

A Cluster-based Multiuser Cooperative Network

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Abstract—A novel cluster-based coherent multi-user (MU) relaying system is proposed, where a number of source/destination pairs communicate concurrently over the same physical channel and a set of amplify-and-forward relays assist them. A secondary short-range wireless system with single/multi-hop unicast or broadcast ability is considered for the channel state information (CSI) dissemination. To reduce the CSI dissemination overhead, the relays are grouped in several independent clusters. Distributed diversity gain is attained at the destinations by applying phase rotations between clusters. The CSI dissemination is analyzed thoroughly, and compared with that of previous works on MU relaying. It is shown that for a given number of clusters and relays, the most uniform distribution of relays to clusters is optimum to minimize the CSI exchange traffic overhead and transmit energy consumption for CSI dissemination; besides it maximizes the average achievable rate for source/destination links. Furthermore, a new solution for a maxmin fairness based relay gain allocation is proposed and formulated as a semidefinite program.

I. INTRODUCTION

Cooperative relaying schemes have been proposed as a promising approach to supersede the conventional multiple-input multiple-output systems to achieve diversity and spatial multiplexing gains. In this work we focus on amplify-and-forward (AF) relays assisted two-hop half-duplex transmission between source/destination (S/D) pairs. In the first phase the sources transmit data to the relays and in the second phase, the relays transmit the scaled and/or rotated data to the destinations. In order to achieve a spatial multiplexing gain, independent data streams are transmitted concurrently in the same frequency band. In [1], the conversion of spatial diversity to temporal diversity by relay phase rotations is proposed. A system with N_r AF relays is shown to achieve the maximum distributed diversity gain of N_r by using time-variant phase-rotations at the relays [2].

A MU relaying scheme for the case of infinite number of relays is proposed in [3]. For finite number of single-antenna relays, a scheme which is based on zero-forcing (ZF) by performing a nullspace projection is presented in [4]. Multiple AF relays assist the communication between multiple single-antenna S/D pairs. Choosing the relay gain factors accordingly, the S/D links are orthogonalized which removes the interference between different S/D pairs. A minimum number of relays is needed to achieve this. When there are more relays, the gain factors can be further optimized [5].

The aforementioned multi-user relaying systems require the so called global channel state information (GCSI) at each relay to determine the relay gain factors, which means that each relay knows not only its local CSI (LCSI), but also

LCSIs of all other relays in the system. The dissemination of these LCSIs can be performed by a secondary short-range low-power wireless technology. Depending on the system features, the LCSIs can be disseminated by unicast or broadcast communication between relays. It has been shown in [6] that unicast communications can be preferable for certain conditions. On the other hand, considering a dense network, the number of required LCSI exchanges (and accordingly the energy consumption) increases polynomially with the number of nodes in the system. Moreover, the topological conditions may prevent to exchange LCSI in such a dense network.

In this paper, we propose a novel cluster based multi-user relaying (C-MUR) system, where the relays are grouped in independent clusters. Applying phase rotations between clusters, distributed diversity gain is attained at the destinations. As opposed to the previous works on multi-user ZF relaying (MUZFR), a new solution for relay gain allocation is proposed which does not require nullspace projection. Considering the fairness between different links, a maxmin link-rate optimization problem is formulated as a semidefinite program (SDP). Furthermore, a LCSI dissemination analysis in terms of traffic overhead and transmit energy consumption is performed and compared to that of MUZFR with GCSI. It is shown that for a given number of clusters and relays, the most uniform distribution of relays to clusters is optimum to minimize the LCSI exchange overhead and energy consumption. Moreover it maximizes the average link rates.

Notation: The superscripts $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ stand for conjugate complex, matrix transpose and conjugate complex transpose, respectively. The operators \odot , $E_{\{x\}}[\cdot]$, $\text{diag}\{\mathbf{x}\}$, $\text{tr}\{\mathbf{X}\}$ denote the element-wise product, expectation with respect to x , a diagonal matrix with \mathbf{x} on its diagonal, and trace of \mathbf{X} , respectively. $x \sim \mathcal{CN}(0, \sigma^2)$ stands for a zero-mean complex normal circularly symmetric distribution with variance σ^2 . The operation $\text{mod}(a, b)$ stands for the modulo operation that finds the remainder of division of a by b .

II. SYSTEM MODEL

We consider a wireless network with $2N + N_r$ single-antenna nodes, where N S/D pairs communicate concurrently on the same physical channel, and N_r amplify-and-forward relay nodes assist the communication in a half-duplex scheme. The relays are grouped together in C clusters each with n_k relays such that $\sum_{k=1}^C n_k = N_r$. Each cluster is assumed to be unaware and independent of the others. It is also assumed that there are no direct links between sources and destinations due to shadowing effects or topological conditions. Fig. 1 summarizes the network configuration.

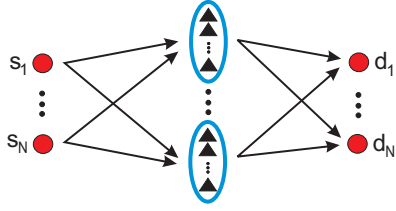


Fig. 1. Cluster based two-hop network configuration with half-duplex relays.

The communication follows a two-hop relay traffic pattern. In the first hop, each source transmits T symbols consecutively. The $n_k \times T$ dimensional received signal block at the k th cluster over T time slots is $\mathbf{R}_k = \mathbf{H}_{\text{sr}}^{(k)}\mathbf{S} + \mathbf{W}_k$, where $\mathbf{R}_k = [\mathbf{r}_k[1] \cdots \mathbf{r}_k[T]]$, $\mathbf{r}_k[i] = [r_{k,1}[i] \cdots r_{k,n_k}[i]]^T$, $r_{k,j}[i]$ is the i th received symbol of the j th relay in the k th cluster; $\mathbf{S} = [\mathbf{s}[1] \cdots \mathbf{s}[T]]$, $\mathbf{s}[i] = [s_1[i] \cdots s_N[i]]^T$, $s_m[i]$ is the i th symbol of the m th source; $\mathbf{H}_{\text{sr}}^{(k)} \in \mathbb{C}^{n_k \times N}$ is the uplink channel matrix from the sources to the k th cluster, and \mathbf{W}_k is the corresponding relay noise matrix with i.i.d. $\mathcal{CN}(0, \sigma_w^2)$ entries.

Before the relays transmit their received symbols through the downlink channel $\mathbf{H}_{\text{rd}}^{(k)} \in \mathbb{C}^{N \times n_k}$ to the destinations, the signals are multiplied with the gain factors $g_{k,j}$ s to obtain $\mathbf{Y}_k = \mathbf{G}_k \mathbf{R}_k$, where $\mathbf{G}_k = \text{diag}\{[g_{k,1} \cdots g_{k,n_k}]\}$. The calculation of the gain factors is explained in Section III. They depend on the uplink and downlink channel matrices. The channel matrices $\mathbf{H}_{\text{sr}}^{(k)}$ and $\mathbf{H}_{\text{rd}}^{(k)}$ have i.i.d. $\mathcal{CN}(0, 1)$ entries and stay constant during at least $2T$ symbol transmission cycles, where we assume block fading channels. Moreover, we assume that each relay knows the CSI of its uplink/downlink channels as well as the other relays' that belong to its own cluster, i.e., $\mathbf{H}_{\text{sr}}^{(k)}$, $\mathbf{H}_{\text{rd}}^{(k)}$ are known to all relays in the k th cluster. We call this local cluster CSI knowledge. Note that since the channel coefficients are constant over $2T$ symbol times, the calculated \mathbf{G}_k stays the same during this period.

If \mathbf{Y}_k is transmitted to the destinations without any further signal processing, the compound channel coefficients from all clusters would add up to a single coefficient which stays constant during T symbol periods. This prevents any temporal diversity. In [1], time variant phase offsets are introduced at the relays to create an artificial time-variant channel, which is then utilized to achieve diversity by an outer code. Here, we use this concept not between the relays but between the clusters. The spatial diversity is transformed to temporal diversity by time variant and cluster specific phase variations, i.e., the same phase variation is used for all relays in the same cluster. Thus, before transmission, \mathbf{Y}_k is multiplied with time/cluster specific $p_{k,i}$ s such that $\mathbf{P}_k \odot \mathbf{Y}_k$ is transmitted to the destinations, where $\mathbf{P}_k = [[p_{k,1} \cdots p_{k,T}]^T \cdots [p_{k,1} \cdots p_{k,T}]^T]^T \in \mathbb{C}^{n_k \times T}$.

The sum transmit power per symbol of all relays in the k th cluster is equal to P_k . It is assumed that each cluster transmits with the same power $P_k = \sigma_s^2 N / C$, where σ_s^2 is the average power of the source transmit symbols. We use this constraint to keep the total power consumption of the network constant when adding relays to clusters.

The destinations receive a superposition of all signals from

C clusters. Hence, the received signal block over T time slots is represented as

$$\mathbf{D} = \sum_{k=1}^C \left(\mathbf{H}_{\text{rd}}^{(k)} (\mathbf{P}_k \odot (\mathbf{G}_k \mathbf{H}_{\text{sr}}^{(k)} \mathbf{S})) + \mathbf{H}_{\text{rd}}^{(k)} (\mathbf{P}_k \odot (\mathbf{G}_k \mathbf{W}_k)) \right) + \mathbf{N},$$

where $\mathbf{D} = [\mathbf{d}[1] \cdots \mathbf{d}[T]]$, $\mathbf{d}[i] = [d_1[i] \cdots d_N[i]]^T$, $d_m[i]$ is the i th received symbol of the m th destination, and \mathbf{N} is the corresponding noise matrix at destinations with i.i.d. $\mathcal{CN}(0, \sigma_n^2)$ entries. In the sequel, it is assumed that $\sigma_w^2 = \sigma_n^2$. The sources do not have CSI knowledge, whereas the destinations know the compound CSI, i.e., $\mathbf{H}_{\text{rd}}^{(k)} \mathbf{G}_k \mathbf{H}_{\text{sr}}^{(k)} \forall k$. There is no symbol exchange between the relays or destinations.

In order to achieve the maximum available diversity we choose $T = C$. A simple choice for the compound phase rotation matrix $\mathbf{P} = [[p_{1,1} \cdots p_{1,C}]^T \cdots [p_{C,1} \cdots p_{C,C}]^T] \in \mathbb{C}^{C \times C}$ is an identity matrix [1]. Choosing \mathbf{P} as an identity matrix, the columns of \mathbf{D} simplify to

$$\mathbf{d}[i] = \mathbf{H}_{\text{rd}}^{(i)} \mathbf{G}_i \mathbf{H}_{\text{sr}}^{(i)} \mathbf{s}[i] + \mathbf{H}_{\text{rd}}^{(i)} \mathbf{G}_i \mathbf{w}_i[i] + \mathbf{n}[i] \text{ for } i = 1, \dots, C.$$

With the chosen \mathbf{P} each time slot is assigned to a single cluster. Thus, the total transmit power of this single cluster becomes $P_k = \sigma_s^2 N$. Since the i th symbol interval is assigned to the k th cluster, in the sequel we use the indices i and k interchangeably. Finally, transmitting over C time slots i.i.d. symbols $s_m[i] \sim \mathcal{CN}(0, \sigma_s^2)$, the mutual information between the m th S/D pair for this artificial time-selective channel is $I_m = \frac{1}{2C} \sum_{i=1}^C \log_2(1 + \text{SNR}_m^{(i)})$, where $\text{SNR}_m^{(i)}$ is the signal-to-noise ratio of the m th link induced by the i th cluster in the i th symbol interval.

III. RELAY GAIN ALLOCATION PER CLUSTER

In the proposed cluster-based MU relaying system, each cluster is independent and unaware of the others. Thus, each cluster per se tries to optimize its gain vector such that the minimum link rate is maximized from its perspective.

Let $\mathbf{h}_{\text{sr}}^{k,l}$ and $\mathbf{h}_{\text{rd}}^{k,l}$ be the l th column vector of $\mathbf{H}_{\text{sr}}^{(k)}$ and the transpose of the l th row vector of $\mathbf{H}_{\text{rd}}^{(k)}$, respectively. Considering the k th cluster, the instantaneous rate of the m th S/D link is $R_m^{(k)} = \frac{1}{2} \log_2(1 + \text{SINR}_m^{(k)})$, and the instantaneous signal-to-interference plus noise ratio $\text{SINR}_m^{(k)}$ from the perspective of the k th cluster is given by

$$\text{SINR}_m^{(k)} = \frac{\sigma_s^2 \mathbf{g}_k^H \mathbf{H}_{m,m}^{(k)} \mathbf{g}_k}{\sigma_s^2 \sum_{n \neq m} \mathbf{g}_k^H \mathbf{H}_{m,n}^{(k)} \mathbf{g}_k + \sigma_n^2 (1 + \mathbf{g}_k^H \mathbf{M}_m^{(k)} \mathbf{g}_k)},$$

where $\mathbf{H}_{m,n}^{(k)} = (\mathbf{h}_{\text{rd}}^{k,m} \odot \mathbf{h}_{\text{sr}}^{k,n})^* (\mathbf{h}_{\text{rd}}^{k,m} \odot \mathbf{h}_{\text{sr}}^{k,n})^T$, $\mathbf{g}_k = [g_{k,1} \cdots g_{k,n_k}]^T$, and $\mathbf{M}_m^{(k)} = \text{diag}\{|\mathbf{h}_{\text{rd}}^{k,m}|^2\}$. Each cluster independently solves the following optimization problem

$$\max_{\mathbf{g}_k} \min_{1 \leq i \leq N} R_m^{(k)} \quad \text{subject to} \quad \mathbf{g}_k^H \mathbf{Q}_k \mathbf{g}_k \leq P_k, \quad (1)$$

where $\mathbf{Q}_k = \sigma_s^2 ((\mathbf{H}_{\text{sr}}^{(k)})^H \mathbf{H}_{\text{sr}}^{(k)}) \odot \mathbf{I}_{n_k} + \sigma_n^2 \mathbf{I}_{n_k}$. The problem (1) is monotonically equivalent to

$$\max_{\mathbf{g}_k} \min_{1 \leq i \leq N} \text{SNR}_m^{(k)} \quad \text{subject to} \quad \mathbf{g}_k^H \mathbf{Q}_k \mathbf{g}_k \leq P_k. \quad (2)$$

Using the relation $\mathbf{g}_k^H \mathbf{H}_{m,n}^{(k)} \mathbf{g}_k = \text{tr}(\mathbf{H}_{m,n}^{(k)} \mathbf{g}_k \mathbf{g}_k^H) = \text{tr}(\mathbf{H}_{m,n}^{(k)} \mathbf{G}_k)$ and introducing a dummy variable τ , (2) is reformulated as

$$\begin{aligned} \max_{\tau, \mathbf{g}_k} \quad & \tau \quad \text{subject to} \quad \text{tr}(\tilde{\mathbf{Q}}_k \mathbf{G}_k) \leq P_k \\ \frac{\sigma_s^2}{\tau} \text{tr}(\mathbf{H}_{m,m}^{(k)} \mathbf{G}_k) - \sigma_s^2 \sum_{n \neq m} \text{tr}(\mathbf{H}_{m,n}^{(k)} \mathbf{G}_k) - \sigma_n^2 \text{tr}(\mathbf{M}_m^{(k)} \mathbf{G}_k) & \geq \sigma_n^2 \forall m \\ \text{rank}(\mathbf{G}) = 1, \quad \mathbf{G}_k = \mathbf{G}_k^*, \quad \mathbf{G}_k \succeq 0, \end{aligned} \quad (3)$$

which is a nonconvex problem because of the rank constraint. Omitting the rank constraint (Lagrangian relaxation), the problem (3) can be converted to a quasi-convex SDP problem. This relaxed problem is equivalent to a SDP feasibility problem when τ is given a priori and can be efficiently solved by SeDuMi [7]. Thus, we use the following bisection algorithm to solve the Lagrangian relaxed problem:

Algorithm: Bisection Algorithm for SDP feasibility problem

initiate: $\rightarrow \tau \in [\tau_{\min}, \tau_{\max}]$

repeat: $\rightarrow \tau = (\tau_{\min} + \tau_{\max})/2$

\rightarrow solve the feasibility problem for τ :

$$\frac{\sigma_s^2}{\tau} \text{tr}(\mathbf{H}_{m,m}^{(k)} \mathbf{G}_k) - \sigma_s^2 \sum_{n \neq m} \text{tr}(\mathbf{H}_{m,n}^{(k)} \mathbf{G}_k) - \sigma_n^2 \text{tr}(\mathbf{M}_m^{(k)} \mathbf{G}_k) \geq \sigma_n^2 \forall m$$

$$\text{tr}(\tilde{\mathbf{Q}}_k \mathbf{G}_k) \leq P_k \quad \mathbf{G}_k = \mathbf{G}_k^*, \quad \mathbf{G}_k \succeq 0.$$

\rightarrow if feasible

$$\tau_{\min} := \tau$$

\rightarrow else

$$\tau_{\max} := \tau$$

until: $\rightarrow \tau_{\max} - \tau_{\min} < \epsilon$

In general, τ_{\min} is set to 0 and τ_{\max} is chosen large enough according to the operation SNR value. Since we drop the rank constraint from the original problem (3), the solution of the relaxed problem may or may not result in a rank-1 $\mathbf{G}_k^{\text{opt}}$ matrix. To choose the $\mathbf{g}_k^{\text{opt}}$ out of $\mathbf{G}_k^{\text{opt}}$ we use the suboptimal randomization technique proposed in [8]. The eigenvalue decomposition of $\mathbf{G}_k^{\text{opt}} = \mathbf{U}\mathbf{\Gamma}\mathbf{U}^H$ is computed, and $\mathbf{g}_k^{\text{opt}}$ is chosen such that $\mathbf{g}_k^{\text{opt}} = \mathbf{U}\mathbf{\Gamma}^{\frac{1}{2}}\mathbf{w}_k$, where $\mathbf{w}_k[t] = e^{j\theta_{k,t}}$ for independently and uniformly distributed phases $\theta_{k,t}$ on $[0, 2\pi)$. Independent from any realization of \mathbf{w}_k , it is assured that $(\mathbf{g}_k^{\text{opt}})^H \mathbf{g}_k^{\text{opt}} = \text{tr}(\mathbf{G}_k^{\text{opt}})$. After generating $\mathbf{g}_k^{\text{opt}}$, any violation of constraints should be checked and $\mathbf{g}_k^{\text{opt}}$ can be scaled to satisfy the most violated constraint accordingly.

IV. THE CSI DISSEMINATION BETWEEN RELAYS

We consider a network with C relay clusters each with n_i , $i = 1, \dots, C$, relays. It is assumed that the relays perfectly estimate their backward and forward channels. With the independency of clusters, each relay in the i th cluster exchanges LCSi only with the other $n_i - 1$ relays in the same cluster.

In a single-hop unicast scheme, each node needs to establish point-to-point links with all the other nodes, which results in a high exchange overhead and transmit energy consumption. Instead, the relays can also establish multiple short-distance hops to disseminate their LCSi (multi-hop unicast). In other words, the relays help each other by forwarding the received LCSi of its neighbours to others. Moreover, if the relays are able to broadcast data, they can establish point-to-multipoint links, which in turn reduces the amount of exchange load w.r.t.

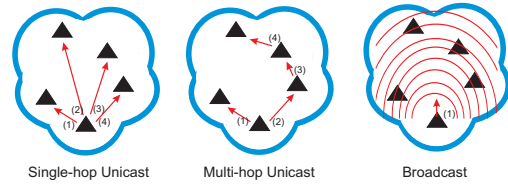


Fig. 2. The schemes to disseminate LCSi between relays: For the current scenario, unicast schemes use 4 channel use, whereas broadcast uses only one.

unicast schemes. See Fig. 2 for the interpretation of different dissemination schemes.

A. The CSI Exchange Load

1) *Unicast:* Assuming a secondary short-range wireless system with only unicast ability (single or multi-hop), the measure of local CSI exchange load (CSIEL) of the i th cluster is defined as $\phi(n_i) = 2\alpha N n_i (n_i - 1)$, where the factor 2 stands for the need for exchanging both uplink and downlink LCSi. N is the number of links, and α is a constant representing the number of units which can be exchanged in each attempt. For simplicity, we assume that exchanges are error-free.

Proposition 1: Given N_r and C , the most uniform distribution of relays to clusters is the optimum cluster-relay configuration (OCC) that minimizes the CSIEL independently of N and α . Thus, if $\text{mod}(N_r, C) = 0$, we have C clusters each with N_r/C relays; otherwise there are $\text{mod}(N_r, C)$ clusters each with $\lceil N_r/C \rceil$ relays, and $C - \text{mod}(N_r, C)$ clusters each with $\lfloor N_r/C \rfloor$ relays.

Proof: Assume that for given C and N_r , there exists a cluster relay configuration $\tilde{C} = \{m_1, \dots, m_C\}$ with the total exchange load $\sum_{i=1}^C \phi(m_i)$ which is smaller than or equal to $\sum_{i=1}^C \phi(n_i)$ obtained by the proposed OCC $C = \{n_1, \dots, n_C\}$. Let $m_i = n_i + r_i$ and $\phi(m_i) = \phi(n_i) + \beta_i$, where β_i is bounded as

$$\beta_i \geq r_i \gamma_1(n_i), \quad r_i \geq 0 \quad \text{or} \quad \beta_i \geq r_i \gamma_2(n_i), \quad r_i < 0,$$

where $r_i \in \mathbb{Z}$. $\gamma_1(n_i) = 4\alpha N n_i$ and $\gamma_2(n_i) = 4\alpha N (n_i - 1)$ are functions of n_i and equal to the absolute CSIEL difference obtained by incrementing or decrementing n_i by 1, respectively. Henceforth, we drop the (n_i) designation of $\gamma_1(n_i)$ and $\gamma_2(n_i)$ for notational simplicity. Rearranging the total load of \tilde{C} as $\sum_{i=1}^C \phi(m_i) = \sum_{i \in \mathcal{P}} \phi(m_i) + \sum_{i \in \mathcal{P}'} \phi(m_i)$, we obtain

$$\begin{aligned} \sum_{i=1}^C \phi(m_i) &= \sum_{i \in \mathcal{P}} (\phi(n_i) + r_i \gamma_1 + \epsilon_i) + \sum_{i \in \mathcal{P}'} (\phi(n_i) + r_i \gamma_2 + \epsilon_i) \\ &= \sum_{i \in \mathcal{P}} r_i \gamma_1 + \sum_{i \in \mathcal{P}'} r_i \gamma_2 + \sum_{i=1}^C \phi(n_i) + \sum_{i=1}^C \epsilon_i \end{aligned} \quad (4)$$

where $\epsilon_i \in \mathbb{R}^+ \forall i$, \mathcal{P} is the index set of clusters with positive r_i s, and \mathcal{P}' is the index set of remainder clusters. Focusing on the first two summands of (4),

$$\begin{aligned} \sum_{i \in \mathcal{P}} r_i \gamma_1 + \sum_{i \in \mathcal{P}'} r_i \gamma_2 &= \sum_{i \in \mathcal{P} \cap \mathcal{N}} r_i \gamma_1 + \sum_{i \in \mathcal{P} \cap \mathcal{N}'} r_i \gamma_1 + \sum_{i \in \mathcal{P}' \cap \mathcal{N}} r_i \gamma_2 + \sum_{i \in \mathcal{P}' \cap \mathcal{N}'} r_i \gamma_2 \\ &= 4\alpha N \left(\sum_{i \in \mathcal{P}} r_i \lceil N_r/C \rceil - \sum_{i \in \mathcal{P} \cap \mathcal{N}'} r_i + \sum_{i \in \mathcal{P}'} r_i (\lceil N_r/C \rceil - 1) - \sum_{i \in \mathcal{P}' \cap \mathcal{N}'} r_i \right) \end{aligned}$$

where \mathcal{N} and \mathcal{N}' are the index sets of clusters according to the OCC with $\lceil N_r/C \rceil$ and $\lfloor N_r/C \rfloor = \lceil N_r/C \rceil - 1$ relays, respectively. Since $\sum_{i=1}^C m_i = \sum_{i=1}^C n_i = N_r$, the condition $\sum_{i \in \mathcal{P}} |r_i| = \sum_{i \in \mathcal{P}'} |r_i|$ should also be satisfied. Introducing this fact, we obtain

$$\begin{aligned} \sum_{i \in \mathcal{P}} r_i \gamma_1 + \sum_{i \in \mathcal{P}'} r_i \gamma_2 &= 4\alpha N \left(\sum_{i \in \mathcal{P}} |r_i| - \sum_{i \in \mathcal{P} \cap \mathcal{N}'} |r_i| + \sum_{i \in \mathcal{P}' \cap \mathcal{N}'} |r_i| \right) \\ &= 4\alpha N \left(\sum_{i \in \mathcal{P} \cap \mathcal{N}} |r_i| + \sum_{i \in \mathcal{P}' \cap \mathcal{N}'} |r_i| \right) > 0 \end{aligned} \quad (5)$$

Combining (5) with (4), we have $\sum_{i=1}^C \phi(m_i) > \sum_{i=1}^C \phi(n_i)$, which contradicts with our assumption that there exists a $\tilde{\mathcal{C}}$ which has a smaller or equal CSI exchange load than \mathcal{C} . \square

Proposition 2: *Given N_r and the design constraint that relays are distributed to clusters as uniform as possible, a network with $C > 0$ clusters has more CSIEL than any network with $C + \beta$ clusters, where $\beta \in \mathbb{Z}^+$.*

Proof (Sketch): The proof follows from the definition of $\phi(n)$. The proposition claims that

$$\sum_{i \in \mathcal{R}} \phi(m_i) + \sum_{i \in \mathcal{R}'} \phi(m_i) > \sum_{i \in \mathcal{Q}} \phi(n_i) + \sum_{i \in \mathcal{Q}'} \phi(n_i)$$

where m_i s and n_i s are chosen according to OCC stated in Proposition 1, and $\mathcal{R}, \mathcal{R}', \mathcal{Q}, \mathcal{Q}'$ stand for the index set of clusters with $\eta_1 = \lceil N_r/C \rceil$, $\eta_2 = \lfloor N_r/C \rfloor$, $\eta_3 = \lceil N_r/(C + \beta) \rceil$, and $\eta_4 = \lfloor N_r/(C + \beta) \rfloor$ relays, respectively. Defining $\xi_1 = \text{mod}(N_r, C)$, $\xi_2 = C - \text{mod}(N_r, C)$, $\xi_3 = \text{mod}(N_r, C + \beta)$, $\xi_4 = C + \beta - \text{mod}(N_r, C + \beta)$, the inequality boils down to

$$\eta_2(N_r - \xi_2) > \eta_4(N_r - \xi_4), \quad (6)$$

which can be further modified to the following expression after some algebraic manipulations and setting $\beta = 1$,

$$\begin{aligned} \left(\lfloor \frac{N_r}{C} \rfloor - \lfloor \frac{N_r}{C+1} \rfloor \right) (2N_r - C) &> C \left(\lfloor \frac{N_r}{C} \rfloor - \lfloor \frac{N_r}{C+1} \rfloor \right) \\ &\times \left(\lfloor \frac{N_r}{C} \rfloor + \lfloor \frac{N_r}{C+1} \rfloor \right) - \lfloor \frac{N_r}{C+1} \rfloor \left(\lfloor \frac{N_r}{C+1} \rfloor + 1 \right). \end{aligned} \quad (7)$$

For any $C > N_r/2$, (7) holds true. Moreover, for any $C \leq N_r/2$ that leads $\lfloor N/C \rfloor = \lfloor N/(C + 1) \rfloor$, (7) can be shown to be true. In order to prove the proposition for any $C \leq N_r/2$ which results in $\lfloor N/C \rfloor \neq \lfloor N/(C + 1) \rfloor$, it is sufficient to show that

$$2N_r/C - 1 \geq \lfloor N_r/C \rfloor + \lfloor N_r/(C + 1) \rfloor. \quad (8)$$

Redefining $\lfloor N_r/(C) \rfloor$ and $\lfloor N_r/(C + 1) \rfloor$ as $N_r/(C) - a$ and $N_r/(C + 1) - b$, $0 \leq a < 1$, $0 \leq b < 1$, respectively, (8) becomes $N_r/C - N_r/(C + 1) \geq 1 - a - b$. This can be shown to hold true immediately, which proves the inequality (6) for $\beta = 1$. The proof is concluded by induction for $\beta > 1$. \square

With Propositions 1-2, the minimum CSIEL is attained by the most uniform distribution of relays to C_{\max} clusters, where $C_{\max} = \lfloor N_r/N_{\min} \rfloor$ and N_{\min} is the minimum allowable cluster size determined by the network designer.

2) *Broadcast:* Assuming the same conditions as in the unicast case, the measure of CSIEL of the i th cluster with broadcast dissemination, can be defined as $\psi(n_i) = 2\alpha N n_i$. $\psi(n_i)$ changes linearly with n_i , and the total network CSIEL is not affected by the size of different clusters.

B. Energy Consumption of LCSII Dissemination

Let E_c be the so called sensitivity level of any node, which is the minimum amount of receive energy that the node needs to decode the received symbols correctly. The transmit energy reduces proportionally with d^{-v} , where d is the distance between the transmitter and receiver, and $v \geq 2$ is the path loss exponent. Thus, the energy consumption per one unit of exchange is $\alpha E_c d^v$.

1) *Single-hop Unicast:* Considering the i th cluster with n_i relays, which are arbitrarily located and use single-hop unicast communication inbetween, the total energy consumption of the cluster for CSI dissemination is $E_{\text{csi}}^{\text{su}}(n_i) = 2\alpha N E_c \sum_{i=1}^{n_i} \sum_{j=1, j \neq i}^{n_i} d_{i,j}^v$ where $d_{i,j}$ is the distance between the i th and the j th relay.

2) *Multi-hop Unicast:* Dissemination with multiple hops and establishing short distance point-to-point links reduce the transmit energy consumption drastically w.r.t. to single-hop unicast. Defining \mathcal{M}_i as the lowest-energy-optimal set of distances of the hops that the i th relay needs to disseminate its LCSII, the general energy consumption expression for multi-hop unicast scheme is $E_{\text{csi}}^{\text{mu}}(n_i) = 2\alpha N E_c \sum_{i=1}^{n_i} \sum_{j \in \mathcal{M}_i} d_{i,j}^v$. Since the derivation of this optimal set \mathcal{M}_i is out of the scope of this paper, we assume that it is readily available and known.

3) *Broadcast:* The total energy consumption of a n_i -relay cluster using broadcast communication for CSI dissemination is $E_{\text{csi}}^{\text{b}}(n_i) = 2\alpha N E_c \sum_{i=1}^{n_i} d_{i,\max}^v$, where $d_{i,\max} = \max(d_{i,j}), \forall j \neq i, j = 1, \dots, n_i$.

To simplify the analysis, we further consider the following scenario for energy consumption calculations. Assume that all relays are located on a straight line with equal distances r between the neighbouring relays as shown in Fig. 1. Then $E_{\text{csi}}^{\text{su}}(n_i)$, $E_{\text{csi}}^{\text{mu}}(n_i)$, and $E_{\text{csi}}^{\text{b}}(n_i)$ can be re-defined as

$$\begin{aligned} \tilde{E}_{\text{csi}}^{\text{su}}(n_i) &= 4\alpha N E_c r^v \left(\sum_{k=1}^{\lceil n_i/2 \rceil - \text{mod}(n_i, 2)} \left(\sum_{i=1}^{n_i-k} (i^v) + \sum_{i=1}^{k-1} (i^v) \right) \right. \\ &\quad \left. + \text{mod}(n_i, 2) \sum_{i=1}^{\lfloor n_i/2 \rfloor} (i^v) \right), \end{aligned}$$

$$\tilde{E}_{\text{csi}}^{\text{mu}}(n_i) = 2\alpha N E_c n_i (n_i - 1) r^v,$$

$$\tilde{E}_{\text{csi}}^{\text{b}}(n_i) = 2\alpha N E_c r^v \left(2 \sum_{i=\lceil n_i/2 \rceil}^{n_i-1} (i^v) + \text{mod}(n_i, 2) (\lfloor n_i/2 \rfloor)^v \right),$$

respectively. This scenario will be used for the corresponding simulations in Section V.

Proposition 3: *Given N_r, C , and r , the OCC minimizes all $\tilde{E}_{\text{csi}}^{\text{su}}(n)$, $\tilde{E}_{\text{csi}}^{\text{mu}}(n)$, and $\tilde{E}_{\text{csi}}^{\text{b}}(n)$ independently of v, E_c, N, α .*

Proof (Sketch): It can be shown that for all $\tilde{E}_{\text{csi}}^{\text{su}}(n)$, $\tilde{E}_{\text{csi}}^{\text{mu}}(n)$, and $\tilde{E}_{\text{csi}}^{\text{b}}(n)$ any $\{n_1, n_2, n_3 \in \mathbb{N} \mid n_3 > n_2 > n_1, n_3 - n_2 = n_2 - n_1 = 1, n_1, n_2, n_3\}$ results in

$$\begin{aligned} \tilde{E}_{\text{csi}}^{\text{su}}(n_3) - \tilde{E}_{\text{csi}}^{\text{su}}(n_2) &> \tilde{E}_{\text{csi}}^{\text{su}}(n_2) - \tilde{E}_{\text{csi}}^{\text{su}}(n_1), \\ \tilde{E}_{\text{csi}}^{\text{mu}}(n_3) - \tilde{E}_{\text{csi}}^{\text{mu}}(n_2) &> \tilde{E}_{\text{csi}}^{\text{mu}}(n_2) - \tilde{E}_{\text{csi}}^{\text{mu}}(n_1), \\ \tilde{E}_{\text{csi}}^{\text{b}}(n_3) - \tilde{E}_{\text{csi}}^{\text{b}}(n_2) &> \tilde{E}_{\text{csi}}^{\text{b}}(n_2) - \tilde{E}_{\text{csi}}^{\text{b}}(n_1). \end{aligned}$$

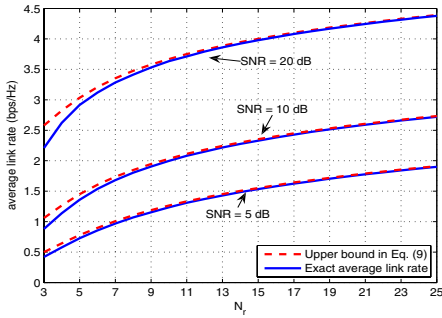


Fig. 3. Comparison of the upper bound and exact link rate with increasing N_r ($N = 2$, $C = 1$).

The proofs directly follow from the definition of the total energy consumptions and their polynomial behaviour. Next, the proposition can be proved by contradiction with similar steps to *Proposition 1*. \square

A similar statement to *Proposition 2* can also be applied to the transmit energy consumption.

C. Performance of optimum cluster configuration

The proposed cluster based MU system unavoidably suffers from an array gain loss, when compared with MU relaying with GCSI. This is due to the fact that the coherent combination of all relays in the system has been disturbed and reduced to coherent combination of the relays in a single cluster. Thus, the less clusters there are in the system, the higher is the array gain and so is the average rate correspondingly. Besides, the interesting point here is the impact of the OCC in *Proposition 1* on the average rate performance of the links. In the following, we show that the OCC also maximizes the average link rates, given that all relays are at the same distance to all sources, the P_k s are equal/fixed and N , σ_s^2/σ_n^2 are fixed.

Using Jensen's inequality, the channel averaged mutual information of the m th link is bounded by

$$E_{\mathbf{H}}\{I_m\} \leq \frac{1}{2C} \sum_{i=1}^C \log_2 \left(1 + E_{\mathbf{H}}\{\text{SNR}_m^{(i)}(\mathbf{H})\} \right) \quad (9)$$

where \mathbf{H} represents $\mathbf{H}_{\text{sr}}^{(i)}, \mathbf{H}_{\text{rd}}^{(i)} \forall i$, $\text{SNR}_m^{(i)}(\mathbf{H})$ represents that it is a function of \mathbf{H} . Next, we focus on the behaviour of $f_{n_i} = \log_2(1 + E_{\mathbf{H}}\{\text{SNR}_m^{(i)}(\mathbf{H})\})$ as the number of relays n_i of the i th cluster increases. It can be assumed that for any set of values $\{v_1, v_2, v_3 \in \mathbb{N} \mid v_1 > v_2 > v_3, v_1 - v_2 = v_2 - v_3 = 1\}$, the following holds: $f_{v_1} - f_{v_2} \leq f_{v_2} - f_{v_3}$. This assumption is equivalent to

$$\frac{1 + E_{\mathbf{H}}\{\text{SNR}_m^{(i)}(\mathbf{H})\}|_{n_i=v_1}}{1 + E_{\mathbf{H}}\{\text{SNR}_m^{(i)}(\mathbf{H})\}|_{n_i=v_2}} \leq \frac{1 + E_{\mathbf{H}}\{\text{SNR}_m^{(i)}(\mathbf{H})\}|_{n_i=v_2}}{1 + E_{\mathbf{H}}\{\text{SNR}_m^{(i)}(\mathbf{H})\}|_{n_i=v_3}} \quad (10)$$

In [3], the authors showed that the capacity of a AF relay network can at most increase *linearly* with the number of relays, otherwise it is *sublinear*. Thus, we can deduce that in the most extreme case the linear behaviour of f_{n_i} sufficiently means that (10) is satisfied with equality. As the rate flattens out with increasing n_i [3], the inequality in (10) will be strictly smaller. In Fig. 3 the behaviour of f_{n_i} and the strictness of the bound are validated numerically for several SNR values. After

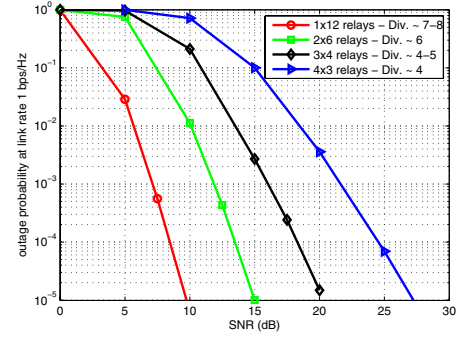


Fig. 4. The outage probability vs. SNR for different cluster configurations with $N_r = 12$, $N = 2$.

validating our assumption, now we prove that OCC is optimum for maximizing the upper bound of average rate performance.

Assume that for given C and N_r , there exists a cluster relay configuration $\tilde{C} = \{m_1, \dots, m_C\}$ with $\sum_{i=1}^C f_{m_i}$ which is larger than $\sum_{i=1}^C f_{n_i}$ obtained by the optimum cluster configuration $C = \{n_1, \dots, n_C\}$. Note that the term $1/(2C)$ is dropped. Let $m_i = n_i + r_i$ and $f_{m_i} = f_{n_i} + r_i \mu_i$. μ_i is defined as

$$\mu_i \leq r_i \lambda_1, \quad r_i \geq 0 \quad \text{or} \quad \mu_i \leq r_i \lambda_2, \quad r_i < 0,$$

where $r_i \in \mathbb{Z}$, $\lambda_1 = f_{n_{i+1}} - f_{n_i}$, $\lambda_2 = f_{n_i} - f_{n_{i-1}}$, and $\lambda_1 \leq \lambda_2$. The last condition for λ_1, λ_2 follows from the claim above. Next, following similar steps as for *Proposition 1*,

$$\sum_{i=1}^C f_{m_i} = \sum_{i \in \mathcal{P}} r_i \lambda_1 + \sum_{i \in \mathcal{P}'} r_i \lambda_2 + \sum_{i=1}^C f_{n_i} - \sum_{i=1}^C \varepsilon_i, \quad (11)$$

where $\varepsilon_i \in \{0, \mathbb{R}^+\} \forall i$. The first two summands of (11) are equal to

$$\sum_{i \in \mathcal{P} \cap \mathcal{N}} |r_i| \lambda_1 + \sum_{i \in \mathcal{P} \cap \mathcal{N}'} |r_i| \tilde{\lambda}_1 - \sum_{i \in \mathcal{P}' \cap \mathcal{N}} |r_i| \lambda_2 - \sum_{i \in \mathcal{P}' \cap \mathcal{N}'} |r_i| \tilde{\lambda}_2 \quad (12)$$

where $\tilde{\lambda}_1 = f_{n_i} - f_{n_{i-1}}$ and $\tilde{\lambda}_2 = f_{n_{i-1}} - f_{n_{i-2}}$. Since $\lambda_1 \leq \tilde{\lambda}_1 = \lambda_2 \leq \tilde{\lambda}_2$, (12) can be written as

$$\sum_{i \in \mathcal{P}} r_i \lambda_1 + \sum_{i \in \mathcal{P}'} r_i \lambda_2 = \kappa \left(- \sum_{i \in \mathcal{P} \cap \mathcal{N}} |r_i| - \sum_{i \in \mathcal{P}' \cap \mathcal{N}'} |r_i| \right) \leq 0 \quad (13)$$

for $\kappa \in \{0, \mathbb{R}^+\}$. Combining (11) and (13), we have $\sum_{i=1}^C f_{m_i} \leq \sum_{i=1}^C f_{n_i}$, which contradicts with our assumption and concludes the proof.

V. SIMULATION RESULTS

In this section, we present Monte-Carlo simulation results. The assumed CSI knowledge per node is as described in Section II. Furthermore, a local phase reference is assumed to be available inside the cluster. The average SNR = $\sigma_s^2/\sigma_n^2 = 20$ dB, and $\sigma_s^2 = 1$.

In Fig. 4, the outage probability versus SNR is plotted for different cluster configurations with $N_r = 12$, $N = 2$, and equally sized clusters. The case $C = 1$ is equivalent to MU-ZF relaying with GCSI, which gives about 8th order diversity. As the network is divided into 2(3) relay clusters with 6(4) relays, the system is supposed to give at least 2nd(3rd) order

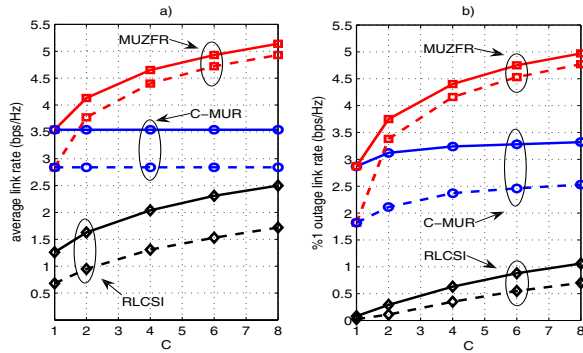


Fig. 5. a) Average rates vs. C b) Outage rates load vs. C ($N = 2$ (solid lines), $N = 3$ (dashed lines), $n_i = 9\forall i$).

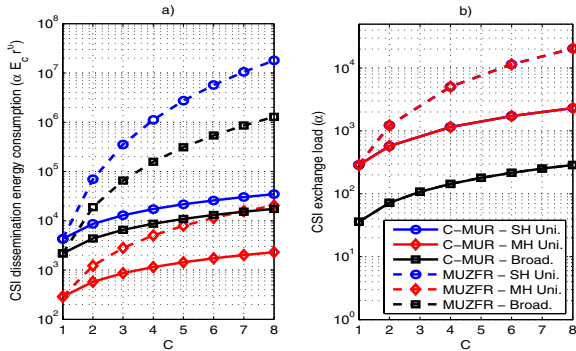


Fig. 6. a) Energy consumption of CSI dissemination vs. C b) CSI exchange load vs. C for unicast and broadcast schemes ($N = 2$, $n_i = 9\forall i$).

of diversity arising from the phase rotations between clusters. The additional $4(2)$ orders of diversity come from the intra cluster gain allocations. This is consistent with the results of [5] presenting that maxmin rate gain allocation maximizes the diversity in MUZFR applications. It is noticeable that for 4 clusters with 3 relays, the system offers only 4th order of phase diversity, but no intra cluster diversity. It is also realized that clustering inevitably suffers from array gain.

Fig. 5 presents a comparison between the proposed C-MUR, MUZFR [5] and relaying with only local CSI (RLCSI) at the relays [4]. Both C-MUR and MUZFR use maxmin rate gain allocations. Equally sized clusters each with 9 relays are considered. For any C , MUZFR and RLCSI use $9C$ relays. Fig. 5-a shows the average link rates, and depicts that C-MUR acts as a balancing system between GCSI and LCS systems. As new clusters are added to the system, the reliability of the C-MUR system increases. This improvement arises from the phase diversity. Also note that adding new clusters does not change the average link rates. It is seen from Fig. 5 that C-MUR has the disadvantage of reduced rates w.r.t. MUZFR. But as expected, its advantage evinces itself in terms of drastically reduced LCS overhead and transmit energy consumption for the dissemination. Fig. 6 depicts this trade-off. For $C = 8$, the C-MUR system offers 3, 2, 1 orders of magnitude energy consumption reduction for single-hop unicast, multi-hop unicast, and broadcast schemes, respectively. Besides, the CSI exchange overhead of C-MUR with the unicast scheme is only 10 percent of that of MUZFR for $C = 8$. On the contrary, both systems have the same overhead with the broadcast

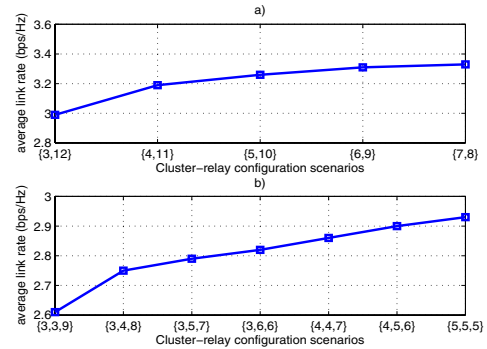


Fig. 7. Average rate vs. cluster-relay configuration: a) $C = 2$, b) $C = 3$ for $N_s = 2$, $N_r = 3$.

scheme. Establishing multiple hops to disseminate LCS does not change the LCS overhead w.r.t the single-hop unicast scheme. Note also that RLCSI has neither CSI exchange load nor energy consumption because the relays do not exchange LCS.

Fig. 7 shows the numerical validation of the proof that the OCC maximizes the average link rates. The average link rate versus the cluster-relay configuration scenarios is plotted for $C = 2$ and $C = 3$. The x-axis consists of all possible cluster-relay configurations $\{n_1, \dots, n_C\}$ for given N_r and C . The maximum average link rates are achieved with the most uniform distribution of relays to clusters which are $\{7, 8\}$, and $\{5, 5, 5\}$ for $C = 2$ and $C = 3$, respectively.

VI. CONCLUSION

We proposed a novel cluster based MU relaying system with phase diversity. The relay gains were allocated per cluster such that the minimum link rate is maximized. A detailed analysis of CSI dissemination has been done. We concluded that for a given number of clusters and relays, the OCC minimizes the CSIEL and dissemination energy consumption; and moreover maximizes the upper bound of average link rates. The cluster based system is a candidate to compensate the disadvantages of MUZFR with GCSI and RLCSI.

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