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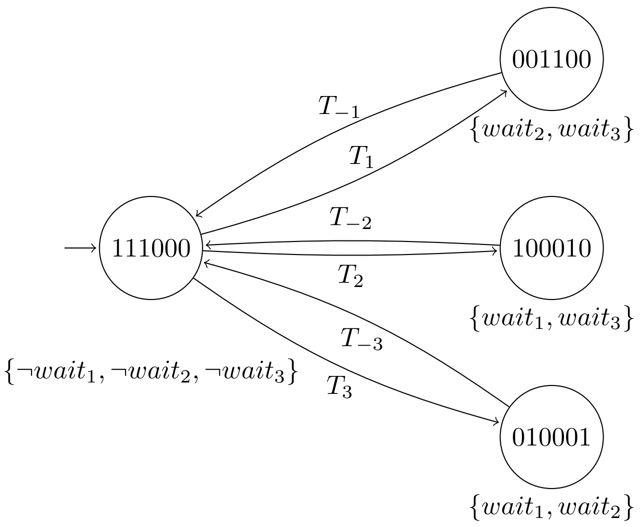
Discrete Event Systems
Sample Solution to Exercise 8

1 Safety Properties und Invarianten

1. A Petri-Net is deadlock-free if there is always at least one enabled transition in the Petri net. To show this, we can model the system as a labelled transition system (LTS) and then show that all states have an outgoing transition.

2. We have a deadlock if all philosophers are waiting for their forks. In our Petri Net, philosopher i waits when he is not eating(i.e., there is no token in eat_i) and there are not enough tokens to fire the transition $takeFork_i$. Let us denote this situation by the proposition $wait_i$. Hence the invariant we want to proof is $\Psi = \neg(wait_1 \wedge wait_2 \wedge wait_3)$.

3. We transform the Petri net into an LTS via state space exploration. We label a state by $d_1d_2d_3d_4d_5d_6$ where each d_i represents one place in the Petri net. The value of d_i shall indicate the number of tokens that are currently in the place corresponding to d_i. In particular, we represent by d_1-d_3 the places $Fork_1$-$Fork_3$ and by d_4-d_6 the places eat_1-eat_3. E.g., $s_0 = 111000$ represents the initial state where all $Fork$ places have one token. To find the next states in our LTS, we explore all possible combinations of transitions that can be fired from s_0. All transitions $takeFork_i$ are enabled, however, only one of them can fire. This gives three possible LTS transitions T_1-T_3 and leads to three new states $s_1 = 001100$, $s_2 = 100010$ and $s_3 = 010001$. T_i can be interpreted as philosopher i taking the two forks. From each of these states, there is only one possible transition. If we are in s_2 ,e.g., there is a token in eat_2 and $Fork_1$. Thus $putForkDown_2$ is the only enabled transition in our Petri net. If we fire this transition, we get to the initial state again. In fact, all states s_1-s_3 have an LTS transition back to s_0. You can interpret a transition T_{-i} as philosopher i putting down his forks.

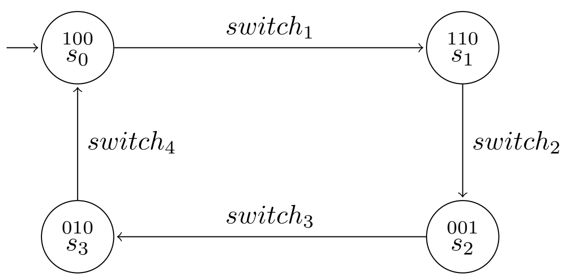


To see that our model is deadlock-free, we just have to realize, that each state in the constructed LTS has at least one outgoing transition. To proof the invariant Ψ, we can translate all states in the LTS to propositions. In the initial state s_0, no philosopher is waiting. In states s_1-s_3, one philosopher eats and the two others are waiting. In any state, Ψ is true. The model is deadlock-free.

2 LTL-Properties

1. Please note that there was one inhibitor arc from state *yellow* to the transition $switch_1$ missing in the exercise sheet. This prevents $switch_1$ from firing when there are tokens in *yellow*.

   - *Transform Petri net into LTS.* We again do a state exploration starting from the initial state where where there is one token in *red*. We denote the LTS states by $d_1 d_2 d_3$ where d_1 corresponds to the number of tokens in *red*, d_2 to the tokens in *yellow* and d_3 to the tokens in *green*.



     The initial state s_0 is 100 and there is one transition, $switch_1$ enabled. We fire $switch_1$ and get one token in *red* and one token in *yellow*, thus $s_1 = 110$. Now $switch_2$ is the only transition enabled. We fire it and get one token in *green*, $s_2 = 001$. Firing $switch_3$ yields one token in *yellow*, $s_3 = 010$. Firing $switch_4$ gets us back to the initial state s_0.

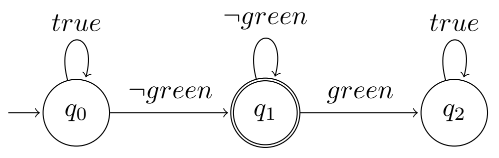
   - *Negate property.* The LTL formulas we want to check are $\Box\Diamond green$ (read:“always eventually green”) meaning that at any point in time, we can be sure that the lights will turn green, and $\Diamond\Box green$ (read “eventually always green”) meaning that after some initial time period, the light will always stay green. We build the inverse using the LTL equivalence rules:

$$\neg\varphi_1 = \neg(\Box\Diamond green) = \Diamond\Box\neg green$$

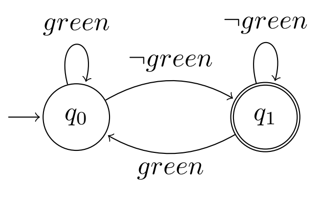
$$\neg\varphi_2 = \neg(\Diamond\Box green) = \Box\Diamond\neg green$$

   - *Construct NBA.* We construct the non-deterministic Buechi automata (NBA) for $\neg\varphi_1$ and $\neg\varphi_2$.

NBA of $\neg\varphi_1$:



NBA of $\neg\varphi_2$:



   - *Construct product automaton.* We construct the Cartesian product of the LTS and the NBA.

     In Figure 1, the automaton on the left depicts the Cartesian product for $\neg\varphi_1$ and the automaton on the right the one for $\neg\varphi_2$. On the vertical axis we have the states of the LTS and on the horizontal we have the states of the NBA. Notice that not all power states are reachable. We construct the product by combinig the proposition that holds in the current LTS state with the possible transitions in the NBA. E.g., in the initial state (s_0, q_0), $\neg green$ holds thus we can take the NBA transitions $\neg green$ and *true*

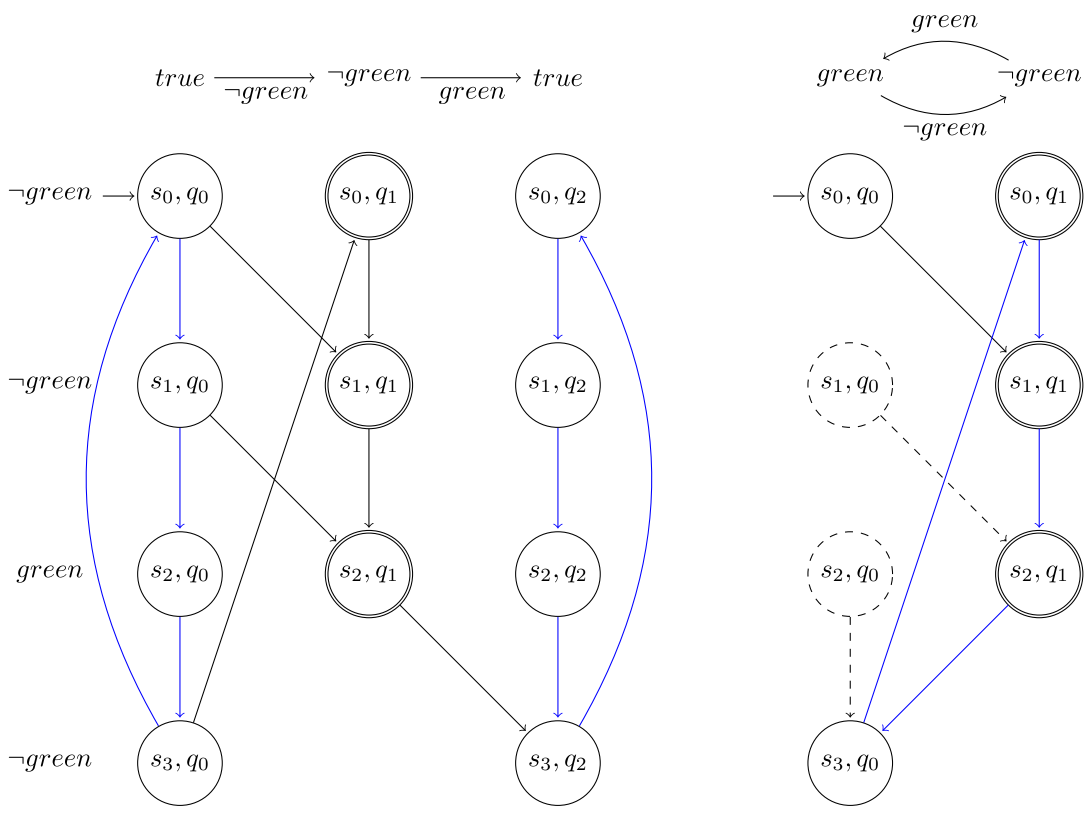


Figure 1: Cartesian product

(since *true* is always true ☺). In the Cartesian product, this results in a transition from state (s_0, q_0) to (s_1, q_1) and to (s_1, q_0) respectively.

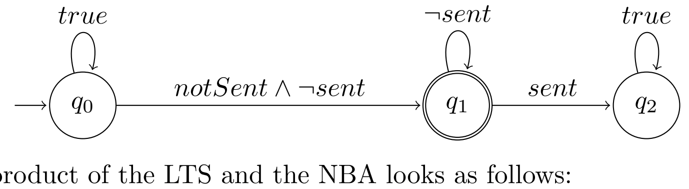
Once we have the Cartesian product automaton, we check whether there exists a cycle which contains an accepting state. If this is the case, we can conclude that the LTL-formula we are checking does not hold. If no cycle contains an accepting state, we may derive that the LTL-formula holds. In the Cartesian product for the formula $\neg\varphi_1$, there are two cycles (colored blue in the illustration). Both do not contain any accepting state, hence we know that φ_1 holds. In the Cartesian product for the formula $\neg\varphi_2$, there is one cycle containing accepting states, hence we know that φ_2 is false.

2. In order to check the LTL-formula Φ, we have to first compute the inverse of Φ.

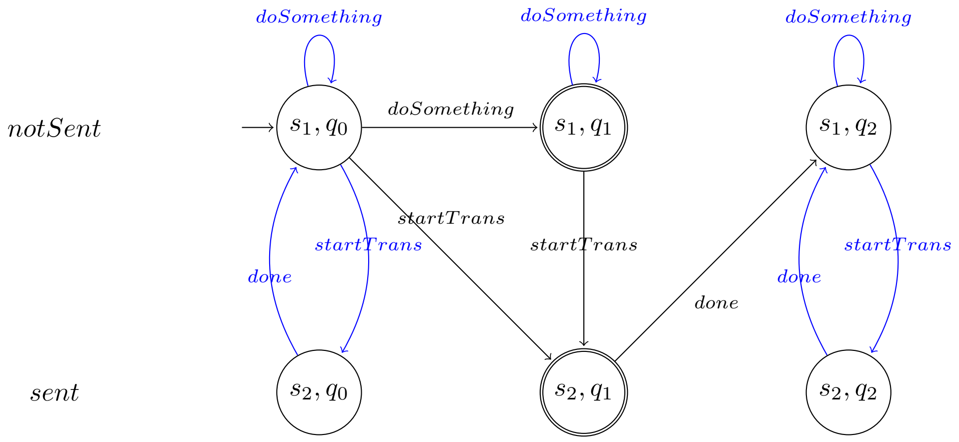
$$\begin{aligned}
\neg\Phi &= \neg\Box(notSent \Rightarrow \Diamond sent) \\
&= \Diamond\neg(notSent \Rightarrow \Diamond sent) \\
&= \Diamond(notSent \wedge \neg\Diamond sent) \\
&= \Diamond(notSent \wedge \Box\neg sent)
\end{aligned}$$

For the third equality, we used the logic equivalence rule $\neg(a \Rightarrow b) = a \wedge \neg b$. Informally, $\neg\Phi$ means that at some point in time, both *notSent* and $\neg$*sent* hold and after that, *notSent* may become false again, but *sent* must stay false forever. We now represent $\neg\Phi$ by an NBA.

NBA of $\neg\Phi$:



The Cartesian product of the LTS and the NBA looks as follows:



The product contains one cycle with an accepting state, namely the self loop of (s_1, q_1). This provides us with an infinite run that fulfills $\neg\Phi$. Hence we have found a counter-example which proves that Φ does not hold. In the LTS, starting in s_1 and taking the transition *doSomething* infinitely often, we will never establish *sent*. This disproves Φ which says that it is always the case that when “a message” is not sent, it will sooner or later be sent.

3 Taskaktivierungsmuster und Timed Automata

See *LoesungAufgabe3.pdf*

4 Scheduling und Timed Automata

See *UppaalModel_Aufg4.xml*