ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing

FS 2025 Prof. R. Wattenhofer

Principles of Distributed Computing
Exercise 3

1 Leader Election in an "Almost Anonymous" Ring

a) Consider a ring of at least three nodes where the nodes cannot distinguish between their neighbors, i.e., when a node v receives a message, v does not know which neighbor has sent the message. All but one of the nodes have the same identifier. Is synchronous deterministic leader election possible in this ring? Either give an algorithm, or prove no algorithm exists.

b) Consider a directed ring where the nodes can receive messages from their clockwise neighbors and send messages to their counterclockwise neighbors. All but two of the nodes have the identifier 0; the remaining two nodes have the identifier 1. For each of the following, either give a synchronous deterministic leader election algorithm, or prove no such algorithm exists.

- The ring has an even number of nodes.
- The ring has an odd number of nodes.

2 Distributed Computation of the AND

Consider an anonymous ring where each node has a single bit input. The nodes can distinguish between their neighbors, i.e., when a node v receives a message, v knows which neighbor has sent the message (Note that the nodes may not know a consistent clockwise or counterclockwise orientation of the ring!).

a) Prove that there is no uniform synchronous deterministic algorithm to compute the AND of all input bits.

b) Present an asynchronous (non-uniform) algorithm for computing the AND. The algorithm should send $O(n^2)$ messages in the worst case. Furthermore, after deciding on an output, a node should be able to halt and not send any further messages.

c) Present a synchronous (non-uniform) algorithm for computing the AND. The algorithm should send $O(n)$ messages in the worst case and have the optimal round complexity.