Exercise 7
Sample Solution
Question: Slotted Aloha

We use slotted Aloha and all machines would like to send in each slot

\[ \Pr(\text{success}) = n \cdot p \cdot (1 - p)^{(n-1)} \]

We do not know the exact number of \( n \) but

\[ A \leq n \leq B \]

What \( p \) is worst cast optimal in this scenario?
Worst Case Optimal?!?

1.) You select a transmission probability \( p \) between 0 and 1

2.) An evil adversary knows what \( p \) you have chosen and is now allowed to decide on the number of machines in the network. (Bounded by A and B)

What \( p \) do you have to chose to get the maximal \( \Pr(\text{success}) \)?
What happens for \( p = 1/A \)

Best case: Adversary chooses \( n = A \)

Worst case: Adversary chooses \( n = B \)
What happens for $p=1/B$

Best case: Adversary chooses $n=B$

Worst case: Adversary chooses $n=A$
What happens for $p=1/160$

Best case: Adversary chooses $n=160$

Worst case: Adversary chooses $n=A$
Which $n$ will the Adversary choose?

The worst case is always $n=A$ or $n=B$. 

![Graphs showing the relationship between $n$ and some function, with annotations indicating the worst case scenarios.](image-url)
Find $p_{opt}$ where $\min\{\Pr(A, p_{opt}), \Pr(B, p_{opt})\}$ is maximized!
Optimizing for the Worst Case

Find $p_{opt}$ where $\min\{Pr(A,p_{opt}), Pr(B,p_{opt})\}$ is maximized!

\[
p = \frac{1}{A}
\]

\[
p = \frac{1}{B}
\]
$p_{\text{opt}}$ is where the minimum of the two curves is maximized.
Gory Mathematical Details

\[
A p_{opt} (1 - p_{opt})^{A-1} = B p_{opt} (1 - p_{opt})^{B-1}
\]

\[
\frac{A}{B} = (1 - p_{opt})^{B-1-(A-1)} = (1 - p_{opt})^{B-A}
\]

\[
p_{opt} = 1 - \sqrt[ B-A ]{ \frac{A}{B} }.
\]

For \( A = 100 \) and \( B = 200 \) we get

\[
p_{opt} = 0.006908
\]