1 Face Routing in Theory

We have seen in the lecture that face routing algorithms rely on planar graphs. The planarity of the graphs is crucial to guarantee that this kind of routing algorithms succeed whenever the source and destination are connected.

a) We first consider networks which are connected as perfect UDG, i.e. two nodes are connected iff their mutual distance is at most 1. Find a (small) network which is not planar and on which face routing algorithms fail if they do not planarize the network. Indicate the source and target node for which face routing fails.

We now consider the more realistic quasi unit disk graphs (QUDG) with parameter \( d \in [0, 1] \). In a QUDG, two nodes \( u \) and \( v \) are always connected if \( uv \leq d \). If \( uv > d \), the two nodes may be connected or not. Assume symmetric links, i.e. if node \( u \) sees node \( v \), then \( v \) sees \( u \) as well.

b) Show that the Gabriel Graph cannot be used directly to planarize a \( d \)-QUDG for \( d < 1 \).

c) Why is there no local algorithm to planarize a \( d \)-QUDG with \( d < \frac{1}{\sqrt{2}} \)?

d) For \( d \geq \frac{1}{\sqrt{2}} \) propose an algorithm to planarize a \( d \)-QUDG.

As a third type of network, we now consider spanning disk graphs (SDG). The connectivity of a SDG fulfills the following property: Whenever two nodes \( u \) and \( v \) are connected, they are connected to all nodes contained in the disk spanned by \( u \) and \( v \), i.e. the smallest disk that touches \( u \) and \( v \). Again, assume symmetric links.

e) Show that face routing algorithms succeed on the Gabriel Graph of any SDG.

2 Face Routing in Practice

a) The face routing algorithms can be implemented locally, i.e. all state of the algorithm can be stored in the message itself. What is the minimum information you need to store for the algorithm to work?

b) Assume that the network is partitioned, and the sender and destination node to be disconnected. How can the face routing algorithm detect this situation?

c) Consider the situation that you are asked to implement a face routing algorithm for an existing wireless network. Of course, real wireless networks are not exactly conform to UDG, QUDG or SDG. As a result, a pure face routing strategy may fail delivering messages. How would you improve your routing algorithm to account for realistic networks?