Principles of Distributed Computing	03/18, 2020

Exercise 5

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1 Regularized Luby's MIS Algorithm

Consider a regularized variant of Luby's MIS algorithm, as follows: The algorithm consists of $\log \Delta + 1$ phases, each made of $O(\log n)$ consecutive rounds. Here Δ denotes the maximum degree in the graph. In each round of the i^{th} phase, each remaining node is marked with probability $\frac{2^i}{10\Delta}$. Different nodes are marked independently. Then marked nodes who do not have any marked neighbor are added to the MIS set, and removed from the graph along with their neighbors. If at any time, a node v becomes isolated and none of its neighbors remain, then v is also added to the MIS and is removed from the graph.

(a) Argue that the set of vertices added to the MIS is always an *independent set*.

(b) Prove that with high probability, by the end of the i^{th} phase, in the remaining graph each node has degree at most $\frac{\Delta}{2^i}$.

(c) Conclude that the set of vertices added to the MIS is a *maximal* independent set, with high probability.

(d*) Reprove item (b) assuming only pairwise independence between the marking events of different nodes, in the same round.

2 Randomized Coloring Algorithm

Consider the following simple randomized $\Delta + 1$ coloring algorithm: Per round, each node selects one of the colors not already *taken away* by its neighbors, at random. Then, if v selected a color and none of its neighbors selected the same color in that round, v gets colored with this color and takes this color away permanently. That is, none of the neighbors of v will select this color in any of the future rounds.

(a) Prove that in the first round, each node has at least a constant probability of being colored.

(b*) Prove that per round, each remaining node has at least a constant probability of being colored.

(b') If item (b) turns out to be complex, you may assume that we use $\lceil 1.02\Delta \rceil$ colors, instead of $\Delta + 1$. Prove that per round, each remaining node has at least a constant probability of being colored.

(c) Conclude that within $O(\log n)$ rounds, all nodes are colored, with high probability.