Principles of Distributed Computing	03/22, 2017
Exercise 5	
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1 Color Reduction in Vertex-Coloring

Exercise

(1a) Design a single-round algorithm that transforms any given k-coloring of a graph with maximum degree Δ into a k'-coloring for $k' = k - \lceil \frac{k}{2(\Delta+1)} \rceil$, assuming $k' \geq \Delta + 1$.

(1b) Use repetitions of this single-round algorithm, in combination with the $O(\log^* n)$-round $O(\Delta^2 \log \Delta)$-vertex-coloring that we saw in class, to obtain an $O(\Delta \log \Delta + \log^* n)$-round $(\Delta + 1)$-coloring algorithm.

2 SuperImposed Codes

Here, we use the concept of cover free families to obtain an encoding that allows us to recover information after superimposition. That is, we will be able to decode even if k of the codewords are *superimposed* and we only have the resulting bit-wise OR.

Exercise

(2a) Concretely, we want a function $Enc : \{0,1\}^{\log n} \to \{0,1\}^m$ — that encodes n possibilities using m-bit strings for $m \geq log_2 n$ — such that the following property is satisfied: $\forall S, S' \subseteq \{1, ..., n\}$ such that $|S| \leq k$ and $|S'| \leq k$, we have that $\vee_{i \in S} Enc(i) \neq \vee_{i \in S'} Enc(i)$. Here $\vee$ denotes the bit-wise OR operation. Present such an encoding function, with a small m, that depends on n and k. What is the best m that you can achieve?