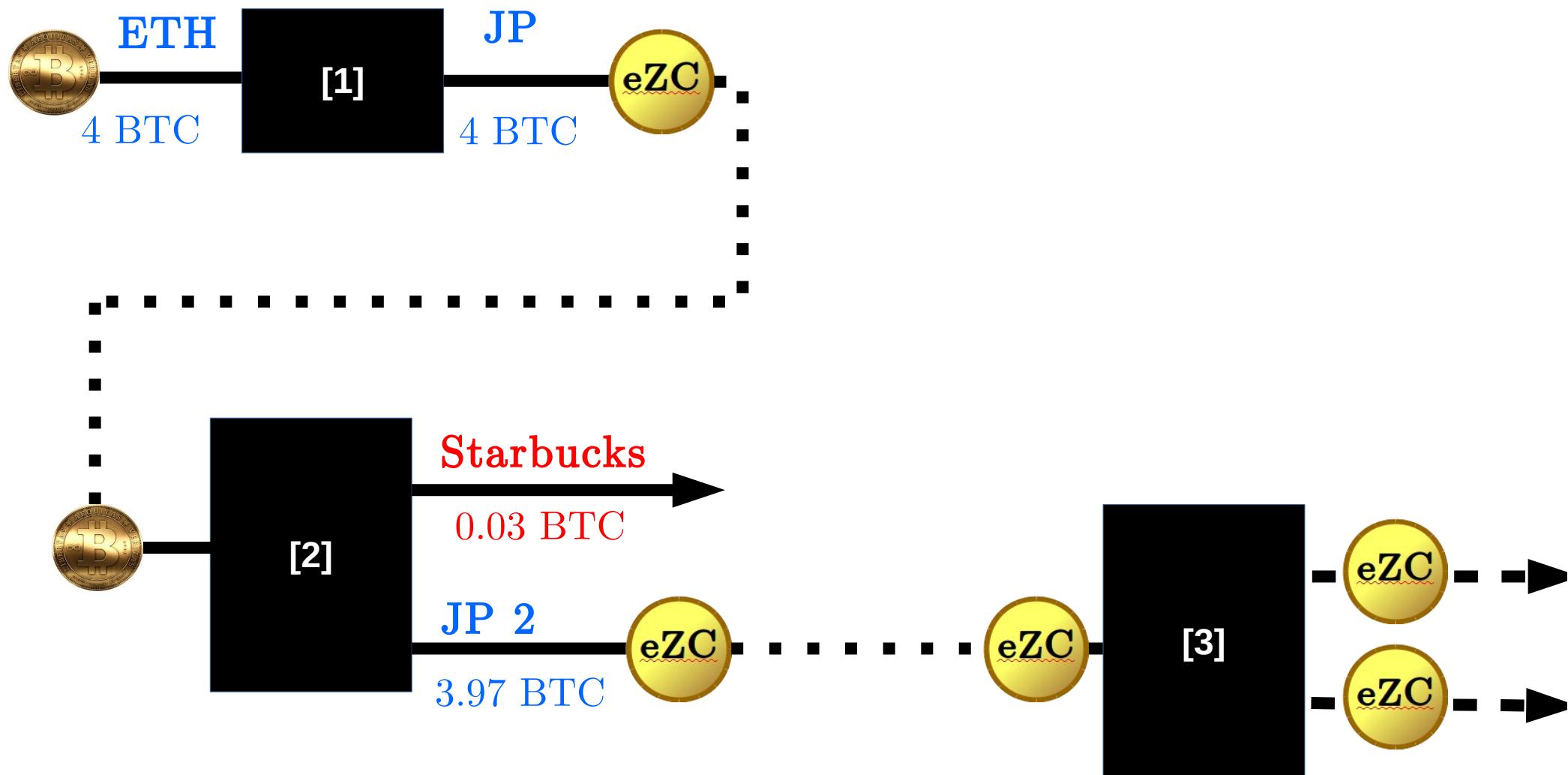
# Transactions Revisited



# Transactions Revisited



# Transactions Revisited



# Conclusion

# Thank You



Tip if you enjoyed it!