1 Maximal vs. Maximum Matching

Let $M$ be a maximal matching and $M^*$ be a maximum matching of a graph $G = (V, E)$. Prove the following two statements.

a) $\frac{|M^*|}{2} \leq |M| \leq |M^*|$.  

b) $|M| \geq \frac{|E|}{2\Delta - 1}$.  

2 Maximal Matching in Bipartite Graphs

Provide an algorithm that finds a maximal matching in a 2-colored bipartite graph in $2\Delta$ rounds.

3 Maximal Matching using Forest Decomposition

a) Recall the 3-coloring algorithm for rooted trees in $O(\log^* n)$ time. Show that the algorithm can be adapted to work in $O(\log^* q)$ time if a $q$-coloring of the tree is provided.

b) Provide an algorithm that decomposes a graph with maximum degree $\Delta$ into at most $\Delta$ many edge-disjoint forests in $O(1)$ time.

c) Suppose that each of the forests is 3-colored. Provide an algorithm that runs in $3\Delta$ rounds and finds a maximal matching in $G$.

d) Devise an algorithm that finds a maximal matching in $O(\Delta + \log^* q)$ time in a $q$-colored graph with maximum degree $\Delta$.

4 Rounding in Non-Bipartite Graphs

Provide an example of a 2-decomposition of a non-bipartite graph in which one rounding step (rounding by a factor 2) leads to a loss $> \frac{1}{\log^{\log^* \Delta}}$.  

Principles of Distributed Computing  
Exercise 12