A simple deterministic distributed MST Algorithm, with Near-Optimal Time and Message Complexities.

Jil Weber

What is a MST?

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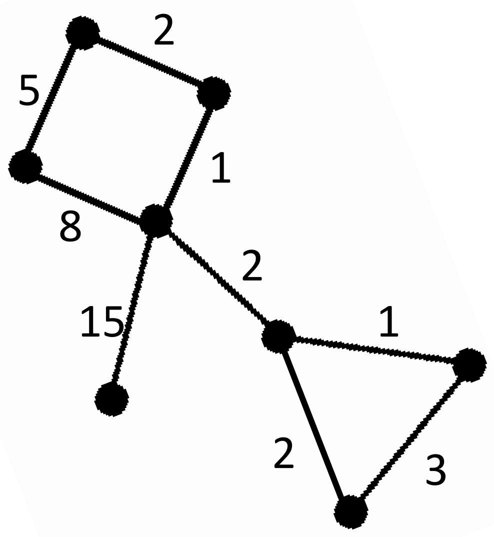
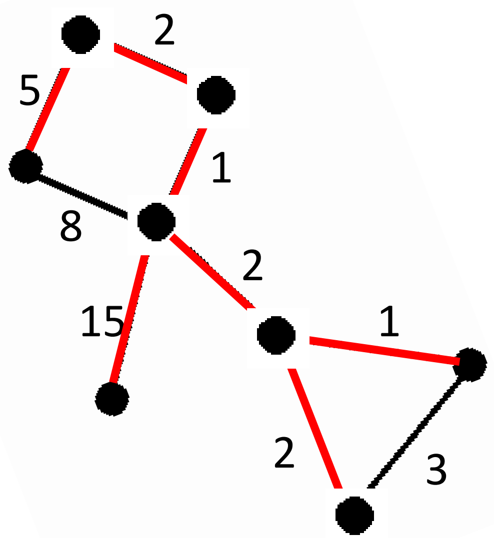
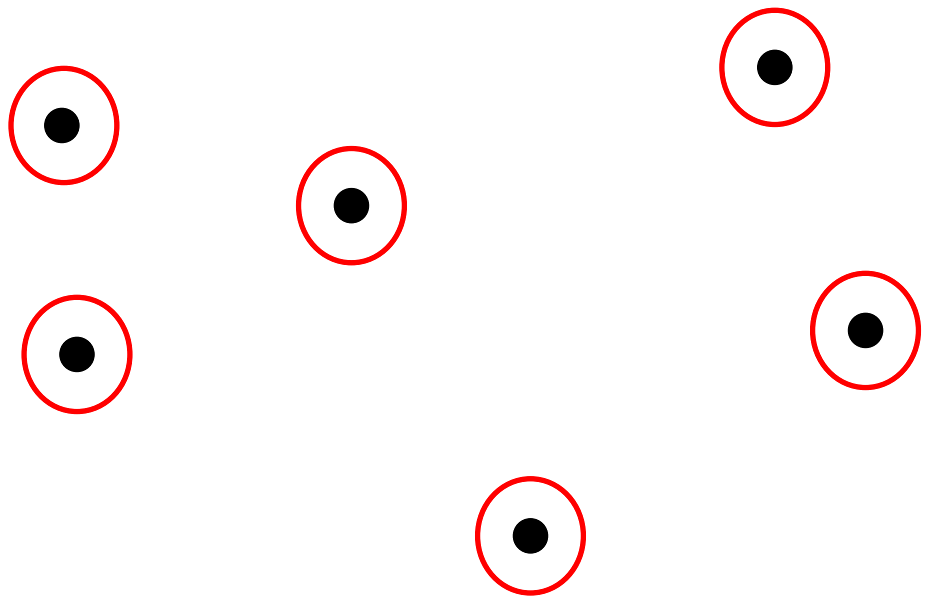
MST = minimum spanning tree





How to construct a MST?

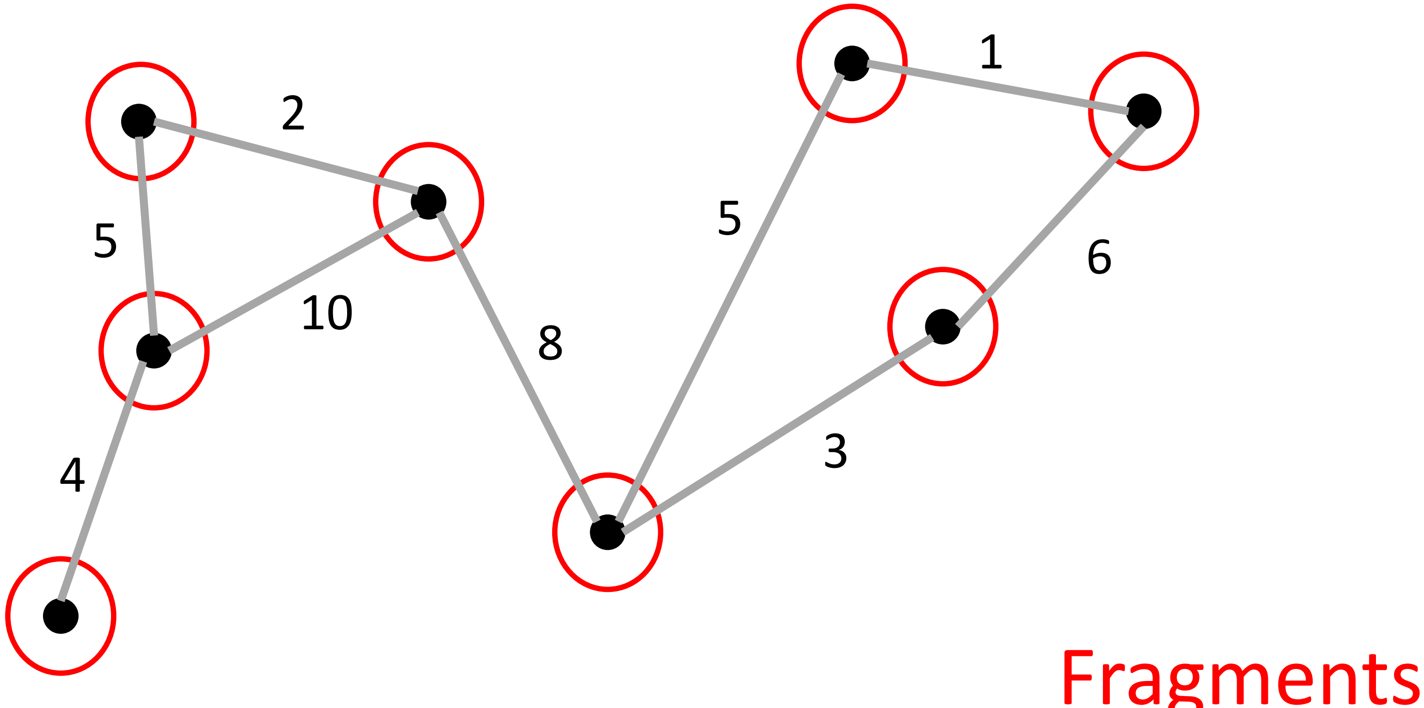
At the beginning: all nodes are their own roots



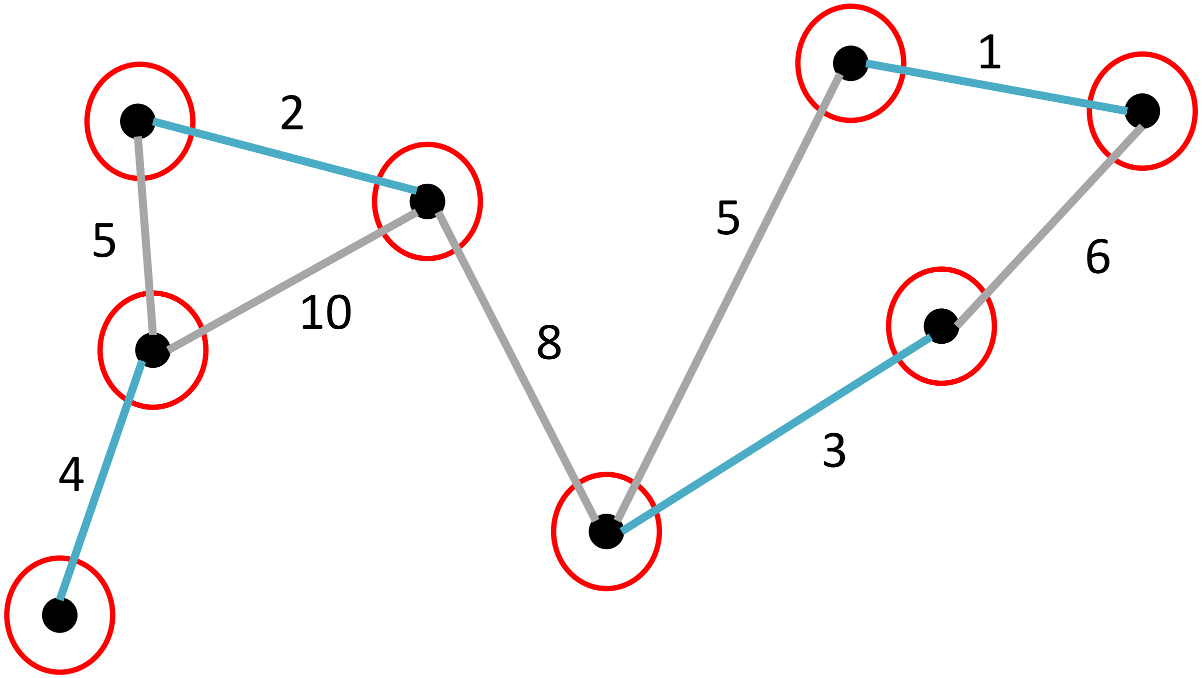
Fragments

How to construct a MST?

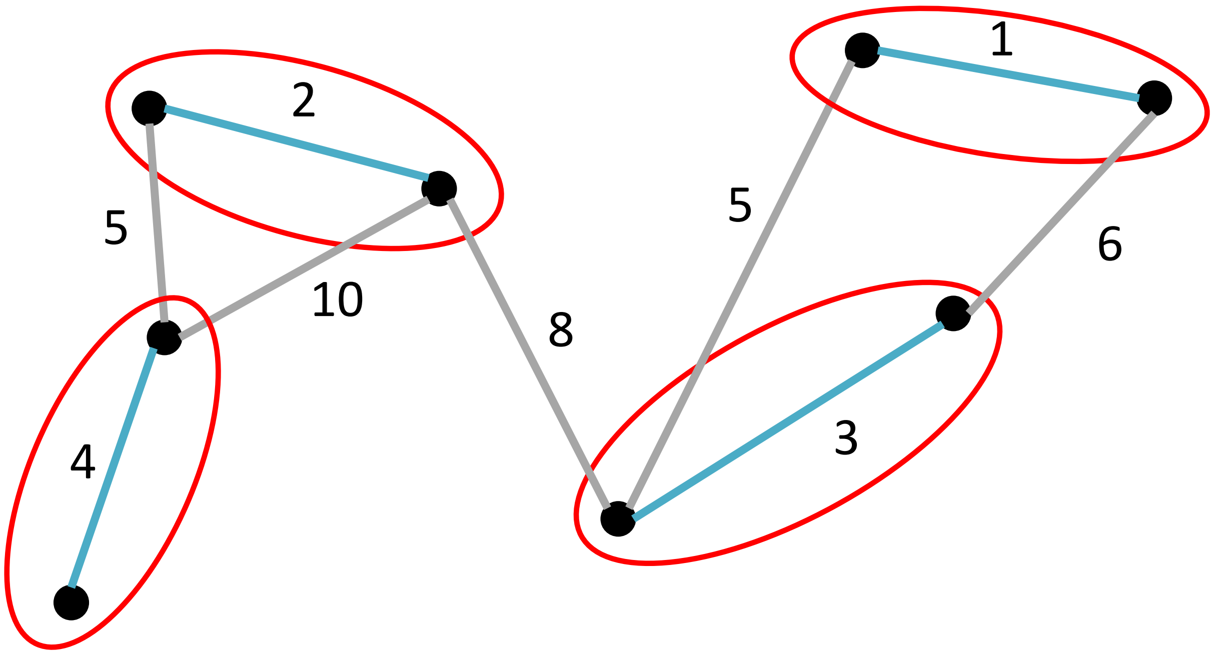
At the beginning: all nodes are their own roots



How to construct a MST?



How to construct a MST?



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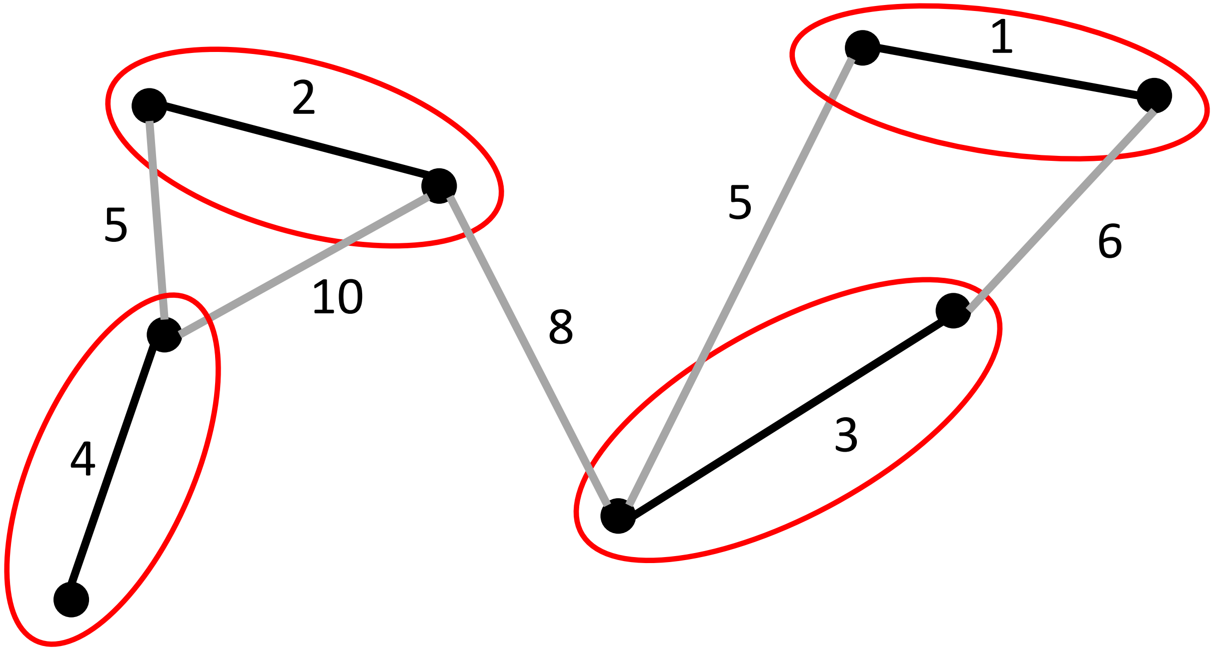

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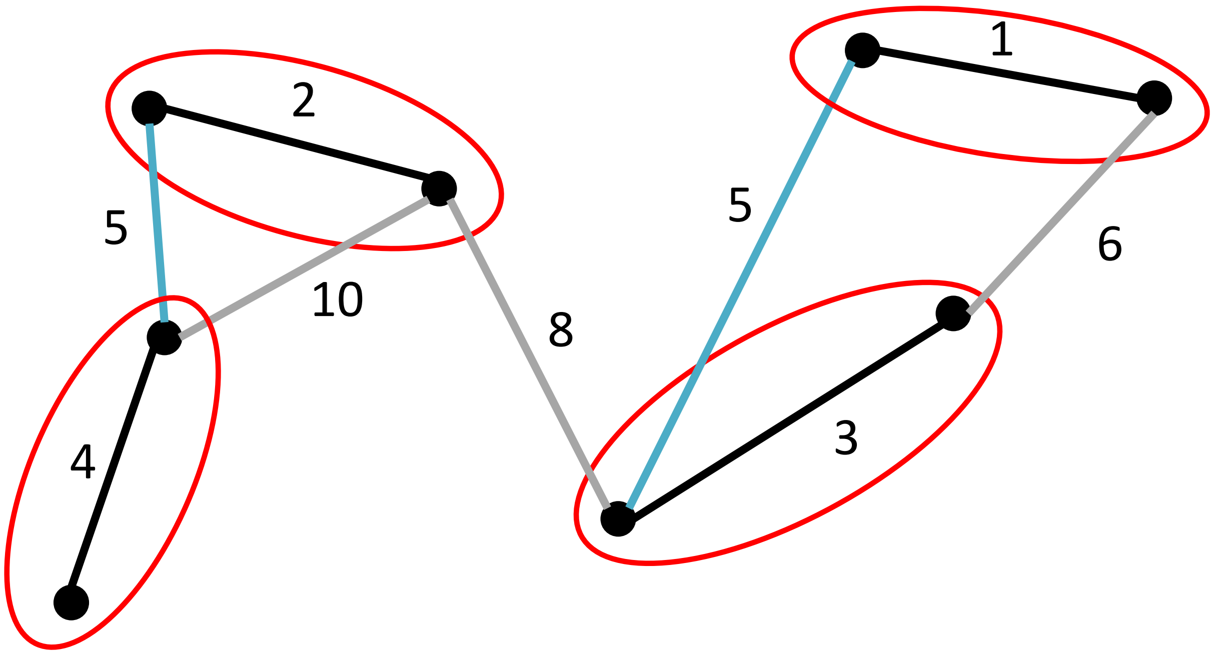

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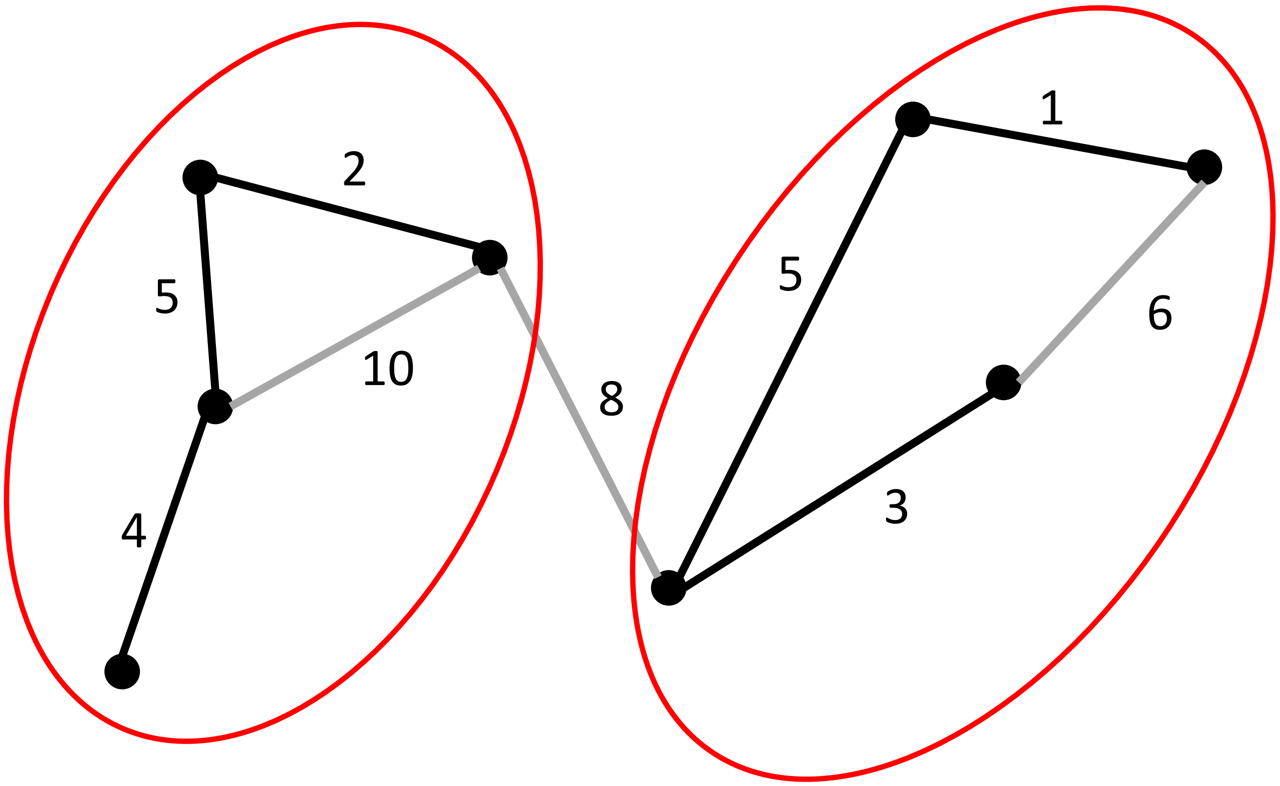

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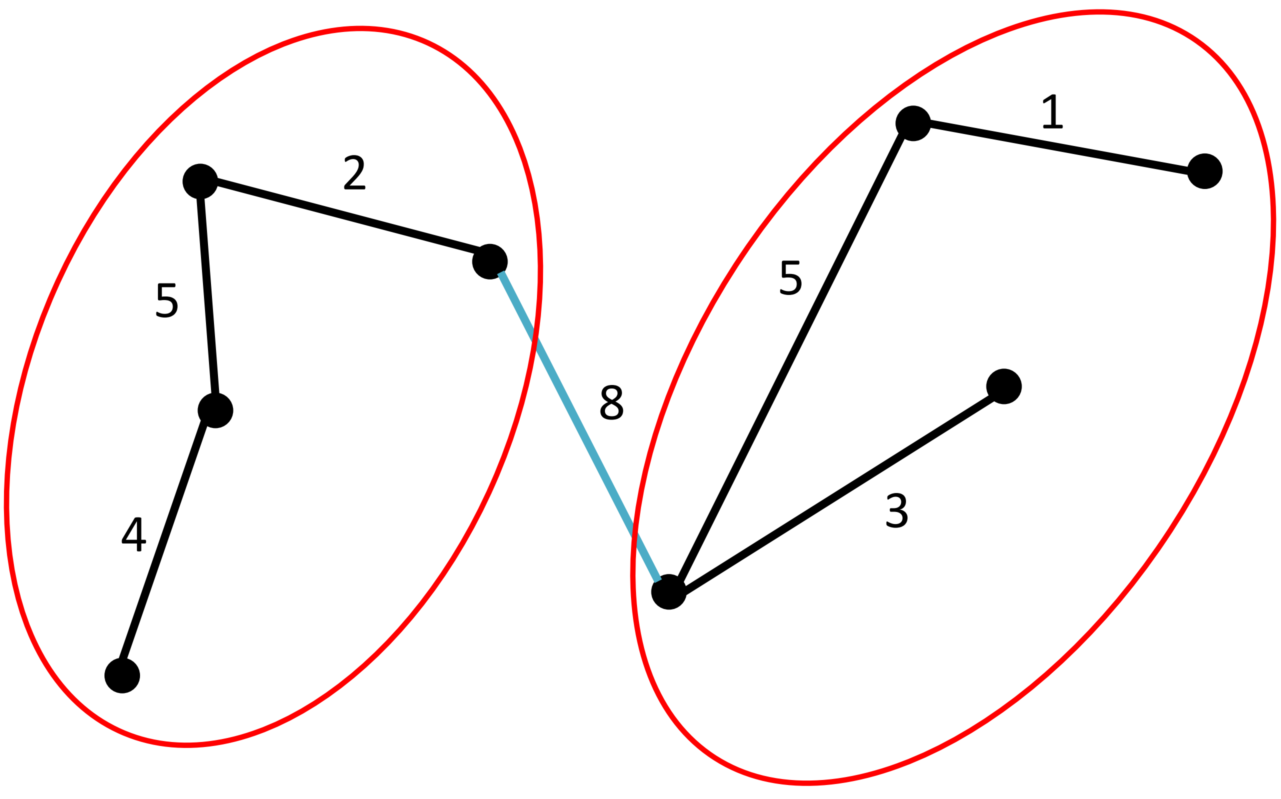
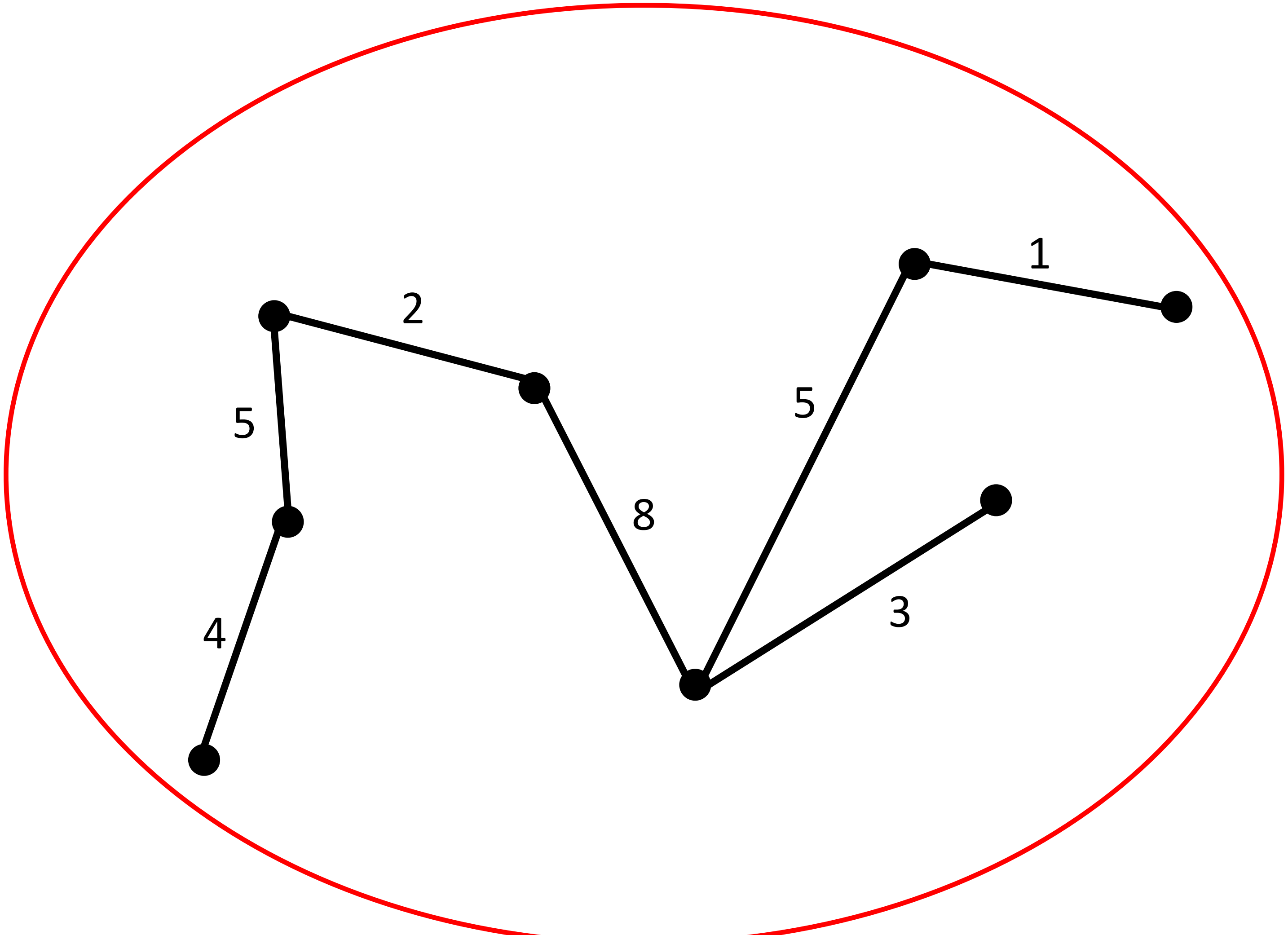
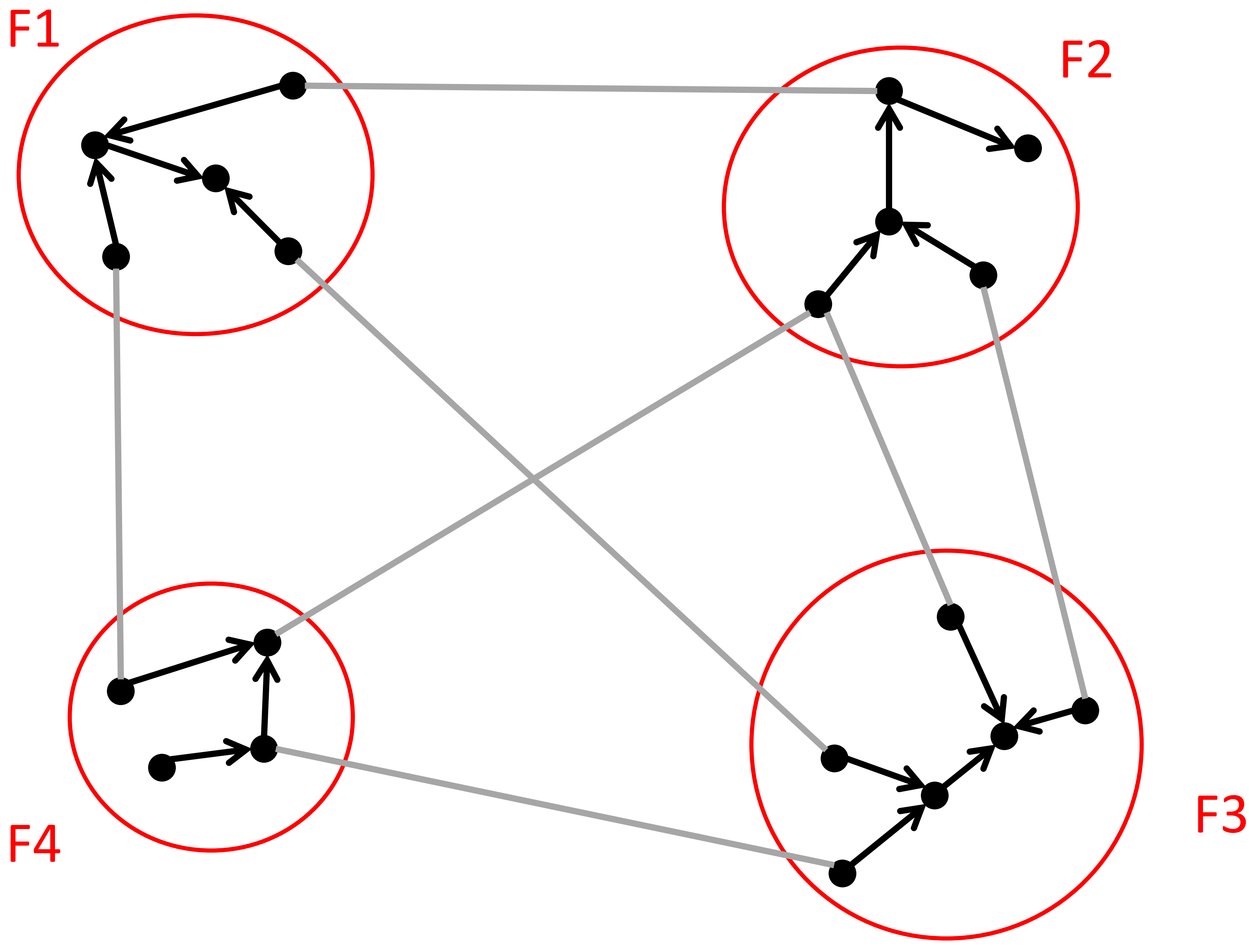
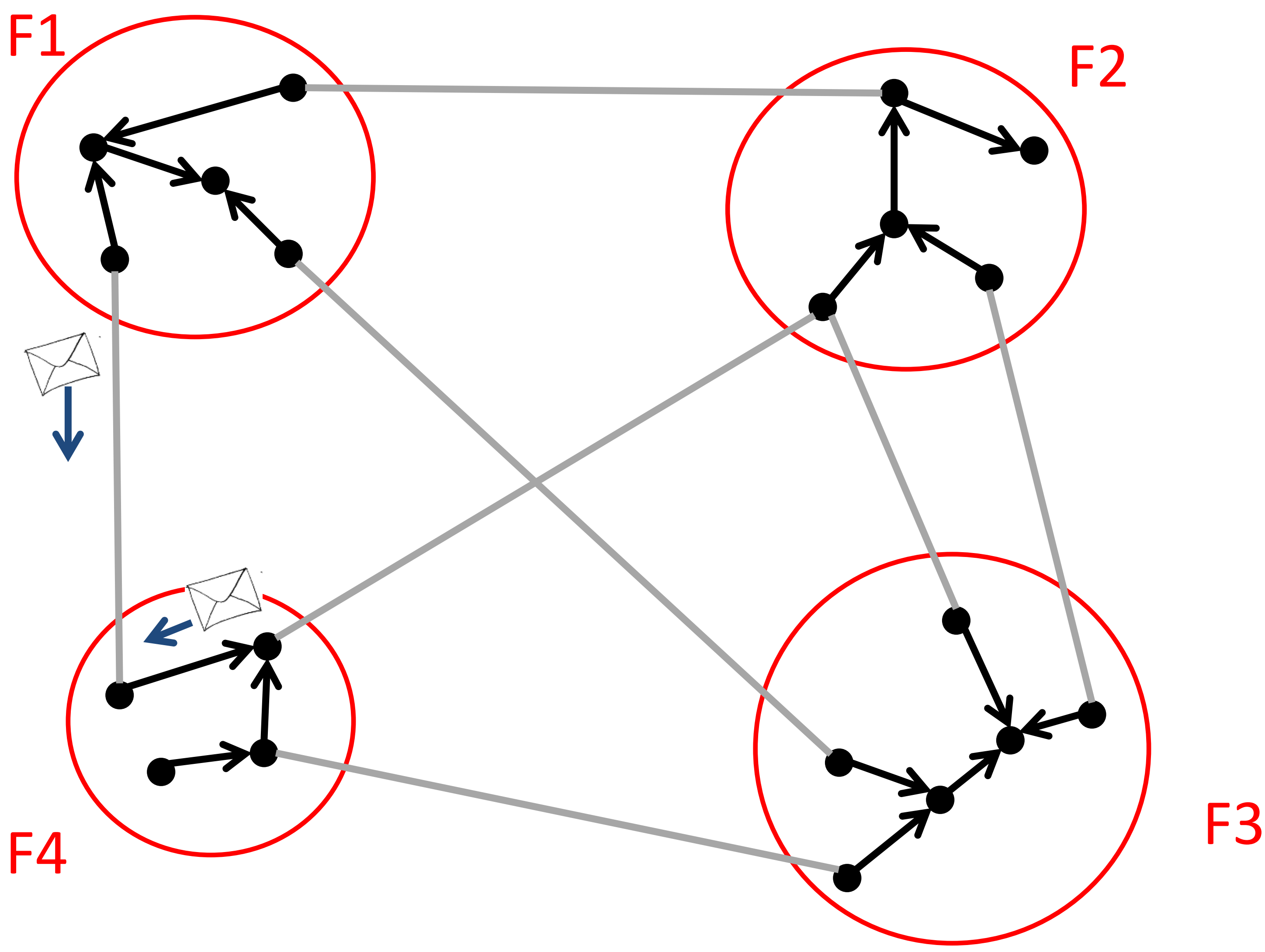


How to construct a MST?

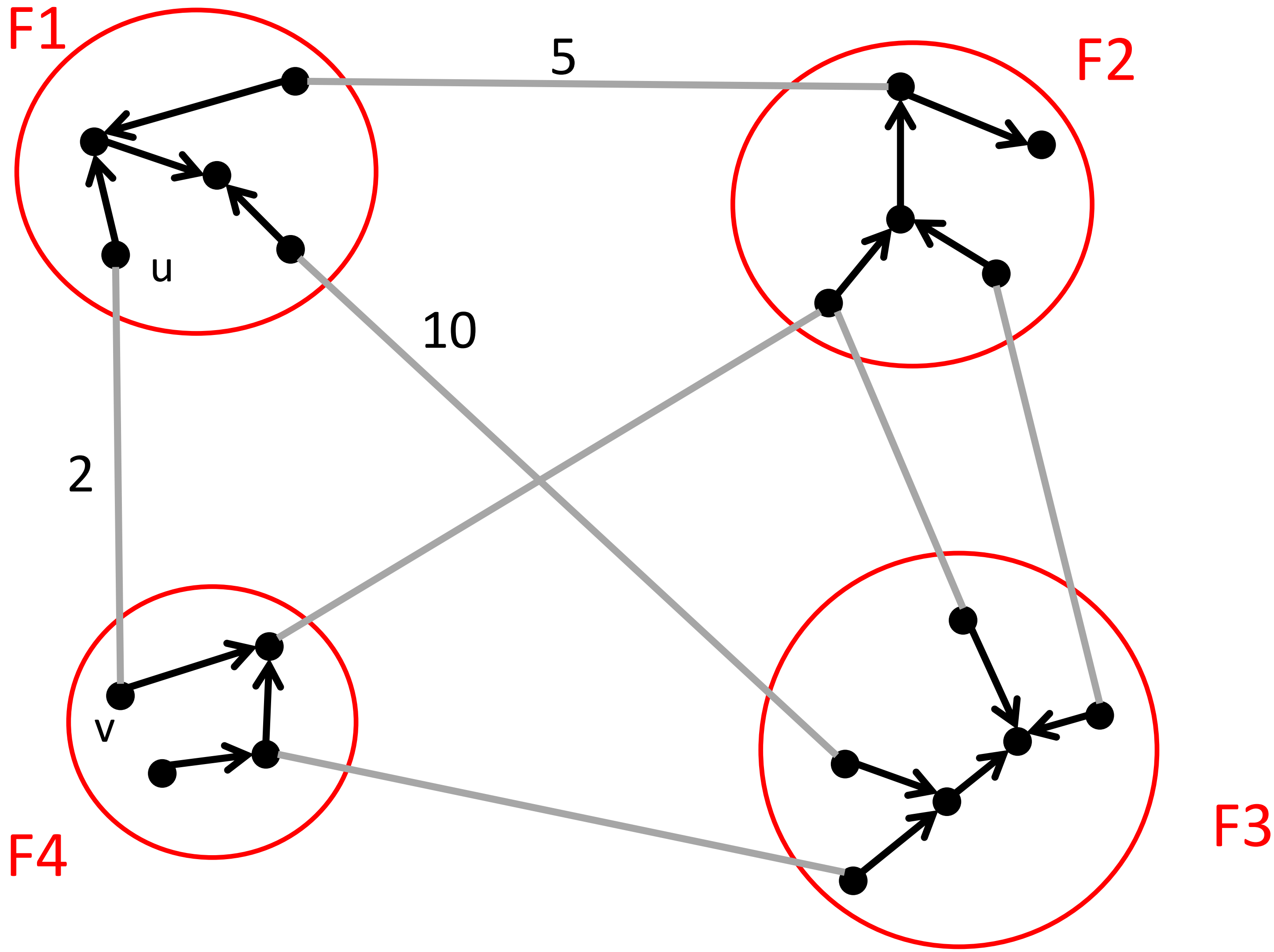


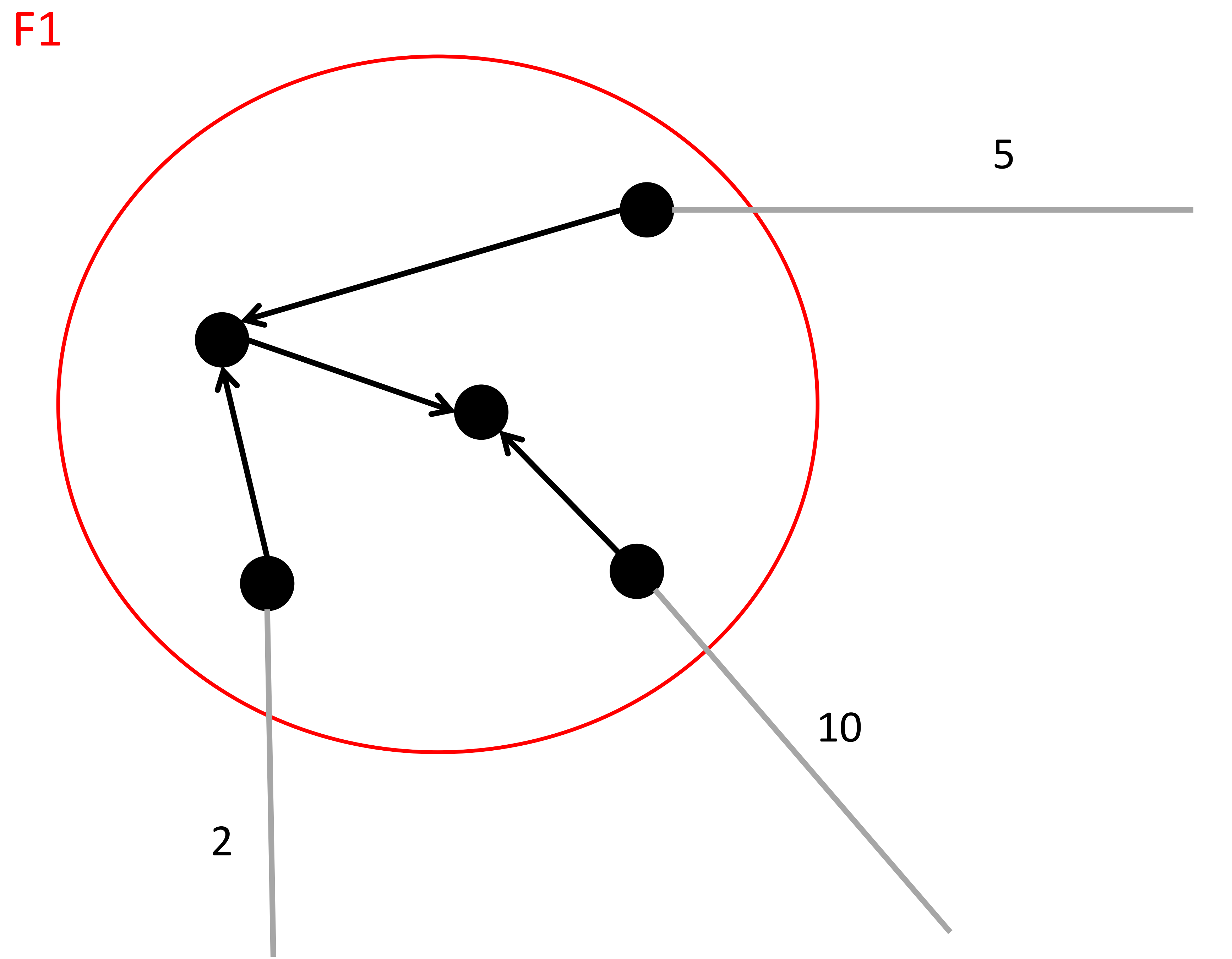

F1
F2
F3
F4

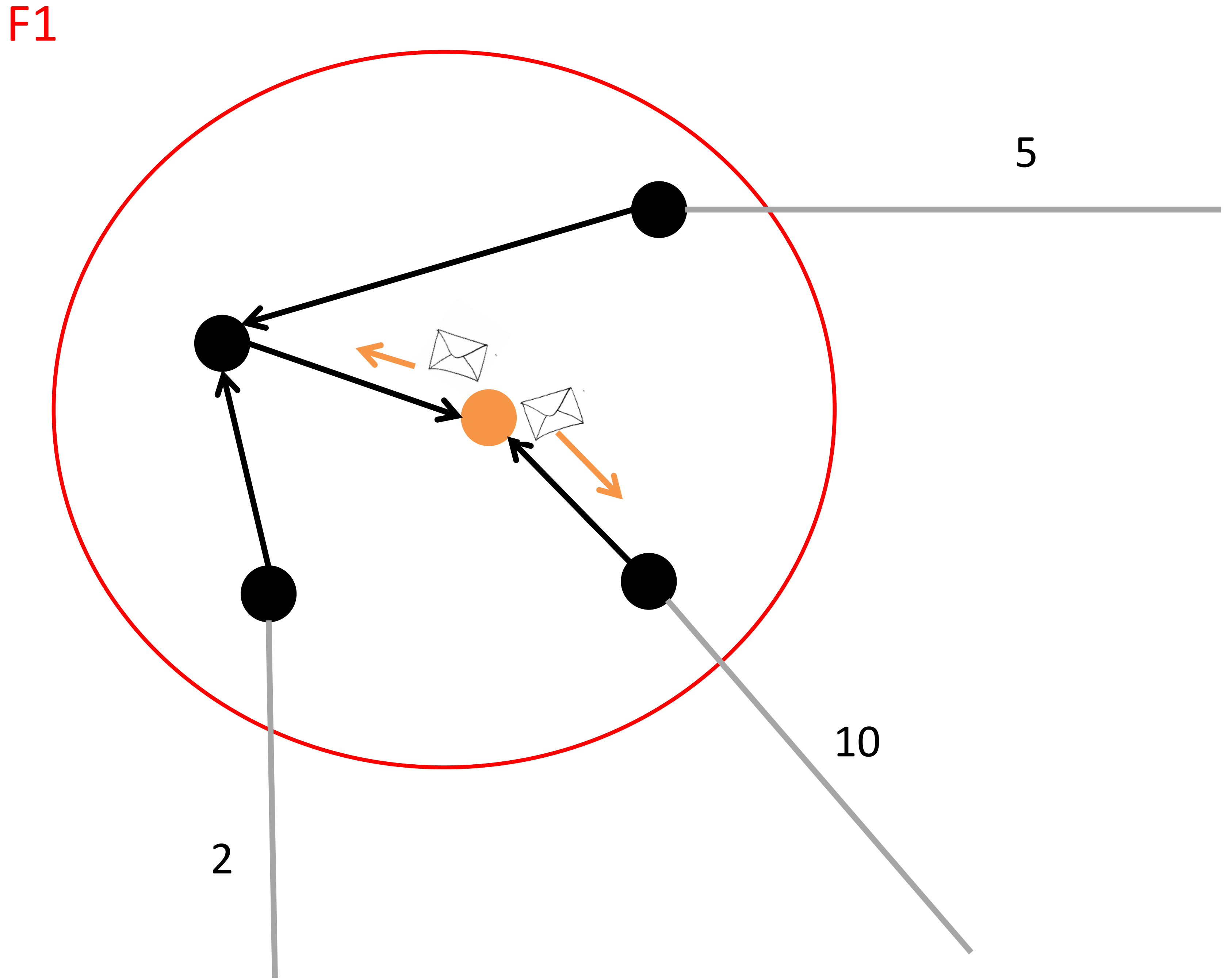
Step 1: Get ID from all neighbours

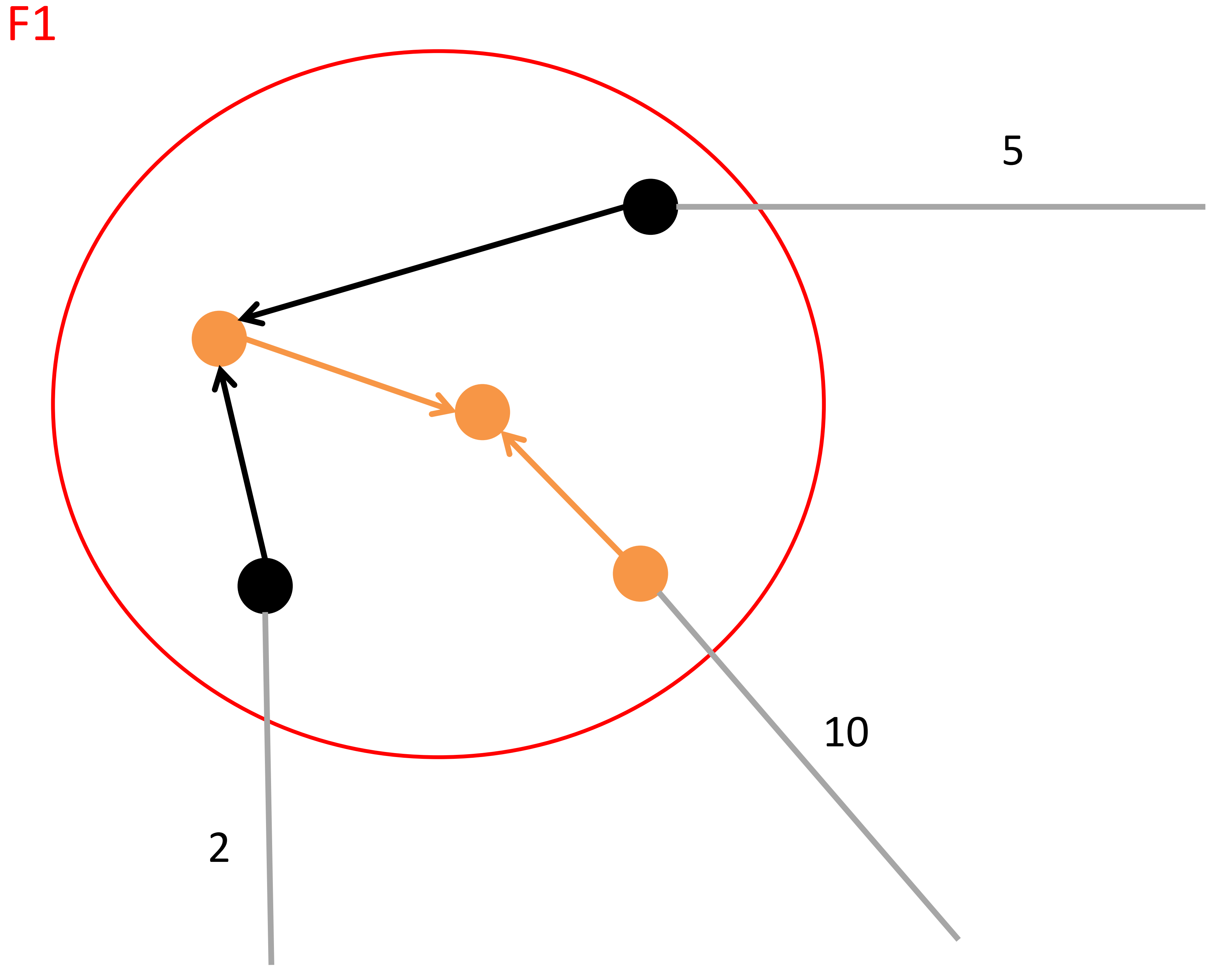


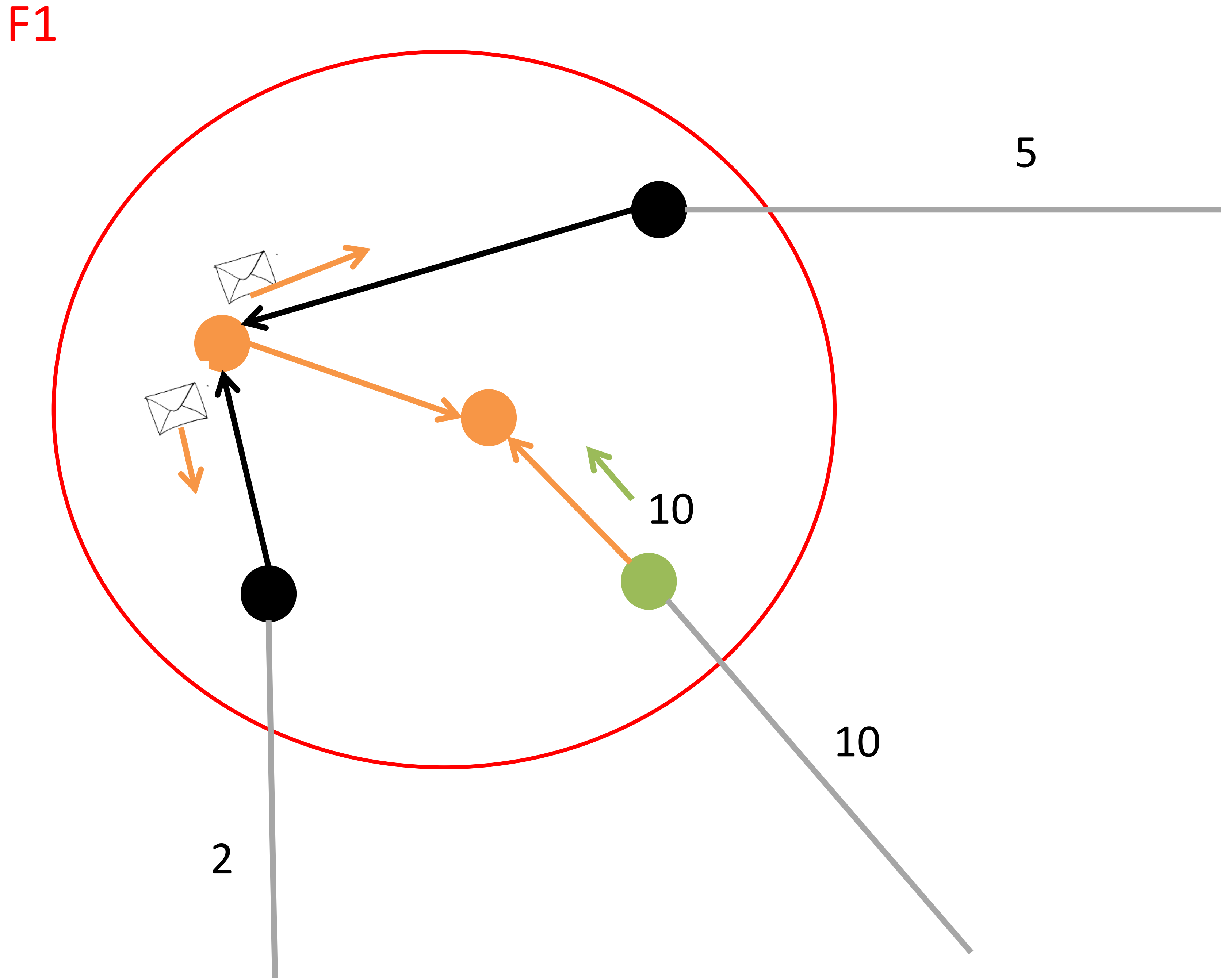
Step 2: Find blue edge



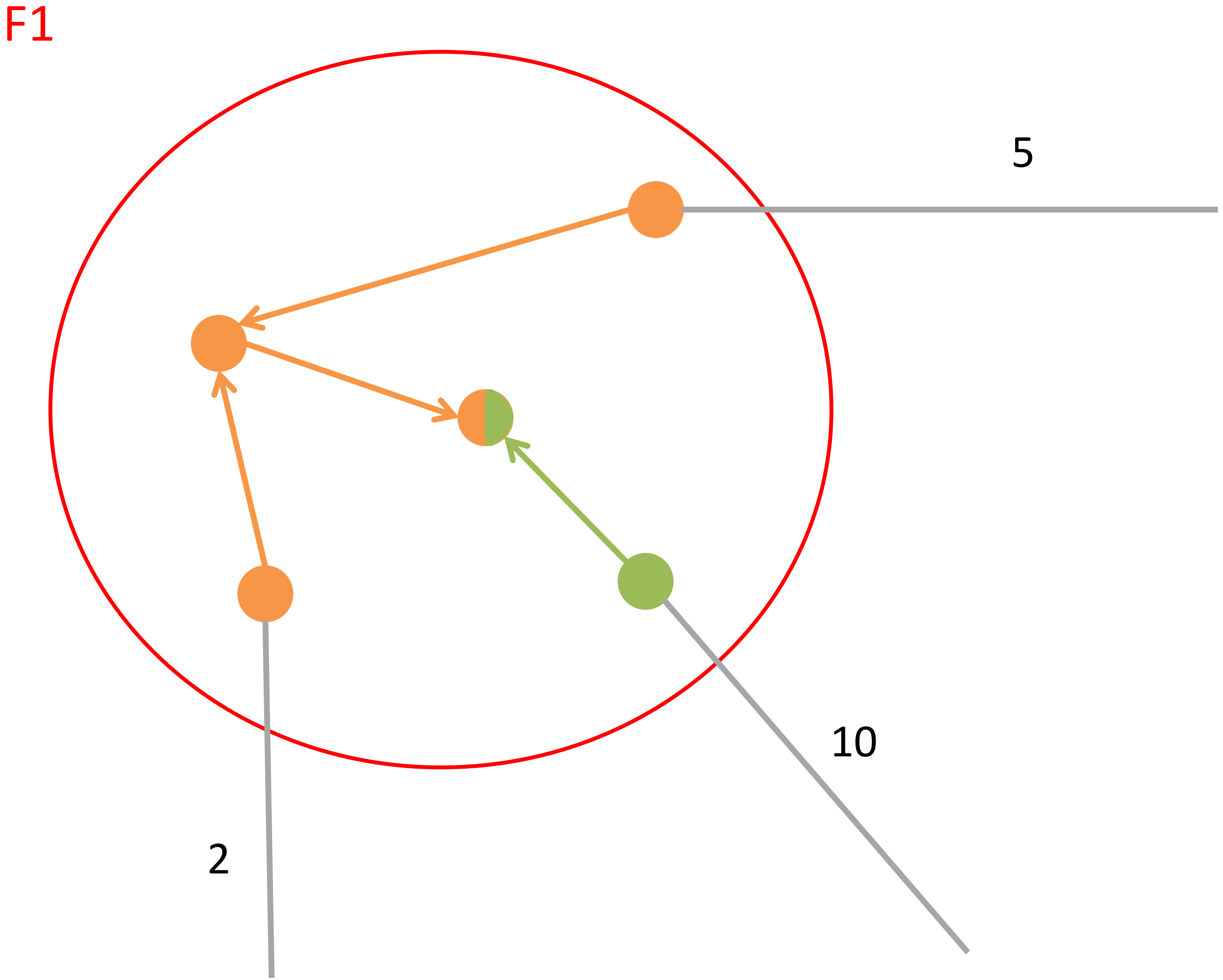

F1
5
10
2

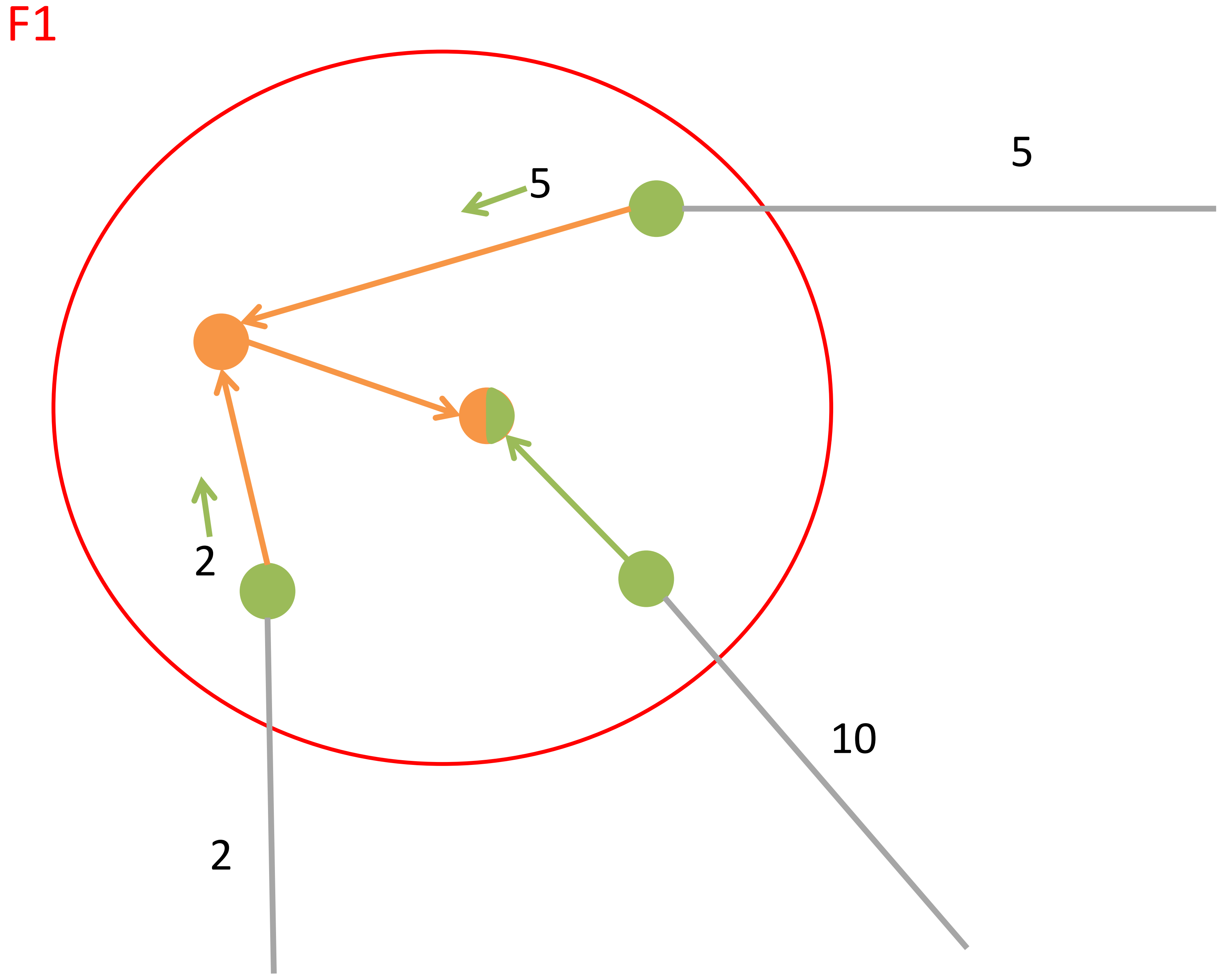

F1
5
10
2


F1
5
10
2

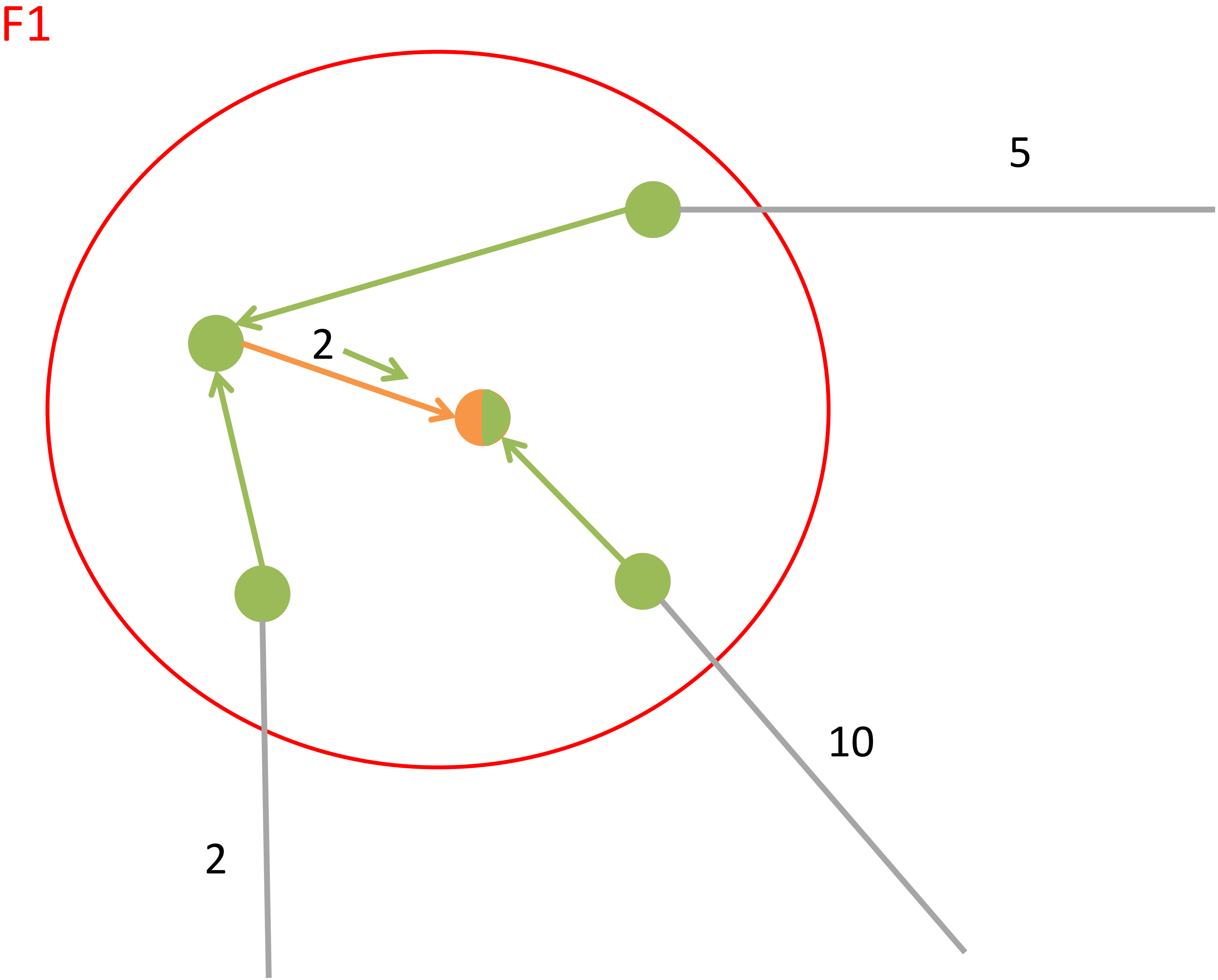


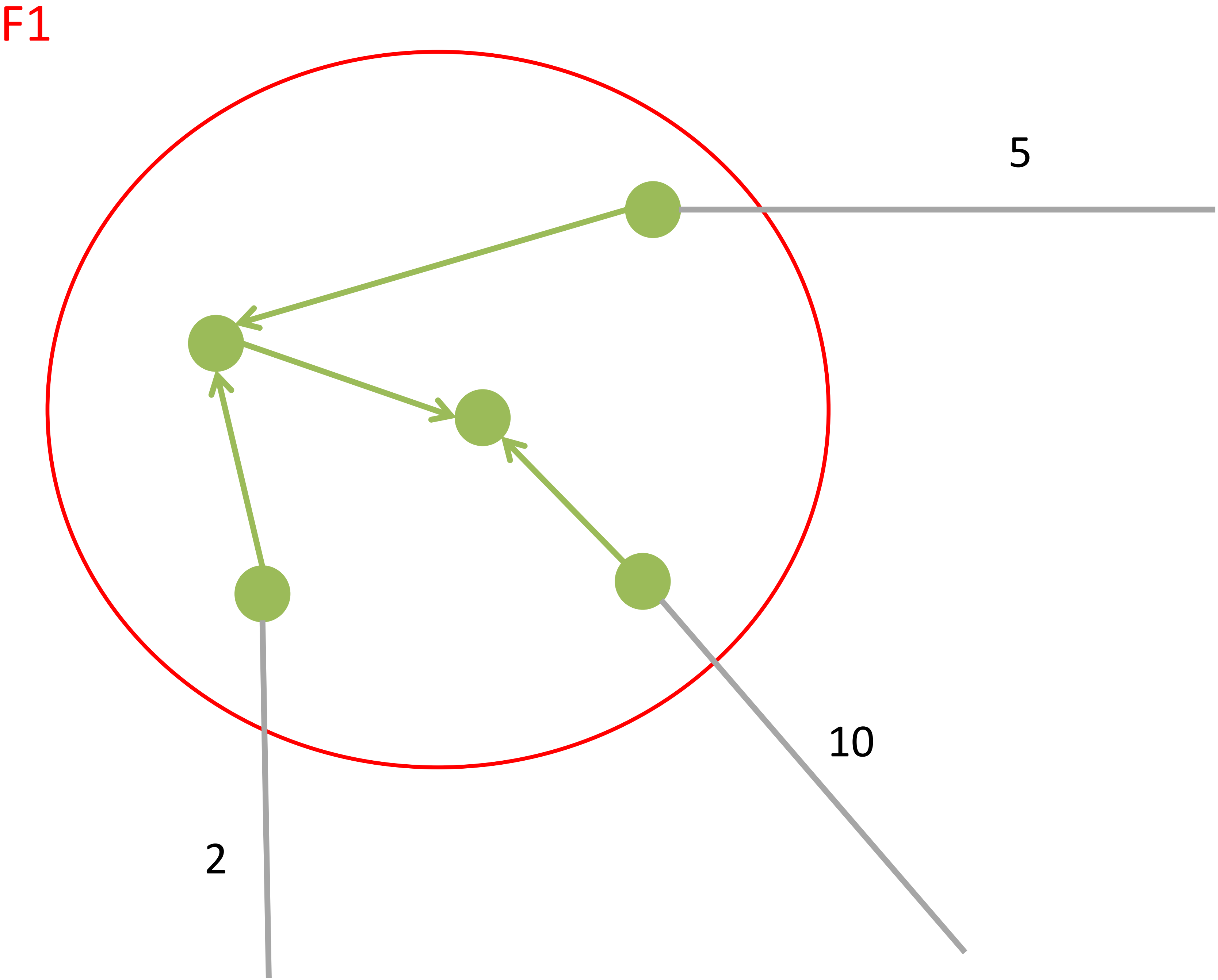
F1
5
10
10
2


F1
5
10
2

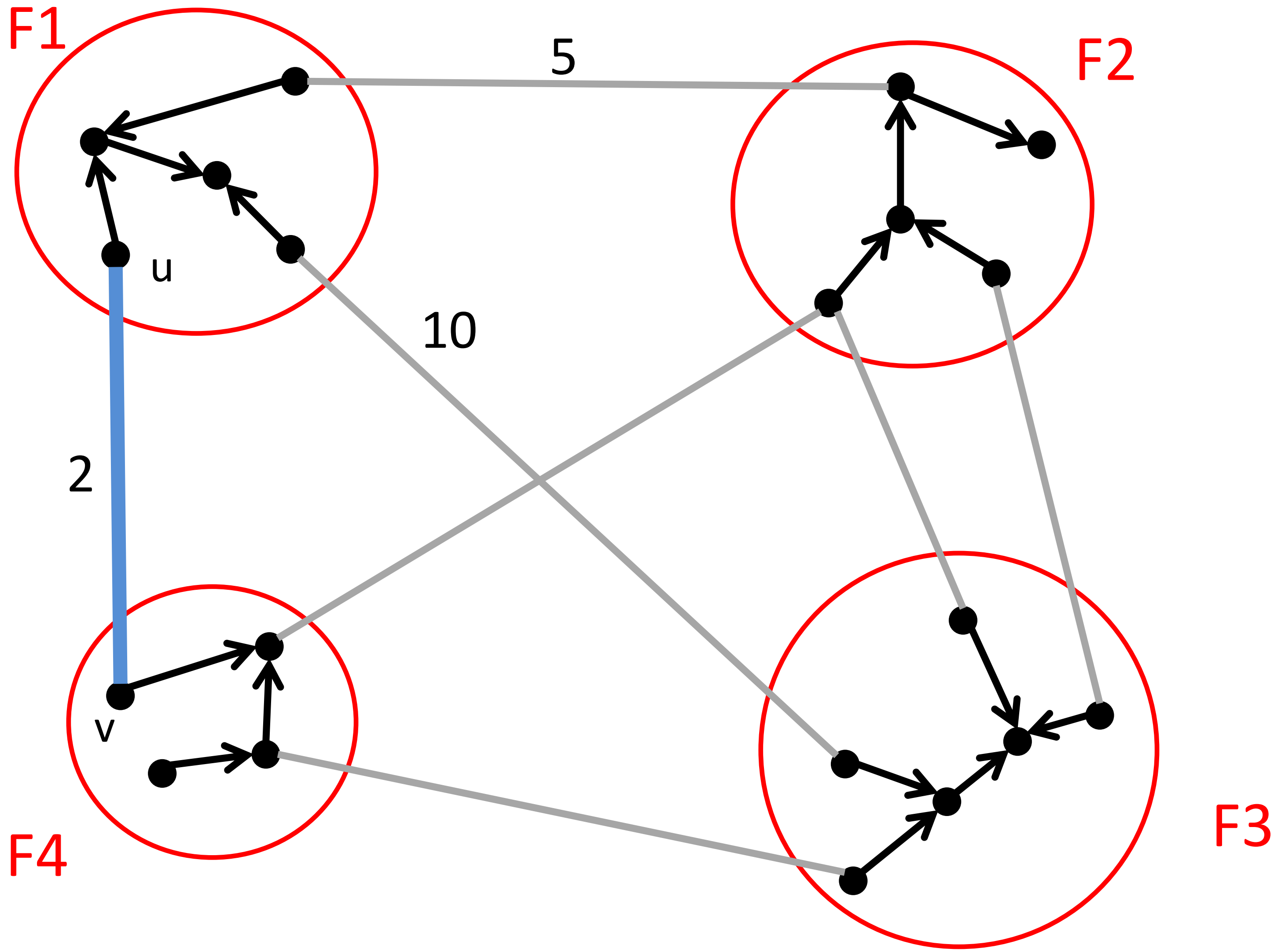


F1
5
5
2
2
10

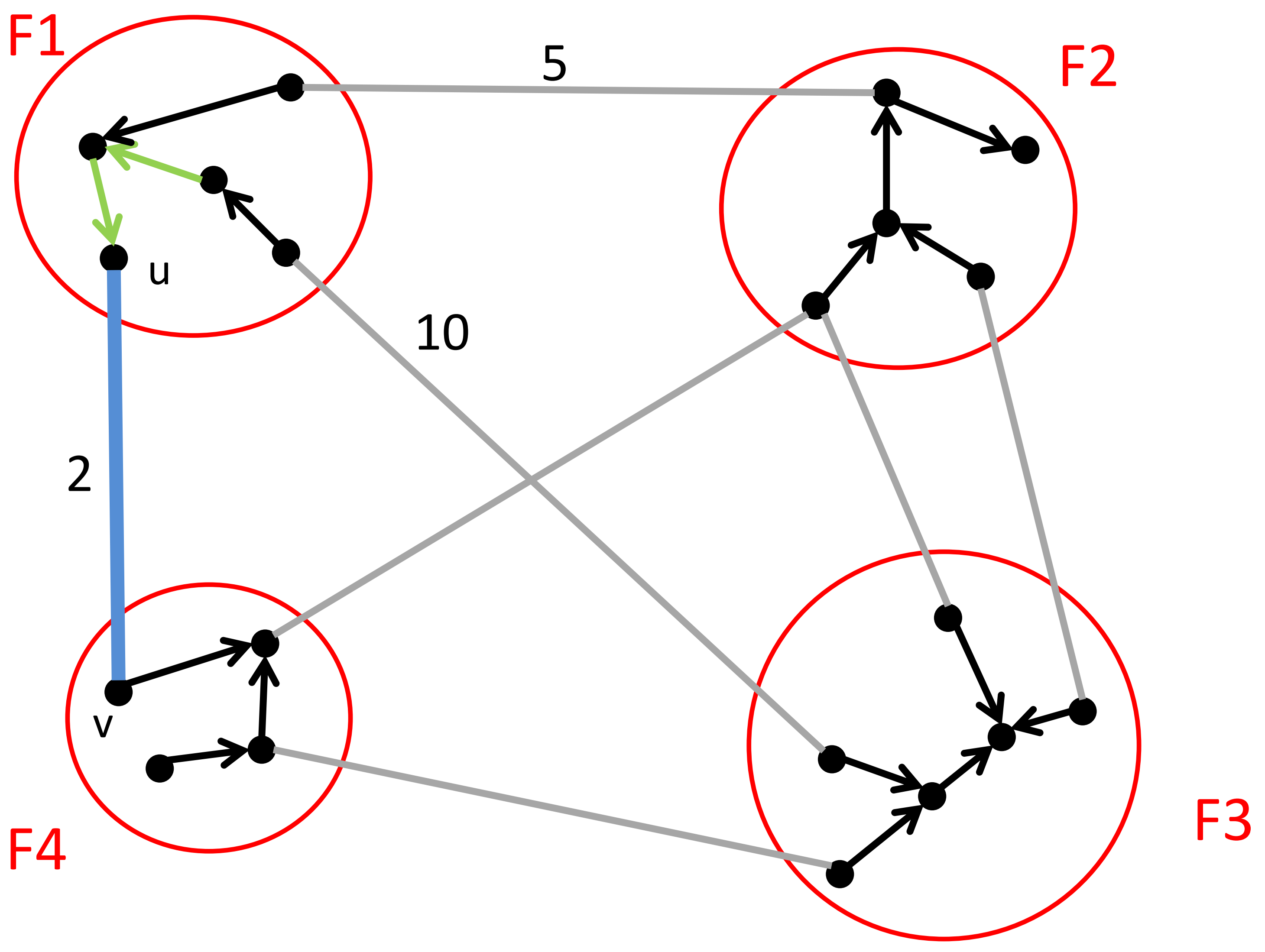

F1
5
2
10
2


F1
5
10
2

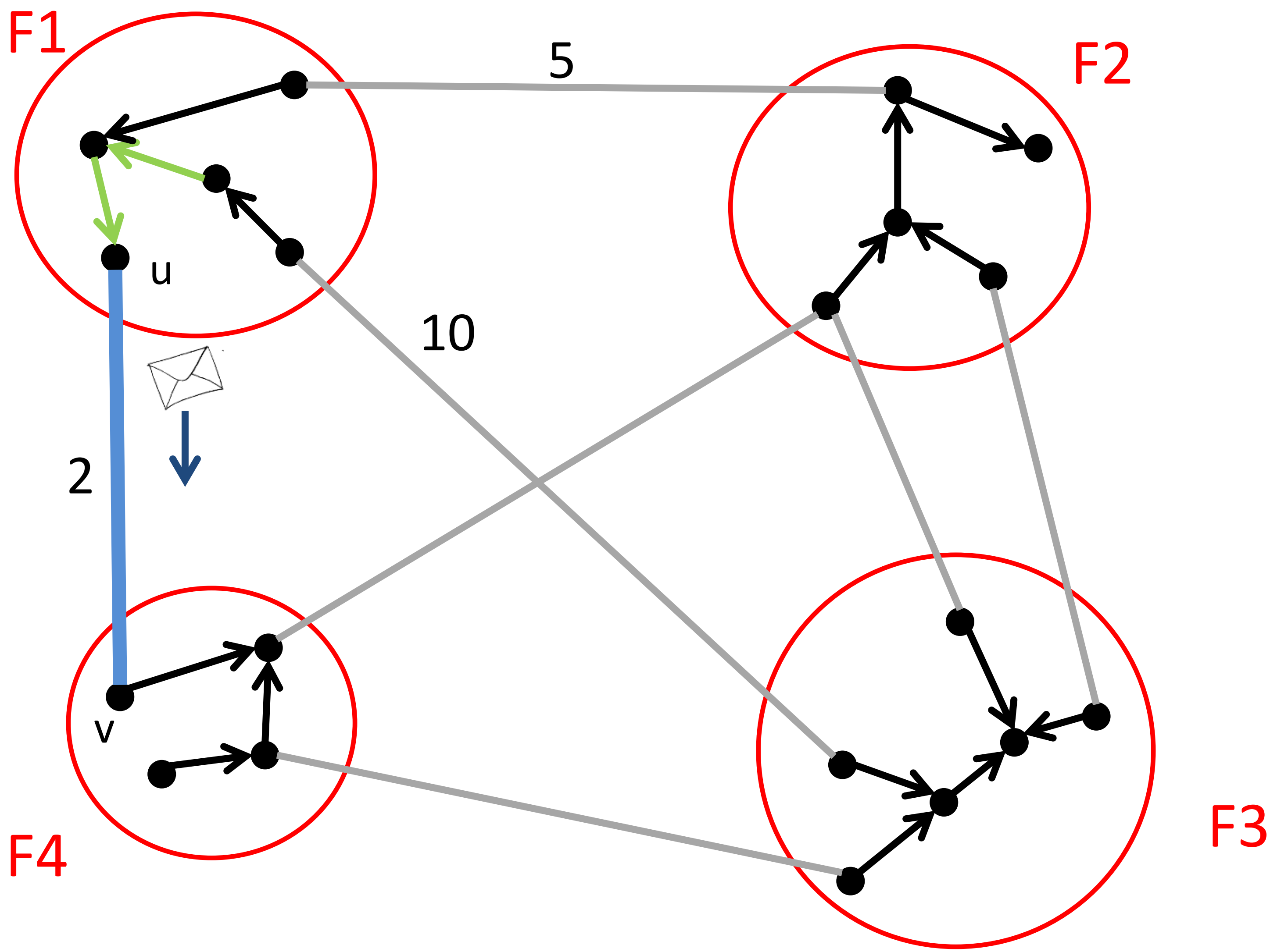
Step 2: Find blue edge



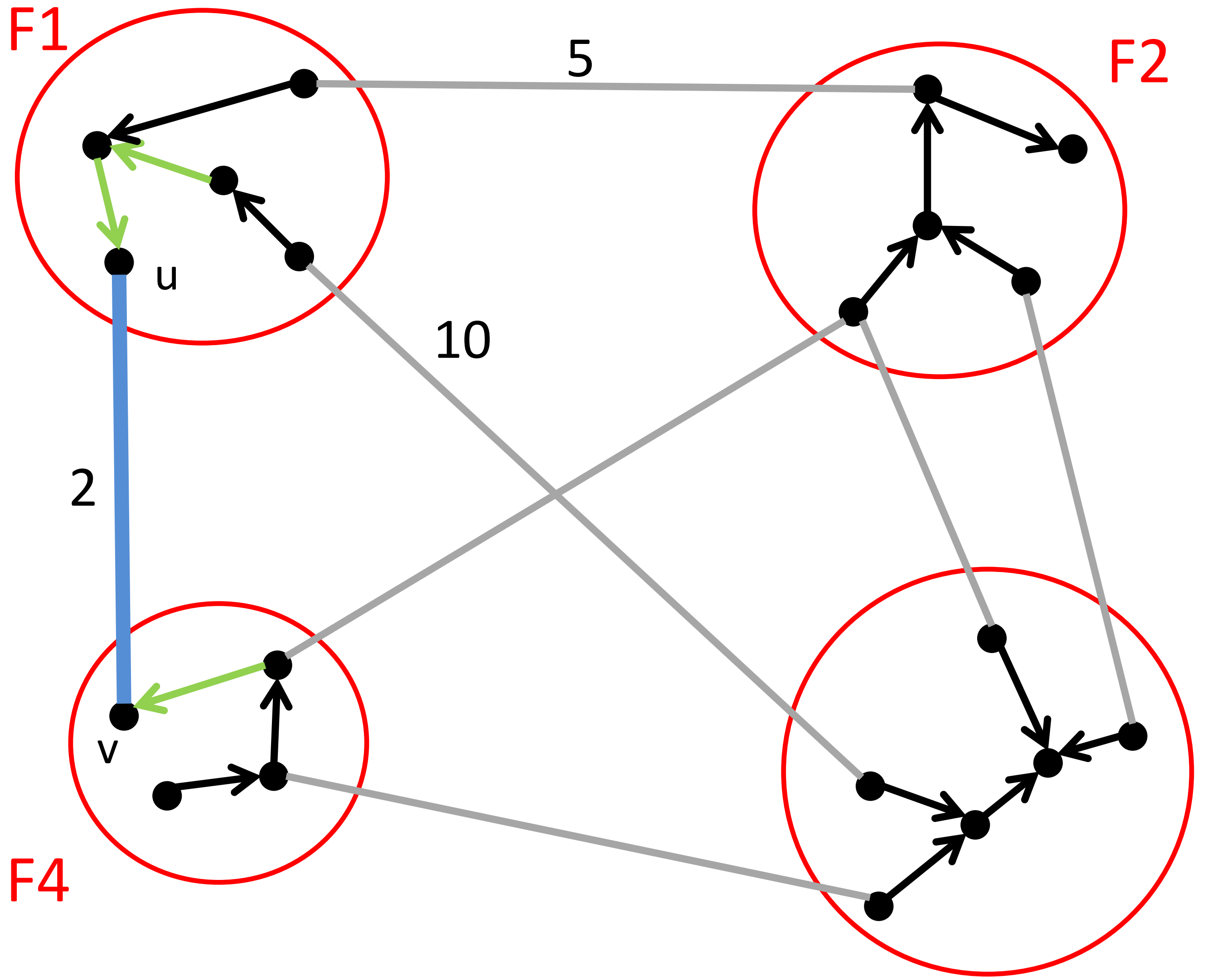
Step 3: Send Message from root



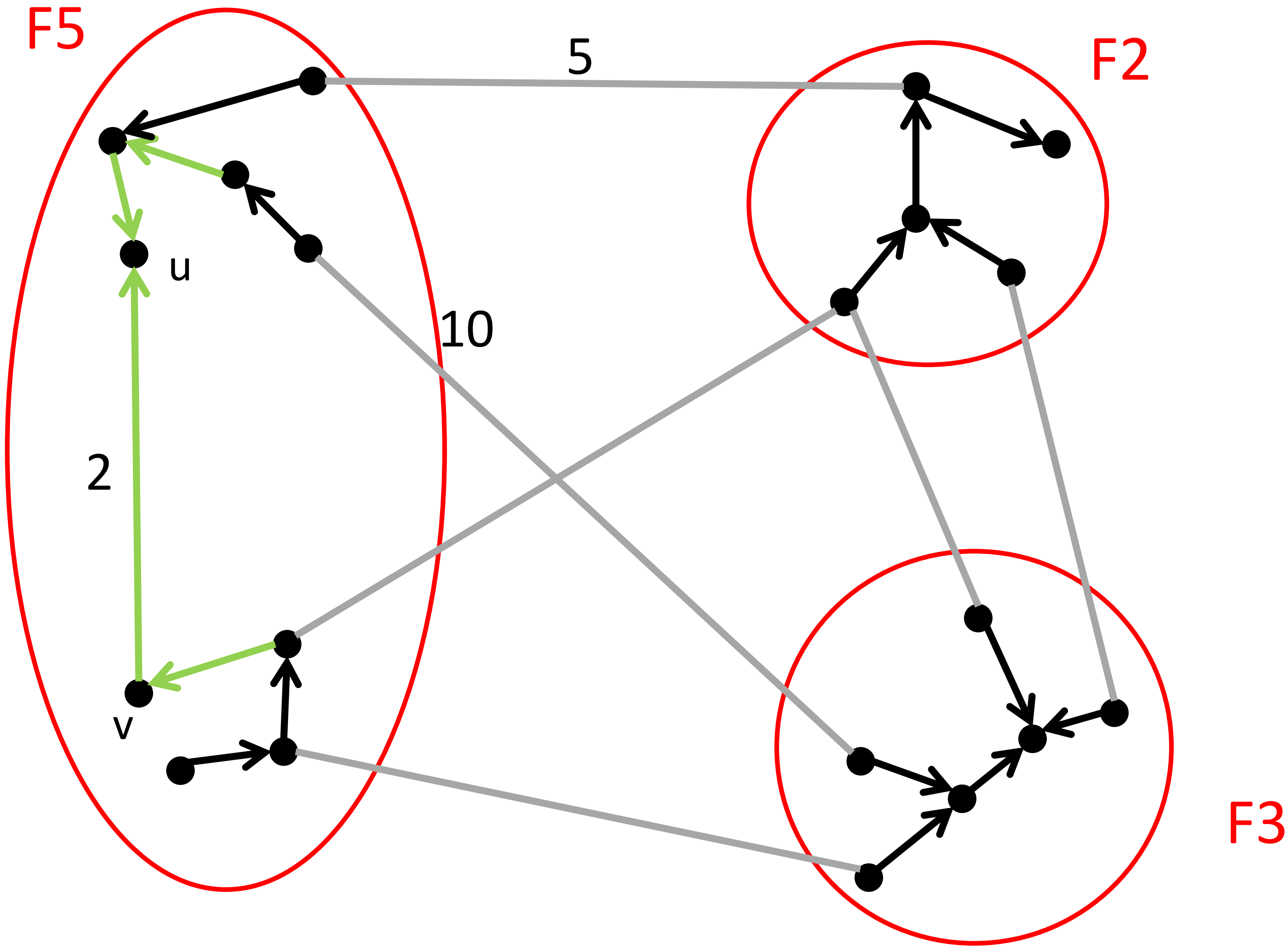
Step 4: Request message



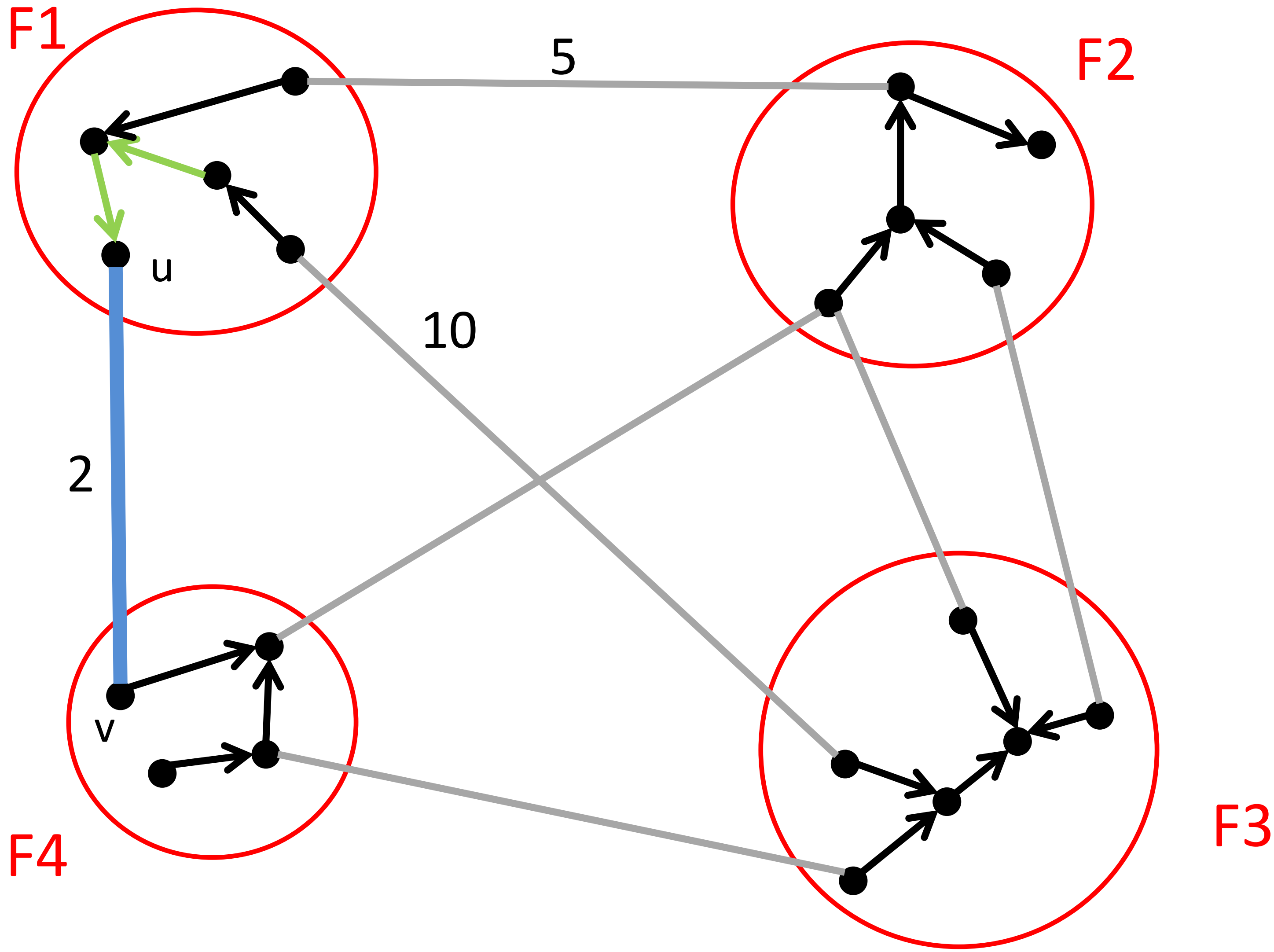
Step 5.1: v sends also request message



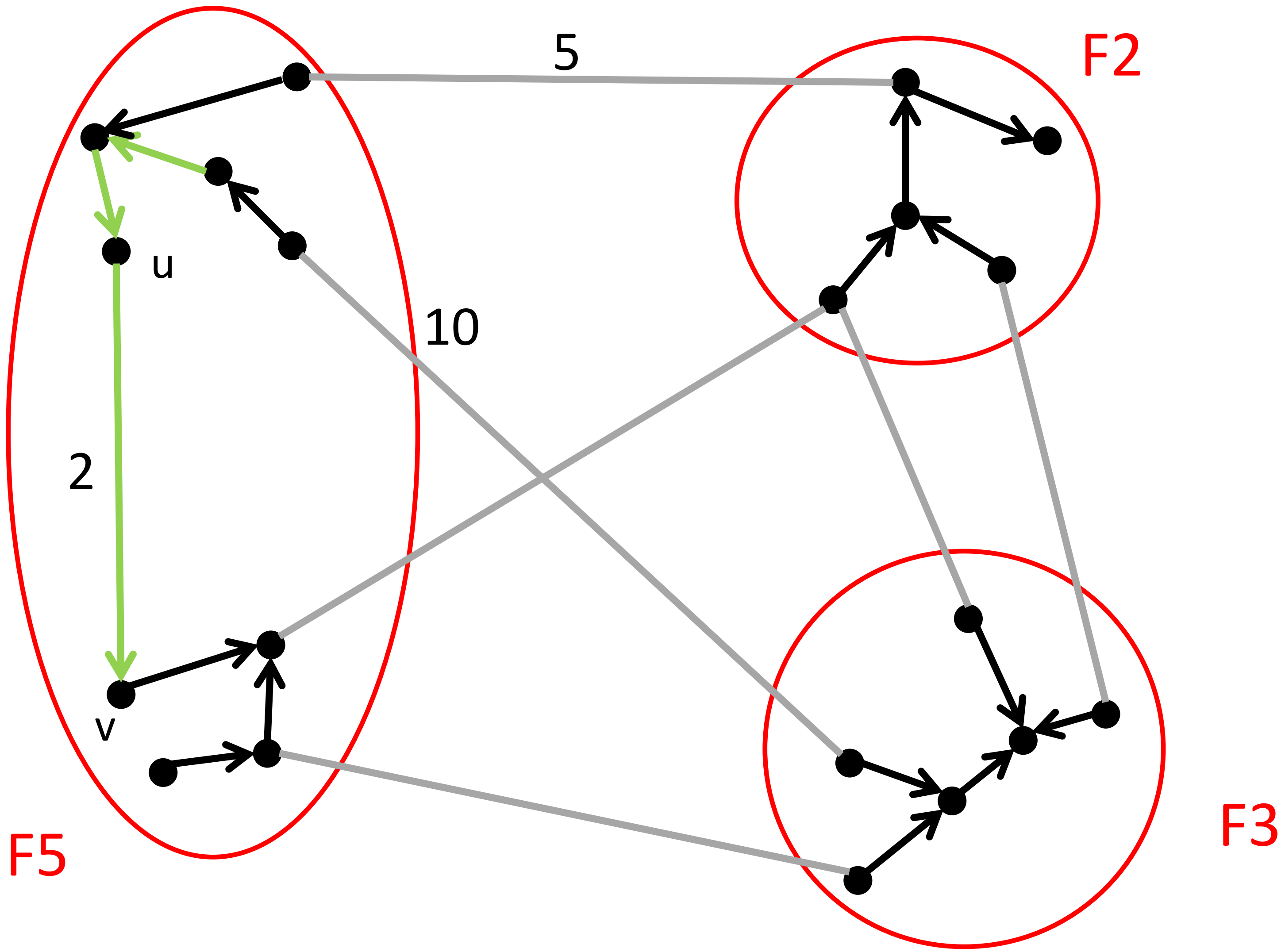
Step 5.1: New root with smaller ID



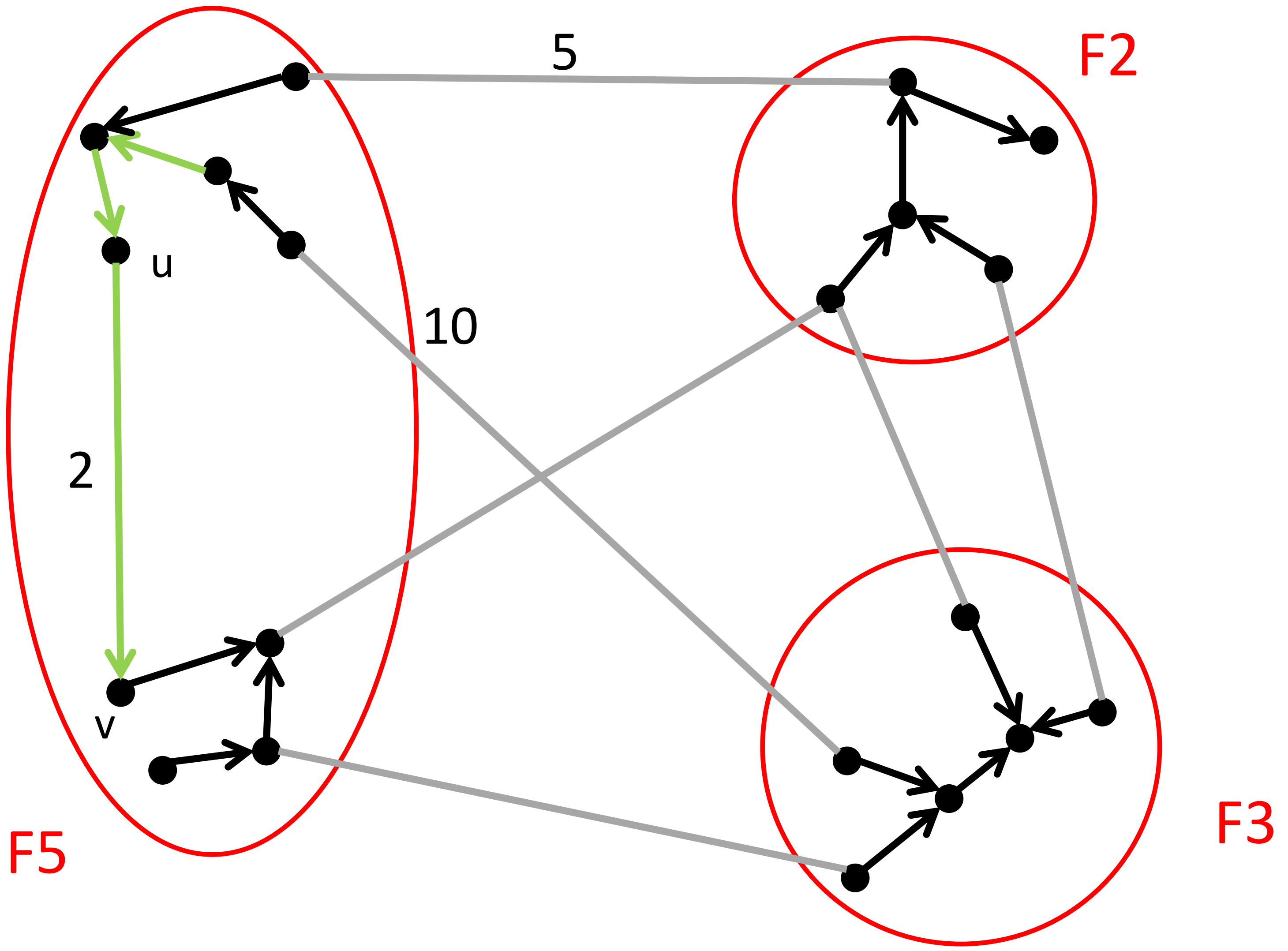
Step 5.2: v parent of u



Step 5.2: v parent of u



Step 6: root sends ID to all nodes of F5



Time and Message Complexity

→One phase: Time: O(n)   Message: O(m)

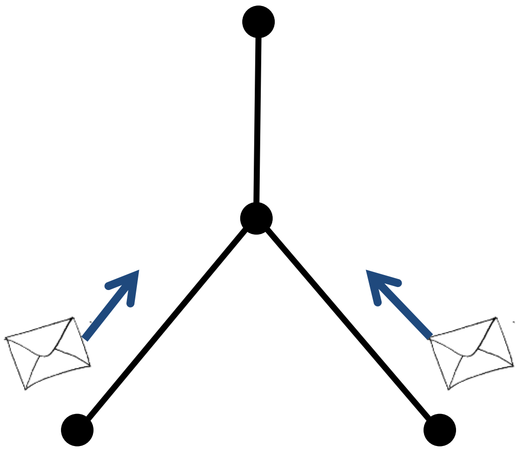
→O($\log(n)$) phases:

Time: O($n \cdot \log(n)$)

Message: O($m \cdot \log(n)$)

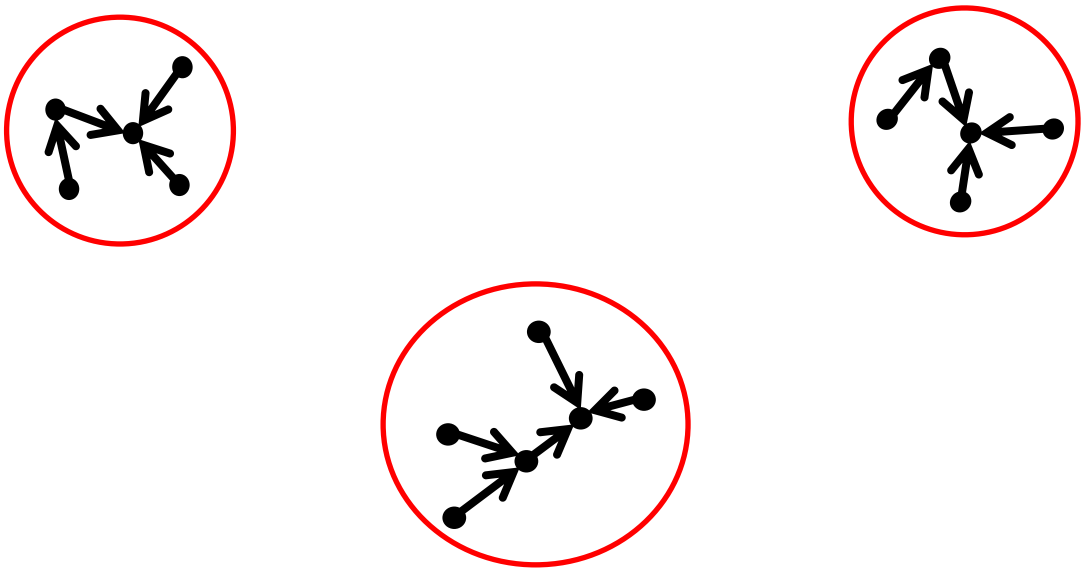
Preliminaries

- Synchronous CONGEST model
  → send message of size O(log(n))
- Edges have weights and the ID of size O(log(n))



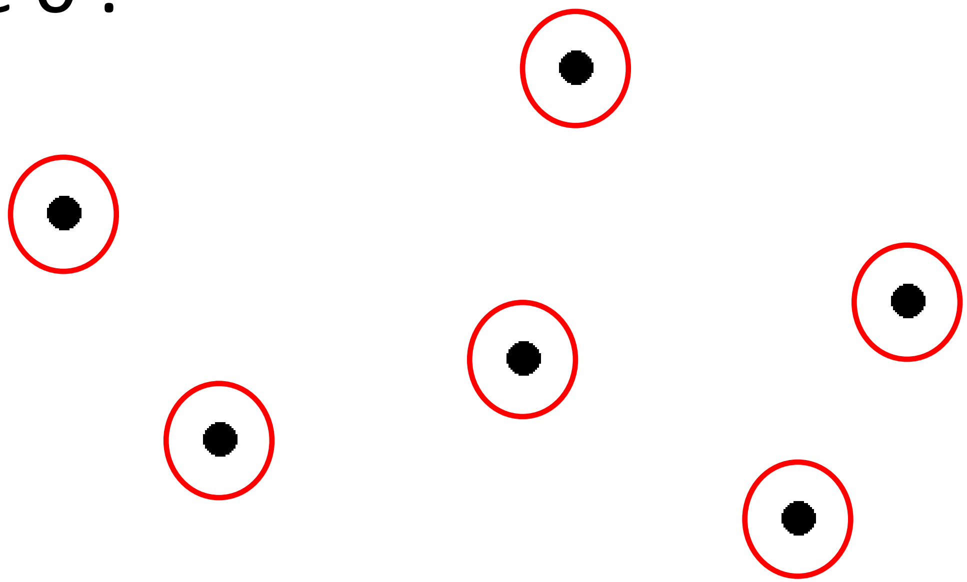
Build base forest

- Construct $(n/k$, O(k))- MST forest:



The different steps

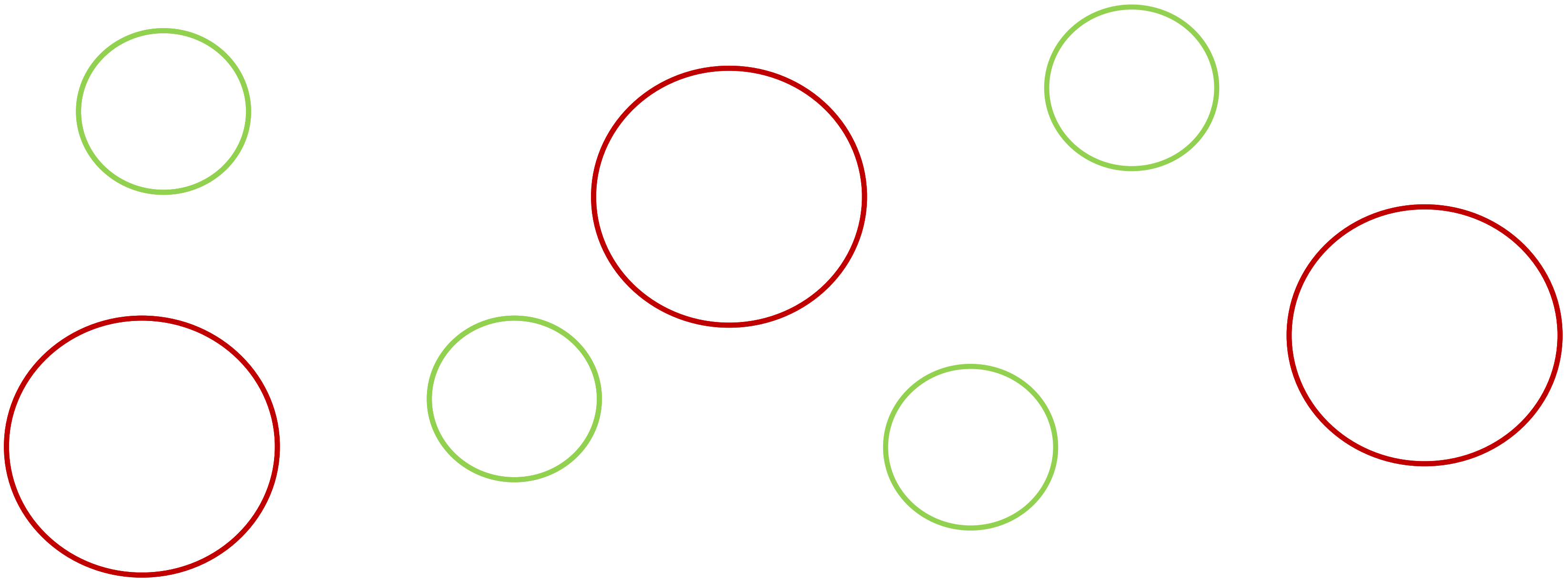
- t = $\log(k)$ phases
- Phase 0 :



- The phase i starts: $(n/2^{i-1}, 6 \cdot 2^{i})$-MST forest

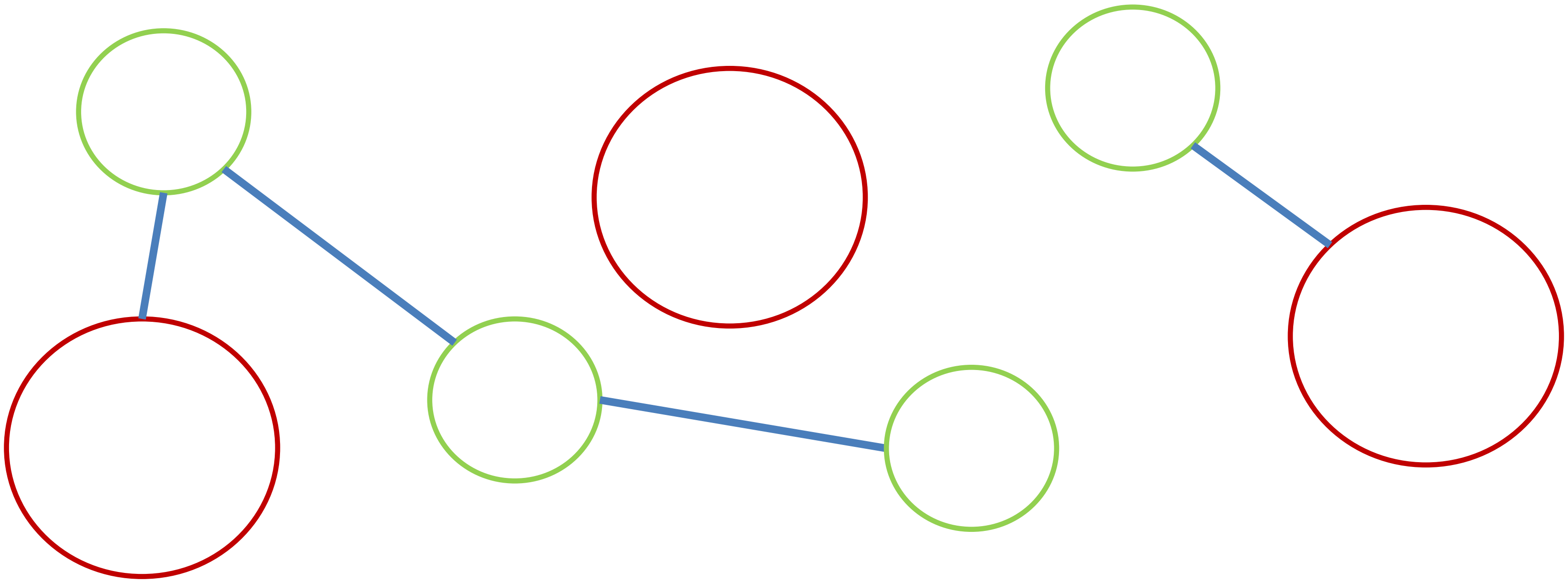
Phase i:

- For each fragment of diameter at most 2^i



Phase i:

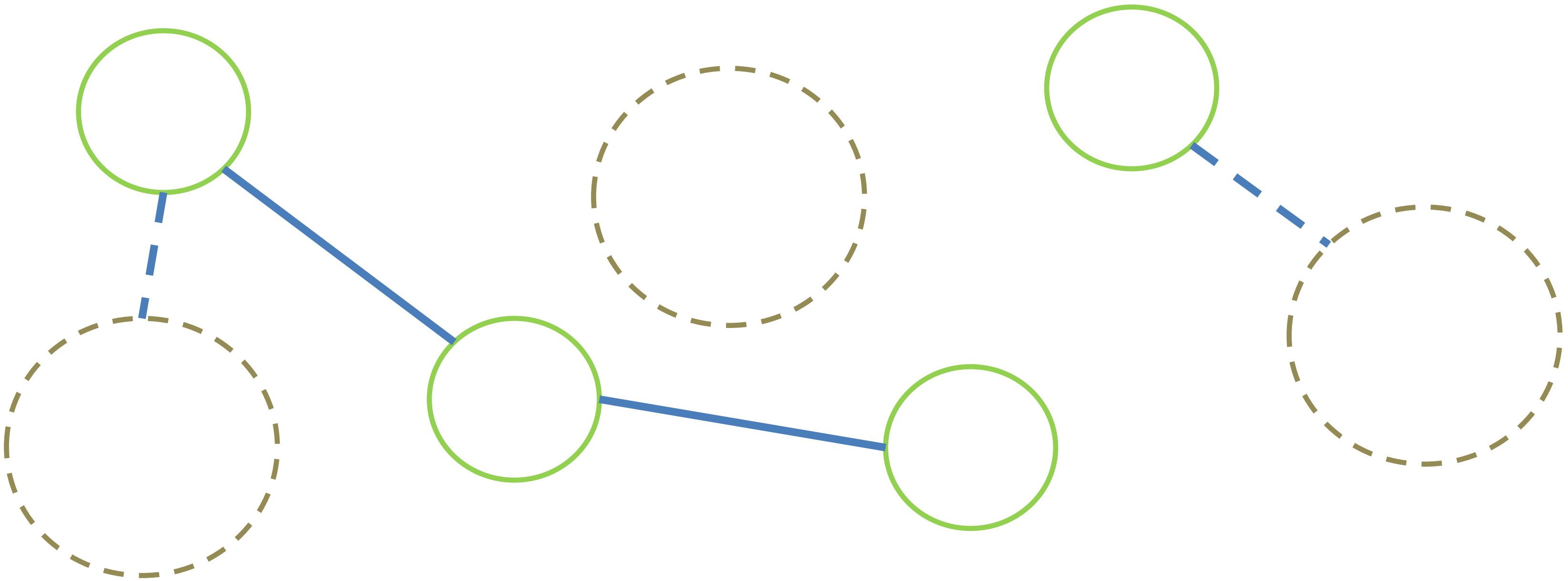
- For each fragment of diameter at most 2^i



- Time: $O(2^i)$ and Message: O(m)

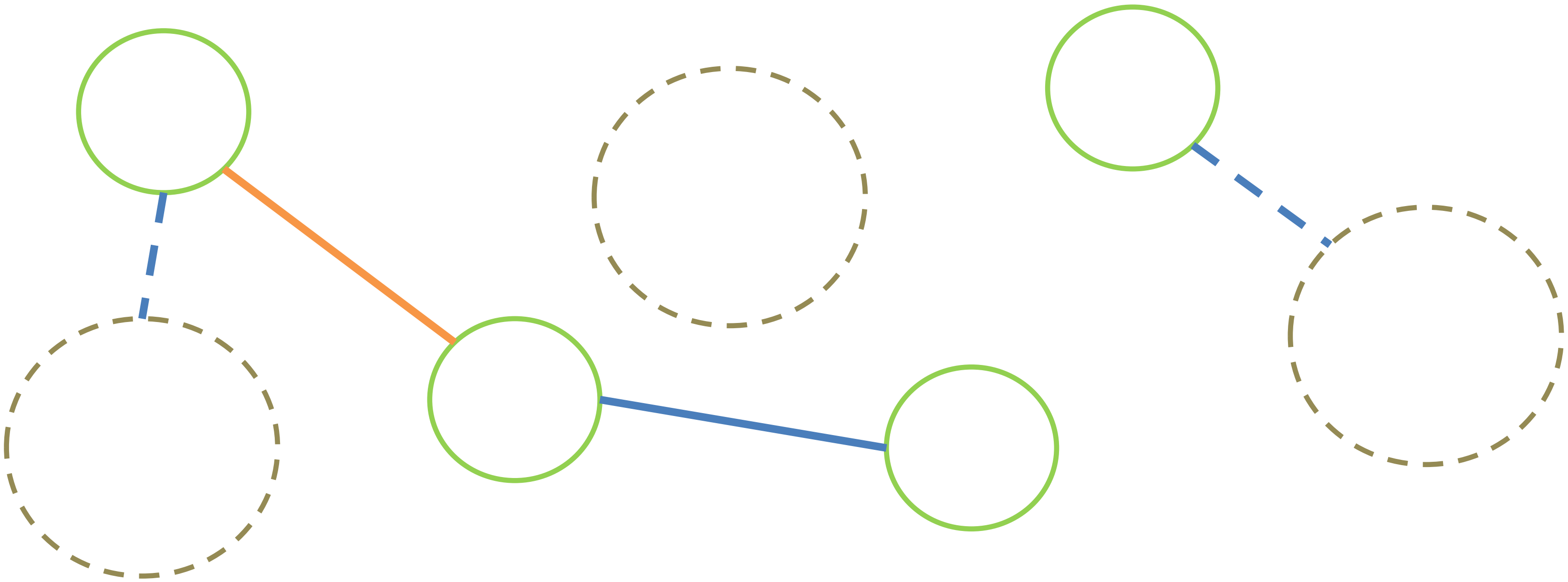
Phase i:

- Candidate fragment graph:



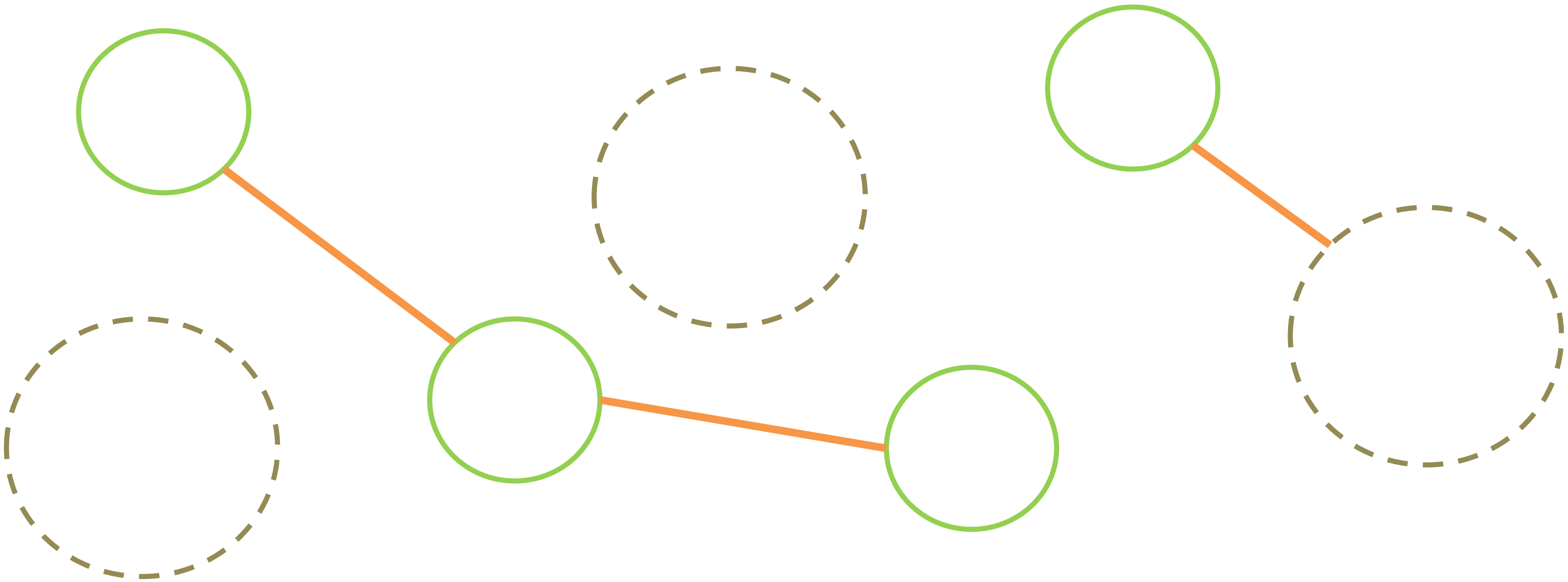
Phase i:

- Maximal Matching:



Phase i:

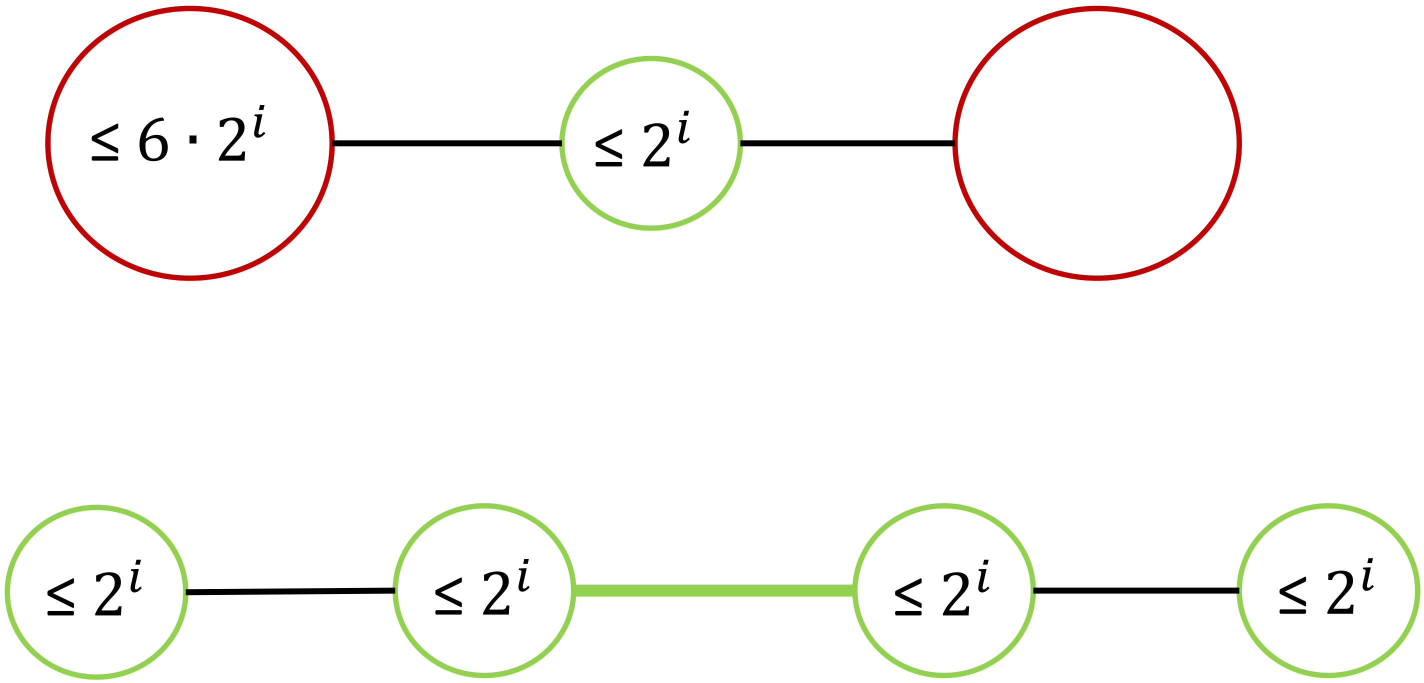
- Merge all green fragments:



- We have a $(n/2^i, 6 \cdot 2^{i+1})$ MST forest

Proof

Diameter of each fragment $\leq 6 \cdot 2^{i+1}$:



Algorithm

- Start with (n/k, O(k))- MST forest

  → Base forest with base fragments

- Step 1 : Construct a BFS tree with root rt

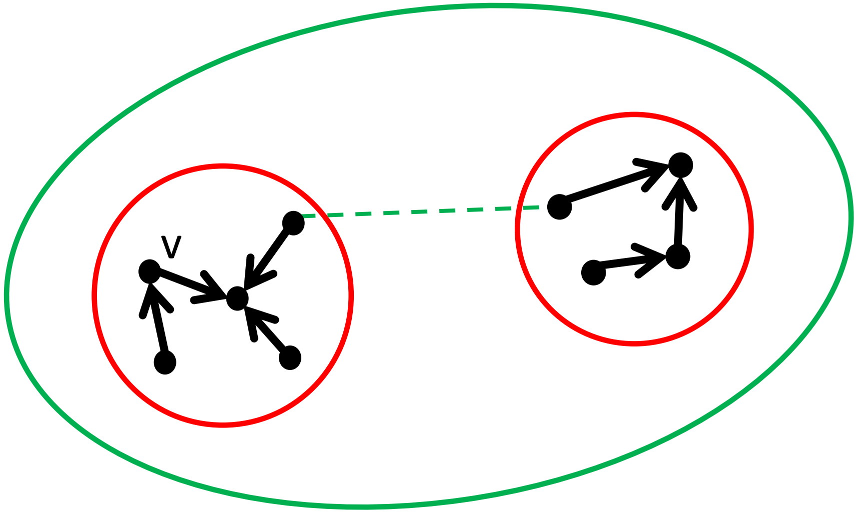
  → Time: O(D) and Message: O(m)

- j Phases already calculated

Phase j+1:

- Node v knows:

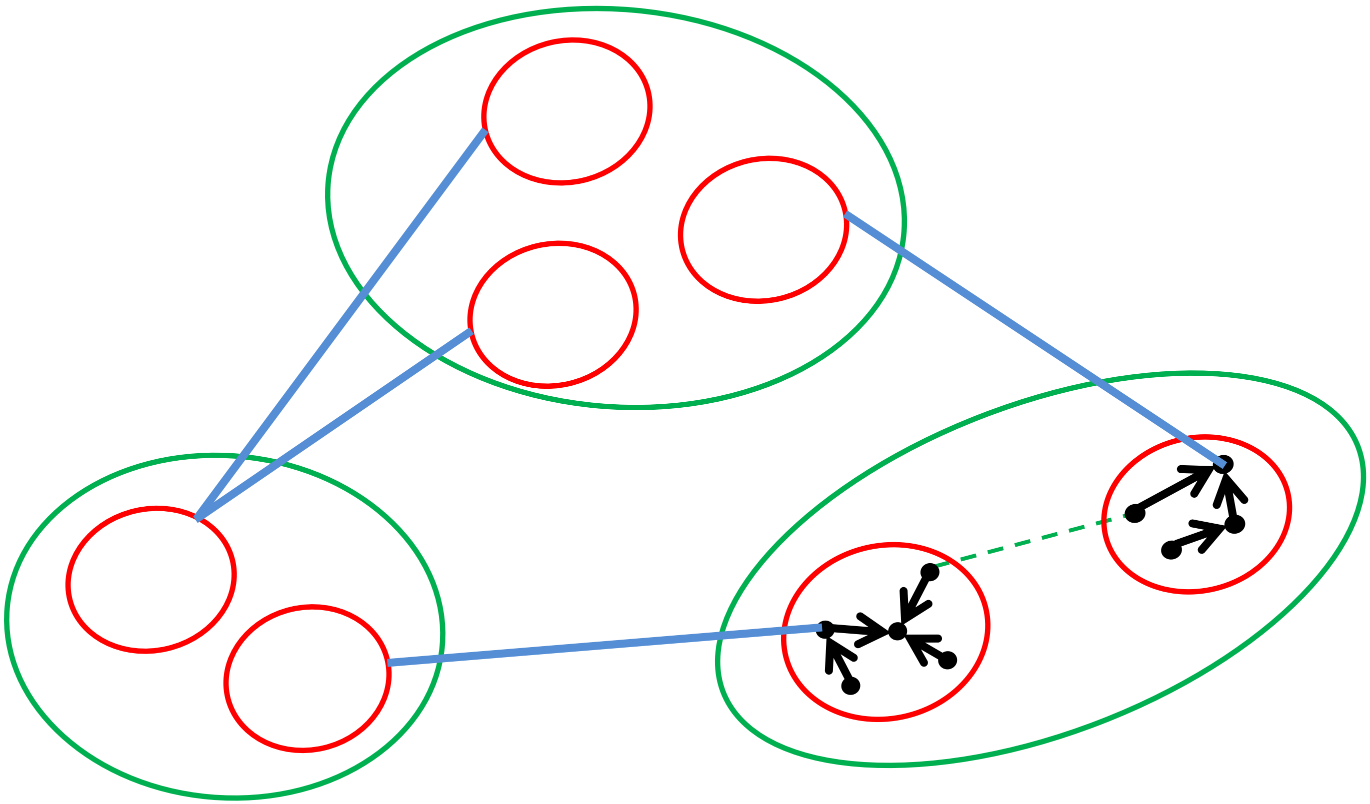
Base Fragments: $\mathrm{F}_{v} \in F_0$



Actual Fragment: $\mathrm{F}'_{v} \in F_j$

Phase j+1:

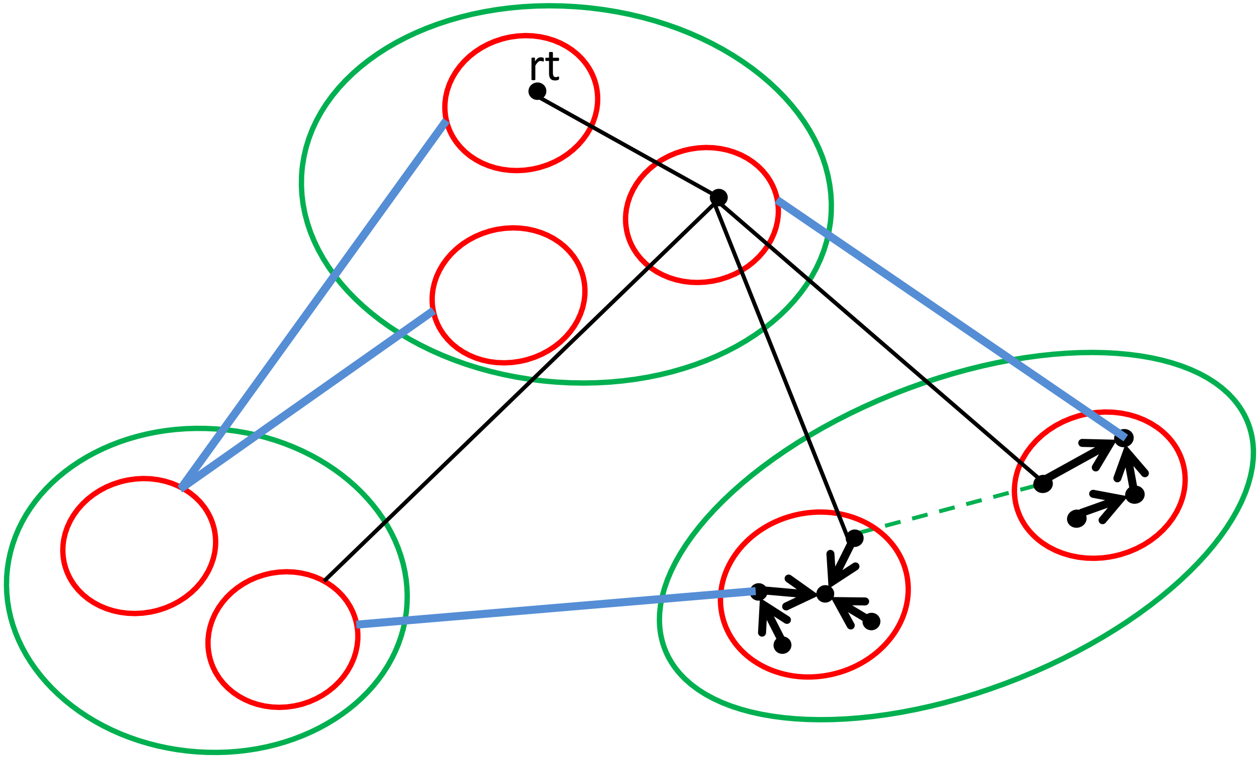
- Step 1: Search minimum weight outgoing edge



- Time: O(k)   Message: O(n)

Phase j+1:

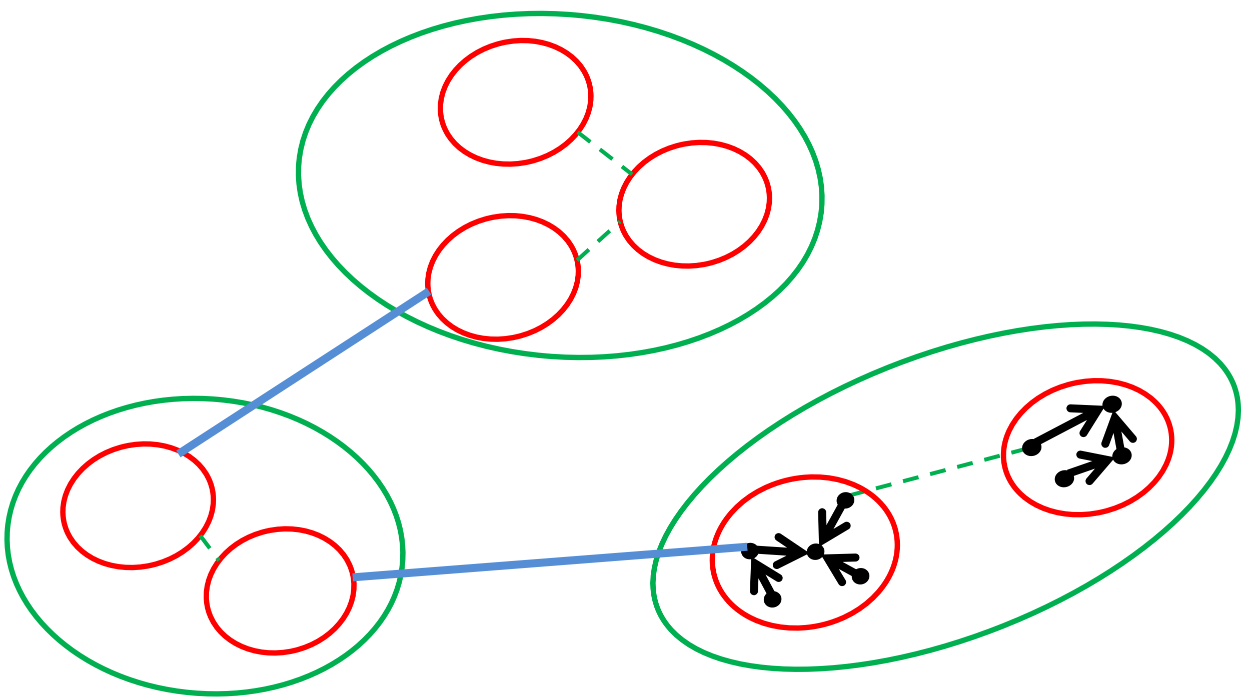
- Step 2: Send blue edge to the root of the BFS tree



- Time: $O(D + |F_j|)$ Message: $O(D * |F_j|)$

Phase j+1:

- Step 3: Root computes the blue edge e for each actual fragment

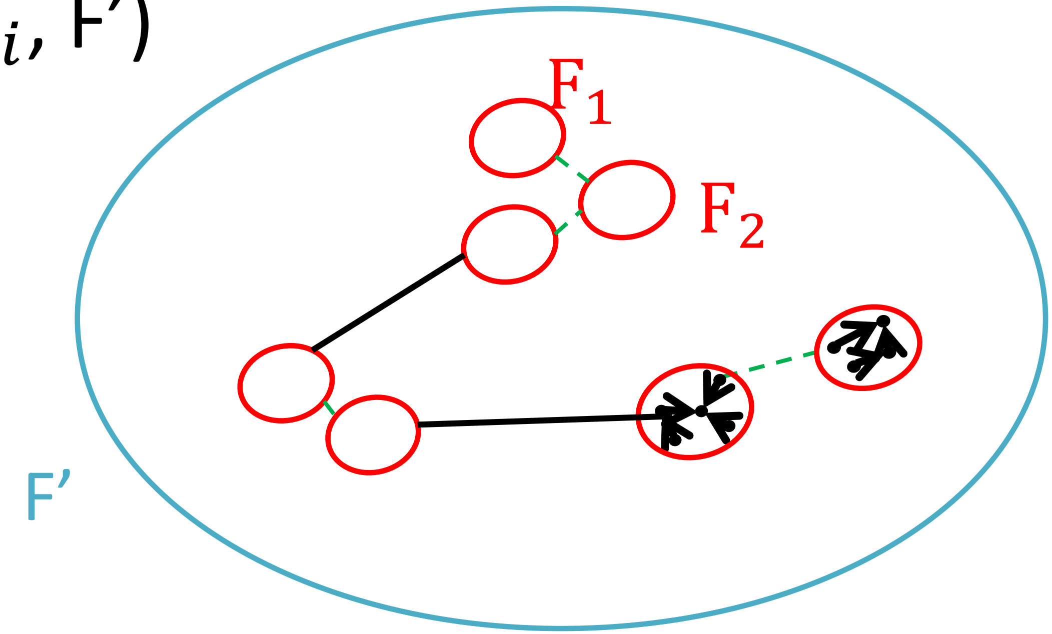


Phase j+1:

- Step 4: Inform all roots of the base fragments, which new actual fragment they include

→Send message (F_i, F’)

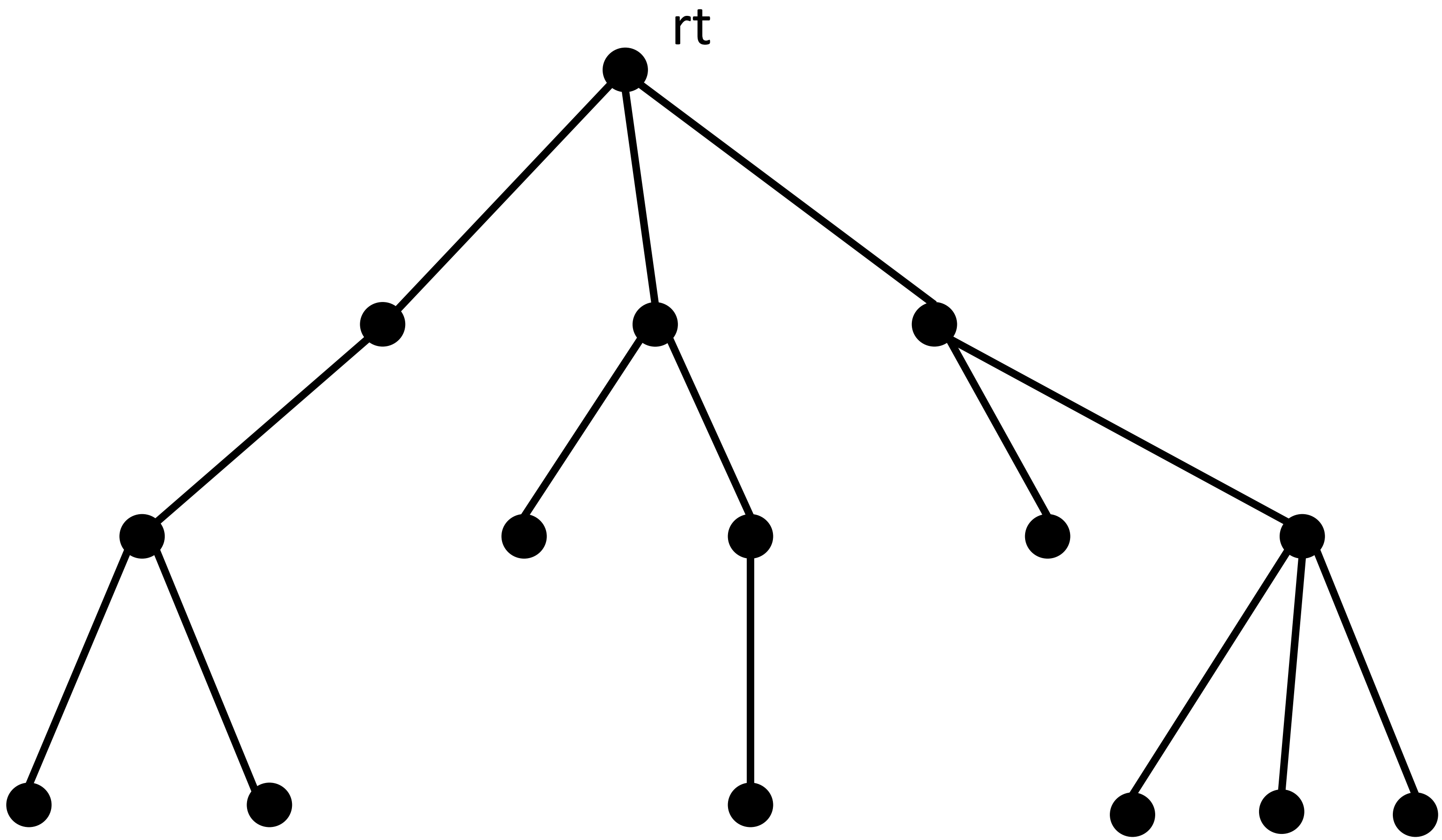
→Time: O(D+ $|F_0|$)

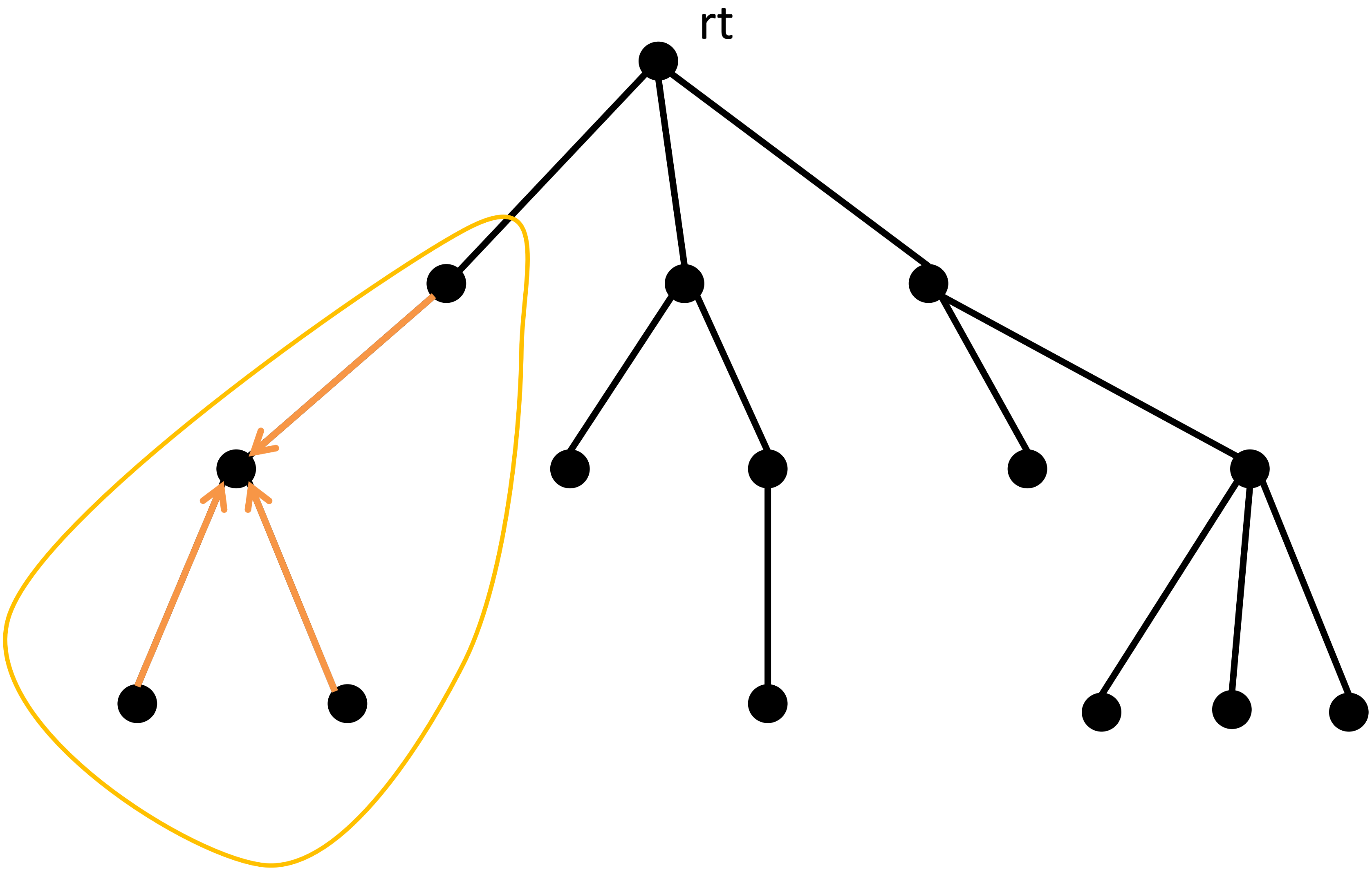


Phase j+1:

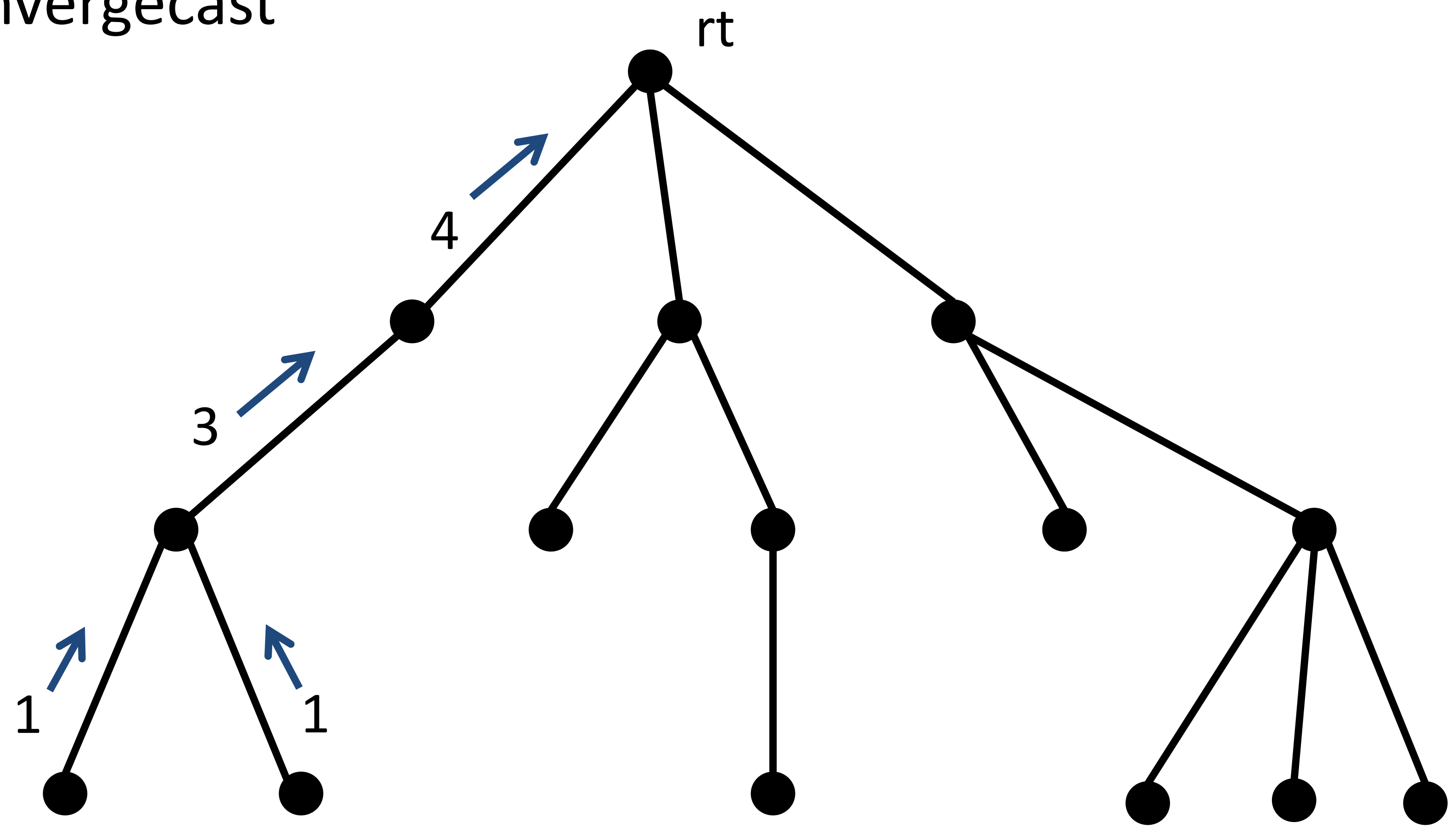
- Step 5: All base roots inform their nodes of the new actual fragment

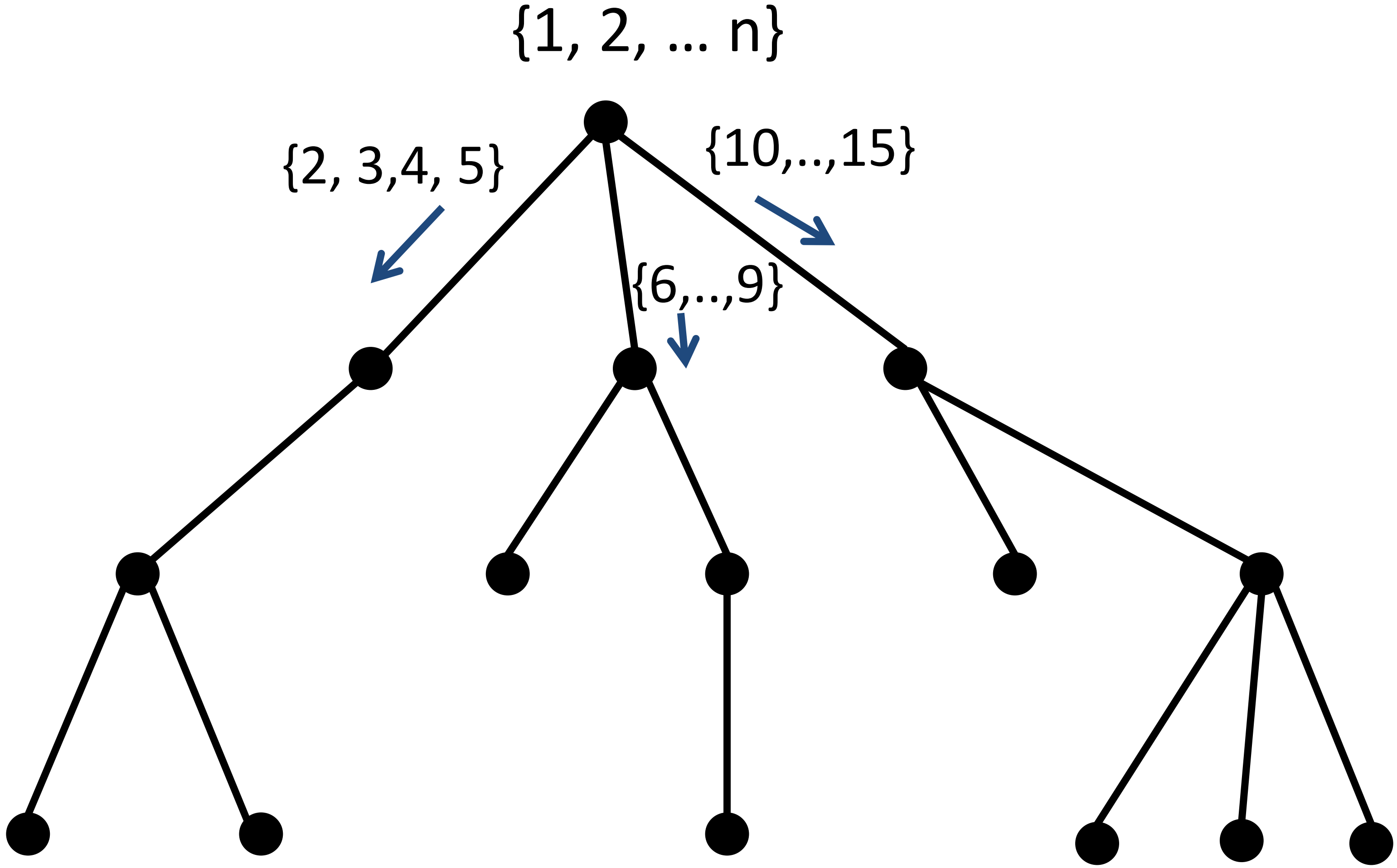
→Time: O(1)   Message: O(m)

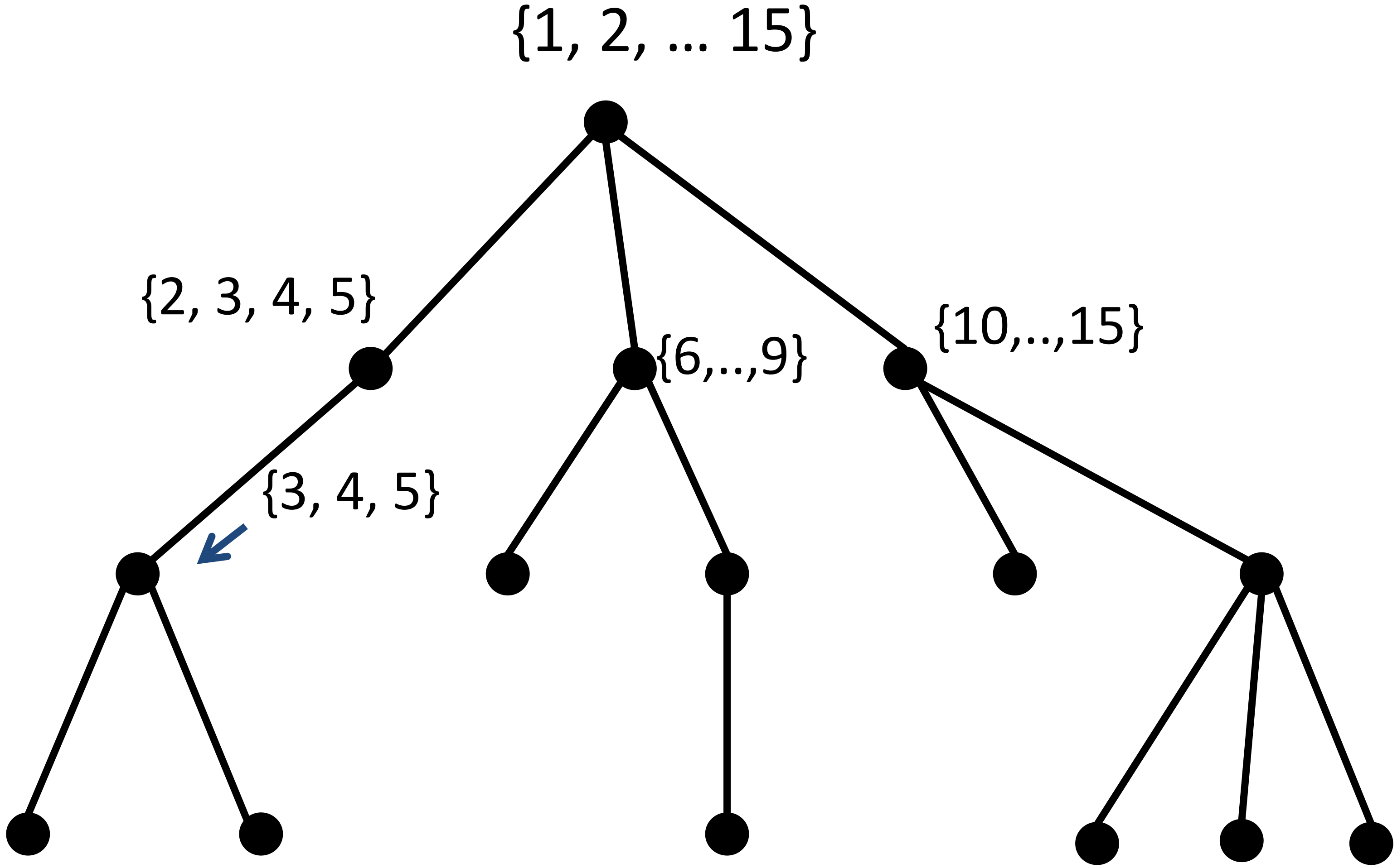

rt

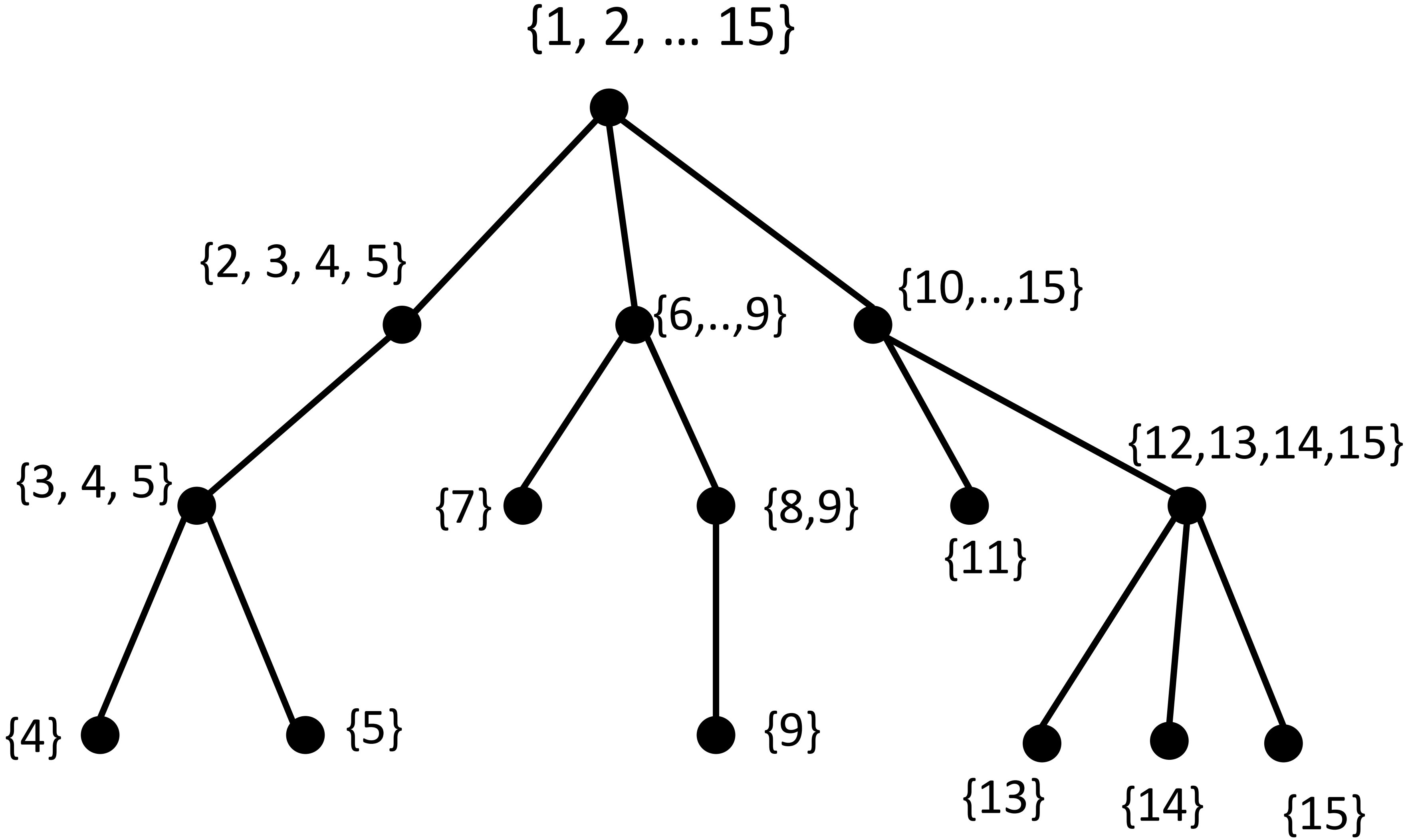

rt

Convergecast

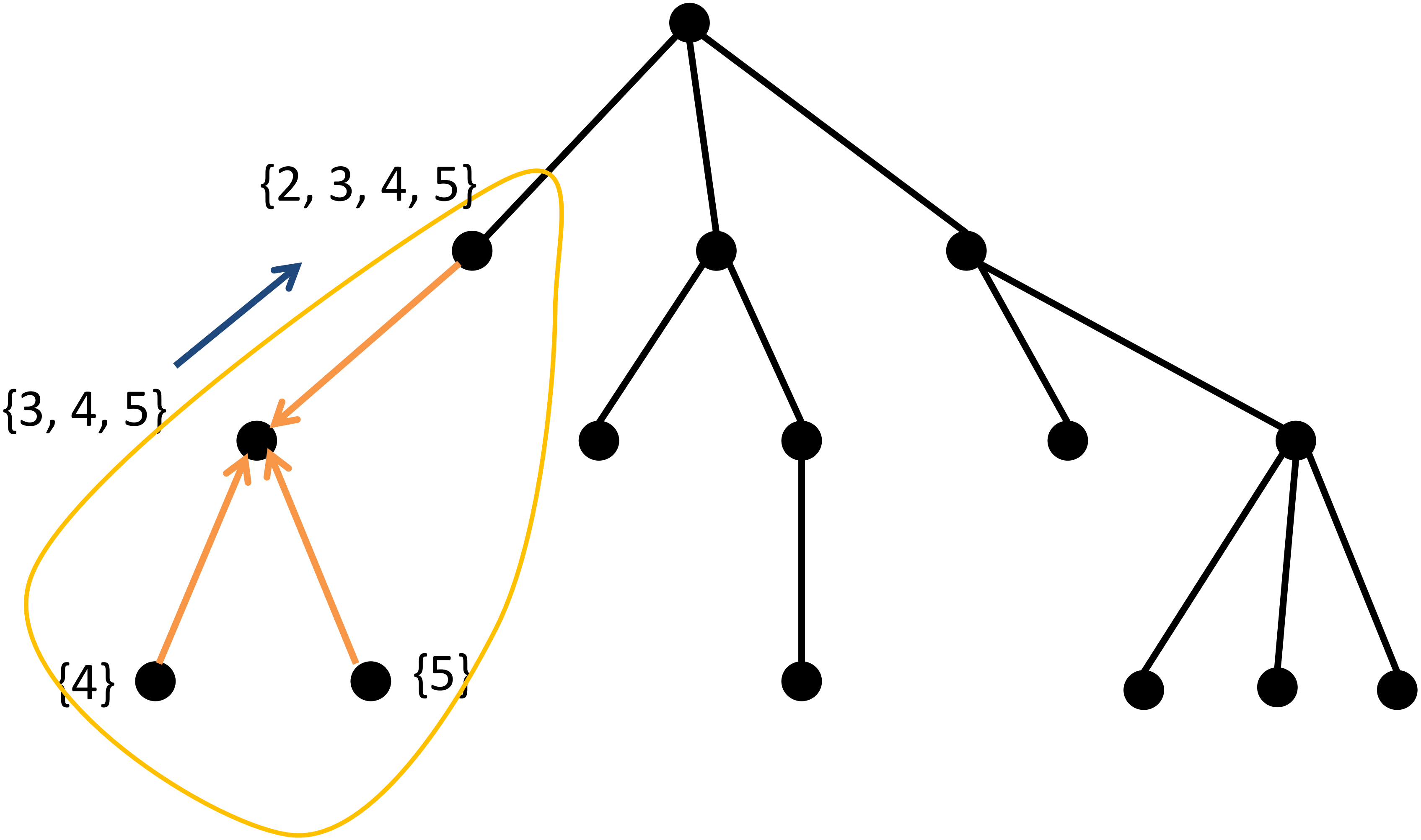



{1, 2, ... n}
{2, 3,4, 5}
{10,..,15}
{6,..,9}


{1, 2, ... 15}
{2, 3, 4, 5}
{6,..,9}
{10,..,15}
{3, 4, 5}


{1, 2, ... 15}
{2, 3, 4, 5}
{6,..,9}
{10,..,15}
{3, 4, 5}
{7}
{8,9}
{11}
{12,13,14,15}
{4}
{5}
{9}
{13}
{14}
{15}

Send intervals of the base fragment's root to the root of the tree



→ Time: $O(D + n/k)$

Time Complexity

- Compute blue edge: O(k)
- Upcast message: O(D+ $|F_j|$)
- Downcast: O(D+ $|F_0|$) = O(D+ n/k)

→ O($\log(n)$) phases

→ O($(\mathrm{D} + \sqrt{n}) \cdot \log(n)$)